СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
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ДУБНА

E7 - 10653

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IONIZATION CURVES OF C^{4+} AND N^{5+} HELIUM-LIKE IONS IN A RANGE OF $0 \leq E_e \leq 30$ KEV

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Ионизационные кривые гелиеподобных ионов C^{4+} и N^{5+} в диапазоне $0 \leq E_e \leq 30$ КэВ

Выполнен расчет ионизационных кривых C^{4+} и N^{5+} при изменении энергии ионизирующих электронов E_e в диапазоне (0–30) КэВ. С этой целью методом графического интегрирования найдены квадраты матричных элементов дипольных переходов $M^2(C^{4+} \to C^{5+}) = (24{,}5 \pm 6) \cdot 10^{-3}$ и $M_i^2(N^{5+} \to N^{6+}) = (17{,}0 \pm 4) \cdot 10^{-3}$. Из полученных данных следует, что измеренные нами сечения ионизации C^{4+} и N^{5+} соответствуют точкам вблизи σ_{max}. Для полной проверки расчетных сечений следует измерить σ_i в нескольких точках по E_e.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1977

E7 - 10653

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Ionization Curves of C^{4+} and N^{5+} Helium-Like Ions in a Range of $0 \leq E_e \leq 30$ KeV

Ionization curves of C^{4+} and N^{5+} have been calculated in the energy range of ionizing electrons $E_e = (0-30)$ KeV. For this purpose the matrix element squares of dipole transitions $M_i^2(C^{4+} \to C^{5+}) = (24.5 \pm 6) \cdot 10^{-3}$ and $M_i^2(N^{5+} \to N^{6+}) = (17.0 \pm 4) \cdot 10^{-3}$ have been found by means of the graphic integration method.

From the data obtained it follows that the ionization cross sections of C^{4+} and N^{5+}, measured earlier by us, correspond to the points in the vicinity of σ_{max}. In order to check completely the calculated values, the cross sections σ_i should be measured at several points by E_e.

The investigation has been performed at the Laboratory of High Energy, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

1. INTRODUCTION

It is shown in paper [1] that the differential cross section of a collision, in which the electron of kinetic energy E_e is scattered with momentum loss hK, is described in the first Born approximation by the relation:

$$\sigma_n(K)dK = \frac{8\pi \cdot a_0^2 R^2}{E_e \cdot E_n} f_n(K)\frac{dK}{K}, \qquad (1)$$

where $a_0 = 5.29 \cdot 10^{-9}$ cm is the radius of the first Bohr orbit of hydrogen atom, $R = 13.6$ eV is the binding energy of an electron in the first Bohr orbit of hydrogen atom, E_n is the excitation energy of the final state "n", $f_n(K)$ is the generalized oscillator strength.

The total excitation cross section is [2]

$$Q_n = \frac{4\pi a_0^2 R^2}{E_e \cdot E_n} \int_{K_{min}}^{K_{max}} f_n(K) d\ln(K^2 a_0^2), \qquad (2)$$

where $K_{max}^2 \cdot a_0^2 = 4\frac{E_e}{R}[1 - \frac{E_n}{2E_e}]$ and

$$K_{min}^2 \cdot a_0^2 = \frac{E_n^2}{4E_e R}[1 + \frac{E_n}{2E_e}]. \qquad (3)$$

For some particular excitation or ionization process the graph of the function $f_n(K)$

against $\ln(K^2a_0^2)$ gives the complete information on the differential and total cross sections. For $Ka_0 \ll 1$ the ordinate transforms into constant which is equal to the optical oscillator strength. The area under the curve $f_n(K)$ between K_{min} and K_{max} is equal to $\frac{Q_n}{4\pi a_0^2 R^2/E_e E_n}$, and it is well approximated by the rectangle with ordinates f_n and 0 and obscissae $\ln(E_n^2/4\ E_e\ R)$ and $\ln(C_n E_n^2/4R)$, where the constant C_n depends only upon the shape of the curve $f_n(K)$.

