ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E6-99-181

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CHARGE RADII
OF NUCLEI NEAR CLOSED NEUTRON SHELLS

Submitted to «Известия РАН, серия физическая»

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1999

INTRODUCTION

The distribution of an electrical charge in a nucleus as radial, and azimutal, is one of its major characteristics. It depends on the internucleon interactions in a nucleus and consequently serves one of the sources of the information about the nuclear forces. The spatial distribution of an electrical charge is represented complex enough and is described by a number of parameters. One of them, describing the linear sizes of a nucleus, is mean-square charge radius MSCR, which is defined by the expression:

$$\langle r^2 \rangle = \frac{\int \rho(r) r^4 dr}{\int \rho(r) r^2 dr}, \qquad (1)$$

where ρ (r) - the density of an electrical charge on the distance r from the centre of a nucleus. The concrete expression for $<r^2>$, depends on a kind of the distribution of the charge in the nucleus. For the nucleus with the sharp border and the homogeneous distribution of the charge (rectangular distribution):

$$\langle r^2 \rangle = \frac{3}{5} R^2, \qquad (2)$$

where R – the radius of a nucleus. In a case of Fermi distribution of a charge (constant density at the centre and smooth recession on border) [1]:

$$\langle r^2 \rangle = \frac{3}{5} c^2 + \frac{7}{5\pi^2} a^2, \qquad (3)$$

where c – half-density radius ($\rho(c) = 0{,}5\rho_0$, ρ_0 - the density of a charge at the centre of a nucleus), and a - parameter of a surface layer (distance, on which density of a charge decreases from $0{,}9\rho_0$ up to $0{,}1\rho_0$). If all electrical charge is concentrated on a surface of a nucleus, that, obviously, $<r^2> = R^2$.

At the constant volume the least value of MSCR appears at the nuclei having the spherical shape. Any deviations from sphere (the appearence of the nuclei deformation) results in the increase of MSCR. This increase of MSCR is defined by the expression [2]:

$$\Delta\langle r^2 \rangle = \frac{5}{4\pi} \langle r^2 \rangle_0 \sum_i \Delta\langle \beta_i^2 \rangle, \qquad (4)$$

where $<r^2>_0$ - MSCR of the spherical nucleus, β_i- the deformation parameters of the various orders (quadrupole β_2, octupole β_3 etc.).

The received on the experience, value of MSCR allows to refer, how the sizes and shape of a nucleus varies with the change of its nucleon structure (for example, with growth of the neutrons number, the excitation energy or the angular moment).

The purpose of the given paper was to research the change of MSCR in the isotopes of the same element near the closed neutron shells. The measurements of the MSCR differences in the nuclei with the neutron magic numbers (28, 50 or 82) and having on one or two neutrons more or less, (at the crossing of the closed shells) were performed.

EXPERIMENTAL METHODS of the MSCR DETERMINATION

The experimental methods of the MSCR determination (and also the other parameters describing the spatial distribution of an electrical charge in a nucleus) are based on the electromagnetic interaction of "test" particles with the nucleus. The results of the measurements will be the most certain at the use of such "test" particles, which undergo only the electromagnetic interaction, for example, electron or muon. These particles are used in two kinds of experiments:

1. Elastic or inelastic scattering on the nuclei.
2. Precise measurement of the stationary states energies in the bound systems electron (or muon) - nucleus.

The level energies of these systems (for example, atoms) depend among the other factors on the final sizes of a nucleus [3]. The energy shift in the comparison with a point nucleus is most significant for s-states (the orbital moment $l = 0$). As is known the point nuclei do not exist in the nature and the theoretical calculation of atomic levels with such nuclei is unsufficiently exact. Therefore the shifts of the levels for various isotopes of the same element are compared in a practice. Thus on the experience compare not the energies of the levels, but the wave-lengths of the radiation transitions between the levels, one of which is sensitive to the spatial distribution of an electrical charge of a nucleus (s-level), and another is not present. In such a way determine the MSCR differences for the whole set of the compared isotopes. Usually the absolute value of MSCR is known for one or several isotopes (they obtained from measurements of the elastic scattering of electron or of the muon levels spectra). Therefore the measured values of the MSCR differences allow to determine the absolute values of MSCR for all set of isotopes.

Firstly such values of the MSCR differences were obtained by the analysis of the atomic or ionic optical spectra. The use in the last years of lasers with the scanned wave-length has raised these measurements on a new step. The unique properties of laser radiation (its intensity and monochromacity, small angular spread) have allowed essentially to improve the accuracy and the sensitivity of the measurements. The main feature of the experiments with the use of the lasers is that the energies of the optical transitions are not measured, but are determined by the wave-lengths of the laser radiation, at which the resonances of the excitation of the atomic levels are observed. Thus the resonant wave-lengths are determined with the high accuracy, and the moments of the resonance appearance are fixed by the various manners. These manners include the measurements of the resonant laser fluorescence, the multistep

ionization of the atoms or the anisotropy of the polarized nuclei radiation. The block-diagram of the experimental set-up based on the method of the resonant laser fluorescence is presented in Fig. 1. This set-up was created in Laboratory of Nuclear Reactions JINR and during a number of years was used for the measurement of the nuclear MSCR for a wide set of elements [4]. This set-up includes the argon-ion pumping laser, the dye laser for the generation of the radiation with the choosed wave-length and the set of the devices for the stabilization, the calibration and the registration of the laser radiation. The use for the atomization of the powerful pulse laser has allowed to carry out the measurements with the refractory elements and their compounds [5].

RESULTS of the MEASUREMENTS

On the described set-up the measurements of the MSCR differences and of the nuclear multipole moments of the long isotope chains of a wide set of elements (from Na up to U) were carried out. A several number of these chains crosses the closed neutron shells (Ti [6], Zr [7], Ce, Nd, Sm [8]). The characteristics of these chains (the range of the mass numbers and the neutron numbers, and also the wave-lengths of the laser radiation used for the excitation of atomic levels) are presented in Tabl. 1. In this table the measured differences of the resonant frequencies appropriate to the excitation of atoms with the magic neutron number nuclei and distinguished from them on one or two neutrons (isotopic shifts) are presented also. The way of the determination of the MSCR differences from the measured isotopic shifts is described in detail [8]. For the majority of isotopic chains the measurement of the isotopic shifts are carried out for the whole set of the wave-lengths, that increased essentially the accuracy and reliability of the received results.

The received in such a way values of MSCR differences of nuclei with the magic neutron number and distinguished from them on one or two neutrons are presented in Tabl. 2. For the completeness of a picture the known data from the other sources are added (they are collected in the reviews [9-10]), and also the absolute values of MSCR of the nuclei with the magic neutron numbers are included [11-12].

The examples of the dependences of the MSCR differences on number of the neutrons in the nucleus at the crossing of the neutron closed shells are shown in Fig. 2. For the comparison the results of the droplet model calculation are presented also (they are discussed below).

DISCUSSION of the RESULTS

From Tabl. 2 and Fig. 2 it is seen, that the character of the MSCR dependence on the neutron number changes essentially at the crossing of the closed shell: the weak growth or even the fall of MSCR at $N < N_{mag}$. is replaced by its strong increase at $N > N_{mag}$. Such break in dependence $\langle r^2 \rangle$ on N is expressed most sharply at N_{mag} = 28 and 50. The nuclei with such number of neutrons have the least values of MSCR in all chain of isotopes. At the every N_{mag} the values $<r^2>$ are identical practically for all elements in a wide range Z, as it is shown in Fig. 3 on an example of the nuclei with the neutron number near N = 82.

The growth of MSCR at the addition of the neutrons depends on the change of its volume and shape:

$$\Delta\langle r^2\rangle = \Delta\langle r^2\rangle_V + \Delta\langle r^2\rangle_\beta, \qquad (5)$$

where $<r^2>_v$ and $<r^2>_\beta$ - the changes of MSCR, induced by the change of the nuclear volume and shape. The volume of the nucleus is determined by the number of the nucleons, however the growth of MSCR with the increase of only the numbers of the neutrons is occured more slowly, than under the known law connecting the radius of a nucleus R and its mass number A:

$$R = r_0 A^{1/3}, \qquad (6)$$

where is usually accepted r_0 = 1,2 fm. Thus the growth of MSCR at the increase of the nucleon number on the ΔA is defined by the expression:

$$\Delta\langle r^2\rangle_V = 0{,}576(\Delta A)A^{-1/3}. \qquad (7)$$

The slower growth of the charge radius in the comparison with expression (7) is explained, apparently, to that the added neutrons are placed mainly on a surface of a nucleus. Such change of MSCR can be described on the basis of a various models distinguished by a choice of the nucleon interaction potential in the nucleus. One of the most widespread is droplet model [14,15]. From the calculation on this model follows, that for nuclei near the closed neutron shells 50 and 82 the increase of MSCR at the pair neutrons addition is 0,12 fm^2, that is twice less, than follows from expression (7). The droplet model dependence $<r^2>_v$ on N is given in Fig. 2 and its strong difference from the experimental data is visible: instead of the smooth dependence the break is always observed at N = N_{mag}.