The Bethe asymptotic formula for total cross section is of the following form [2]:

$$Q_n = \frac{4\pi a_0^2 R^2}{E_e\ E_n} \cdot f_n \ \ln(4E_e \cdot C_n/R)\ . \qquad (4)$$

Schram et al.[3] used the following expression:

$$Q_i = \frac{4\pi a_0^2 R}{E_e} \cdot M_i^2 \ \ln C_i\ E_e \qquad (5)$$

to calculate total ionization cross sections.

In the case of excitation of some discrete level, the constant $M_i^2 = \frac{M_d^2}{a_0^2} = \frac{R}{E_n} \cdot f_n$, where M_d^2 is the dipole matrix element square, f_n is the optical oscillator strength, and E_n is the excitation energy. In the case of ionization it is necessary to integrate over continuum, i.e., $M_i^2 = \int_I^\infty \frac{df}{dE} \cdot \frac{R}{E}\, dE$, where I is the ionization potential. In order to verify experimental results, the function $Q_i E_e/4\pi a_0^2 R = f(\ln E_e)$ is generally used which for optically allowed (dipole) transitions is represented by a straight line with a definite slope. The values of M_i^2 and C_i

are calculated by means of the least-square method from the slopes of the straight line (M_i^2) and intersection points ($M_i^2 \ln C_i$).

In particular, the corresponding data for atoms of He, Ne, Ar, Kr and Xe[3] are given in Table 1.

Table 1

Atom	M_i^2	c_i [eV^{-1}]
He	0.489 ± 0.05	0.108 ± 0.05
Ne	1.87 ± 0.01	0.0319 ± 0.0006
Ar	4.50 ± 0.04	0.049 ± 0.002
Kr	7.51 ± 0.03	0.037 ± 0.001
Xe	11.75 ± 0.08	0.035 ± 0.001

Similar results are then compared both with theoretical and experimental data on the ionization by α-particles, protons, and photons.

In an analysis of the obtained data from the theoretical point of view the numerical integration method of photoionization cross sections is used or the sum rule for oscillator strengths is applied.

Vriens[4] has shown that in a more general case for arbitrary values of E_e, when the conditions for applicability of the Born approximation are violated, the equations (4) and (5) should not be used. In this case the shape of the dependence of $f_n(K)$ on $\ln(K^2 a_0^2)$ is changed.

The following formula has been proposed by Vriens as one of the possible approximations:

$$Q_i = A \cdot F \cdot \frac{E_e - I}{E_e^2} \ln[1 + C(E_e - I)], \qquad (6)$$

where $A = 4\pi a_0^2 R M_i^2 = 47.8 \cdot 10^{-16} \cdot M_i^2$ and

$$F = 1 + (1 - C \cdot I)(0.025 + \frac{1.6}{C \cdot I})(1 - \frac{I}{E_e})(\frac{I}{E_e})^{3/2}.$$

For $E_e \gg I$, $F \approx 1$, for small E_e at $C \cdot I \neq 1$ we have $F < 1$.

The formula (6) has been successfully utilized to calculate the ionization cross section of helium-like ions Li^+, B^{3+}, O^{6+}, Ne^{8+} and Mg^{10+} at $E_e \leq 30$ KeV [5].

2. CALCULATION TECHNIQUE

It is well-known that

$$M_i^2 = \int_0^\infty \frac{R}{I + \epsilon} \frac{df}{d\epsilon} d\epsilon, \qquad (67)$$

where ϵ is the energy of an atomic electron ejected during ionization collision. Bell and Kingston [6] have calculated the continuous oscillator strength $df/d\epsilon$ for the photoionization of He, Li^+, B^{3+}, O^{6+}, Ne^{8+} and Mg^{10+} using Hartree-Fock wave functions.

In paper [5] the curves $\frac{R}{I+\epsilon}\frac{df}{d\epsilon}$ were used versus ϵ to find M_i^2 by the graphic integration method. The parameter C was determined for Li^+ from the experimental data. To evaluate C for other targets, the empirical (calculated) cross sections with $E_e = 20 \cdot I$ were used [7–9].

The values of M_i^2 and C obtained in this way are shown in Table 2.

Table 2

Ion	$M_i^2 \cdot 10^3$	C $[eV^{-1}]$	σ [7-9] at $E_e = 20 \cdot I$
Li^+	137.900	0.1330	0.2063
B^{3+}	35.650	0.1196	0.1253
O^{6+}	11.320	0.0842	0.0860
Ne^{8+}	7.180	0.0430	0.0420
Mg^{10+}	4.784	0.0312	0.0312

Based on the values of M_i^2 and C , the ionization curves of helium-like ions Li^+ , B^{3+} , O^{6+} , Ne^{8+} and Mg^{10+} have been calculated up to $E_e = 30$ KeV [5] .

3. IONIZATION OF C^{4+} AND N^{5+}

In the present paper similar calculations have been realized for ions C^{4+} and N^{5+} with constants C determined from the Lotz equation.

Using the data of Table 4 from paper [6] , we find that for carbon $M_i^2 (C^{4+} \rightarrow C^{5+}) = (24.5 \pm 6) \cdot 10^{-3}$ and for nitrogen $M_i^2 (N^{5+} \rightarrow N^{6+}) = (17.0 \pm 4) \cdot 10^{-3}$.

The values of C , obtained from the Lotz equation $\sigma_i = \frac{9 \cdot 10^{-14}}{E_e \cdot I} \ln E_e/I$ at $E_e/I = 20$,

Table 3

Ion	I [eV]	$M_i^2 \cdot 10^3$	C [eV^{-1}]	C /5/
Li^+	75.6	137.9	0.2063	0.1330
B^{3+}	259	35.6	0.1258	0.1196
C^{4+}	392	24.5	0.0572	-
N^{5+}	552	17.0	0.0532	-
O^{6+}	739	11.3	0.0860	0.0842
Ne^{8+}	1196	7.2	0.042	0.043

have been determined by the relation

$$\ln[1 + C(E_e - I)] = \frac{59.4}{M_i^2 \cdot I} \quad \text{(see Table 3).}$$

The calculated data for the ionization curve $C^{4+} \to C^{5+}$ are given in Table 4 and for the ionization curve $N^{5+} \to N^{6+}$ - in Table 5.

From the obtained data it follows that the previously measured ionization cross sections of helium-like ions C^{4+} and N^{5+} correspond to the points in the vicinity of σ_{max} . A good agreement between the experimental and calculated cross sections for N^{5+} is observed. In the case of C^{4+}, $\sigma(C^{4+})_{calc.} =$

Table 4

Ionization curve for C^{4+} ions at $E_e \leq 30$ keV

E KeV	F	σ_1 [10^{-20} cm^2]	$\sigma_{exp.}$ /10/
0.5	0.692	7.09	-
0.8	0.638	15.5	-
0.9	0.662	16.9	-
1.0	0.692	18.0	-
1.5	0.796	19.5	-
2.0	0.856	23.8 (max)	-
2.5	0.892	17.2±6	30 ± 7
3.0	0.915	15.8	-
4.0	0.942	13.5	-
5.0	0.958	11.8	-
10.0	0.984	7.1	-
20.0	0.994	4.1	-
30.0	0.996	3.1	-

$= (17.2 + 6) \cdot 10^{-20}$ cm^2 and $\sigma(C^{4+})_{exp} = (30-7) \cdot 10^{-20}$ cm^2 are close. In order to verify the proposed formula, measurements of σ_i at several points by E_i should be made.

Table 5

Ionization curve for N^{5+} ions at $E_{el} \leq 30$ KeV

E [KeV]	F	σ_i [10^{-20} cm^2]	$\sigma_{exp.}$ /10/
0.8	0.599	5.0	-
0.9	0.579	6.1	-
1.0	0.585	6.8	-
1.5	0.681	9.2	-
2.0	0.763	9.8 (max)	-
2.1	0.776	9.7 ± 3	9 ± 2
2.5	0.818	9.6	-
3.0	0.854	9.2	-
4.0	0.900	8.2	-
5.0	0.926	7.3	-
10.0	0.972	4.6	-
20.0	0.990	2.7	-
30.0	0.994	1.9	-

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Received by Publishing Department
on May 10,1977