One of the reasons of this distinction is the change of the nuclei shape with growth of the neutron number. At the magic neutron number the shape of the nucleus is closest to spherical, and at the change N the deviations from this shape are observed. These deviations are described by the deformation parameters of the various orders. The most essential of them is the quadrupole deformation, at which the nucleus gets the shape close to a prolate or oblate ellipsoid. This deformation includes a statical part and dynamical one, growing out from zero fluctuations of the nuclear surface.

In the case of the even-even nuclei the value of deformation parameters β can be obtained from the reduced probabilities of the electric quadrupole transitions

between the first level 2^+ and the ground state 0^+ - B(E2):

$$\beta = \frac{4\pi}{3ZR^2}\sqrt{B(E2)}\,. \qquad (8)$$

The collection of the experimental values β, obtained in such a way, is presented in review [16]. At the absence of experimental values β the empirical ones were used obtained from the dependence of the deformation parameter on the neutron number in the nucleus [17]. Using these values β and the expression (4) it is possible to determine the MSCR change at the transition from the spherical nucleus with the magic neutrons number N to the nuclei with N + 2 and N - 2. The values of the deformation parameters of the considered nuclei and the MSCR changes, induced by them, are presented in Tabl. 3. It is seen, that the obtained values $<r^2>_\beta$ are rather insignificant and, as a rule, cannot explain the observed deviations $<r^2>$ from the smooth dependence on the neutron number in a nucleus (Fig.2). The inclusion of the higher orders deformations does not improve practically a situation. The octupole deformation parameters (collected in the review [18]) in the nuclei near the magic neutron numbers are the same order, as the quadrupole deformation parameters and their changes are even less. The hexadecapole deformation, as a rule, is much less, than these two previous ones. Therefore it is possible to believe, that the dependence of the nuclei volume and, hence, $<r^2>_v$ is not the smooth function of the neutron number in the nuclei near the closed shells, and the change $<r^2>_v$ depends on quantum numbers of neutrons and on the degree of shell filling.

It is interesting to look the change $<r^2>_v$ at crossing the various closed neutron shells. These values $<r^2>_v$ can be obtained as the difference between the experimental values of the MSCR changes and the corrections on the change of the quadrupole deformation presented in Tabl. 3:

$$\Delta\langle r^2\rangle_V = \Delta\langle r^2\rangle_{exp} - \Delta\langle r^2\rangle_\beta. \qquad (9)$$

The obtained in such a way the values $<r^2>_v$ are placed in Fig. 4. It is seen, that the values $\Delta<r^2>_v$ differ near the different magic neutron numbers, and also the values $\Delta<r^2>_v$ at $N_{mag.}$ + 2 are higher, than at N_{mag} - 2. It means, that volume of a nucleus grows much faster at the beginning of a neutron shell filling, than at its end. However this difference decreases with the raising Z and N of the nucleus, and for isotopes Hg and Pb vanishes practically. But in the light nuclei (Ar and Ca) the picture is opposite: $\Delta<r^2>_v$ is higher at the end of the shell, than in its beginning.

In a number of cases (at N_{mag} = 20, 28 and 50) $\Delta<r^2>_v \sim 0$, i.e. the volume of the nucleus and MSCR does not change practically at the fulffilling of the shell, and all changes are connected with the quadrupole deformation of the nucleus. The same situation was marked earlier on the border of the spherical and deformed nuclei (at N = 60 and 90) [8,19], when the sharp growth of MSCR was induced completely by the jump of the quadrupole deformation.

TABLE 1

ISOTOPIC SHIFTS OF THE OPTICAL LINES IN THE ATOMIC SPECTRA OF STUDIED NUCLEI.

Element	$N_1 - N_2$	λ nm	$\Delta\nu^{N,N-2}$ MHz	$\Delta\nu^{N,N-1}$ MHz	$\Delta\nu^{N,N+1}$ MHz	$\Delta\nu^{N,N+2}$ MHz
Ti	22-28	588,0	846(5)	402(5)		
		588,9	1417(9)	668(11)		
		591,9	839(10)	403(10)		
		592,2	1664(11)	787(9)		
		593,8	1401(10)	661(9)		
		586,6	1670(10)	787(9)		
		594,2	1678(9)	788(6)		
		598,1	1667(5)	784(6)		
Zr	50-56	573,6			-97(7)	-237(2)
		579,8			-93(4)	-222(2)
		588,6				-226(1)
Ce	78-84	577,3	175(9)			-1740(10)
		577,4	-32(9)			-1365(6)
		578,9	192(7)			-1946(10)
		580,4	177(6)			-1217(9)
Nd	82-90	572,9				574(3)
		581,4				-1456(5)

TABLE 2

CHARGE RADII DIFFERENCE OF THE NUCLEI DISTINGUISHED ON ONE OR TWO NEUTRONS

N	Nuclei	$< r^2 >^{N}$ fm^2	$< r^2 >^{N-2,N}$ fm^2	$< r^2 >^{N+2,N}$ fm^2	$< r^2 >^{N-1,N}$ fm^2	$< r^2 >^{N+1,N}$ fm^2
20	^{38}Ar	3,4020(20)	-0,082(26)	0,169(32)	-0,081(17)	0,044(12)
	^{39}K	3,4367(22)		0,112(37)	-0,056(44)	0,021(19)
	^{40}Ca	3,4827(17)		0,215(5)	-0,127(16)	0,0032(25)
28	^{47}K	3,4551(26)	0,046(12)		0,015(12)	
	^{48}Ca	3,4831(17)	0,145(18)	0,295(48)	0,009(10)	
	^{50}Ti	3,5760(30)	0,165(17)		0,031(13)	
	^{52}Cr	3,6550(26)	0,073(22)	0,159(40)		0,062(18)
50	^{86}Kr	4,1840(20)	0,042(12)	0,282(53)	0,009(11)	0,125(26)
	^{87}Rb	4,1990(20)	0,033(60)	0,283(57)	0,025(34)	0,127(41)
	^{88}Sr	4,2036(58)	0,050(8)	0,277(12)	0,007(4)	0,124(5)
	^{90}Zr	4,2733(20)		0,224(26)		0,137(16)
82	^{136}Xe	4,7908(20)	-0,052(12)	0,254(20)		0,105(10)
	^{137}Cs	4,8085(70)	-0,057(7)	0,270(50)	-0,064(10)	0,117(20)
	^{138}Ba	4,8348(7)	-0,034(4)	0,269(15)	-0,066(5)	0,119(8)
	^{139}La	4,8550(10)	-0,055(27)		-0,080(10)	
	^{140}Ce	4,8773(14)	-0,020(4)	0,265(12)		
	^{142}Nd	4,9145(32)	-0,019(12)	0,269(26)	-0,059(12)	0,118(12)
	^{144}Sm	4,94900(14)	-0,006(5)	0,261(20)	-0,038(10)	0,115(7)
	^{145}Eu	4,9798(116)	-0,026(5)	0,250(14)	-0,048(6)	0,114(7)
	^{147}Tb			0,207(11)		0,084(9)
	^{148}Dy	5,0462(235)	-0,013(2)	0,243(22)		
126	^{206}Hg	5,4799(19)	-0,107(5)		-0,071(5)	
	^{207}Tl	5,4895(32)	-0,103(10)			0,099(15)
	^{208}Pb	5,5071(8)	-0,109(3)	0,195(3)	-0,068(3)	0,087(2)
	^{212}Rn		-0,110(10)		-0,082(8)	
	^{213}Fr		-0,099(3)		-0,064(3)	
	^{214}Rn		-0,091(15)		-0,061(8)	

TABLE 3

CHANGE OF MSCR, INDUCED BY THE QUADRUPOLE DEFORMATION OF NUCLEI.

$N = N_{mag.}$			$N_1 = N_{mag.} - 2$		$N_2 = N_{mag.} + 2$	
$N_{mag.}$	Nuclei	$\beta(N_{mag.})$	$\beta(N_1)$	$\Delta < r^2 >^{N,N_1}$ fm^2	$\beta(N_2)$	$\Delta < r^2 >^{N,N_2}$ fm^2
20	^{38}Ar	0,162(6)	0,273(16)	0,223(10)	0,251(15)	0,169(40)
	^{40}Ca	0,122(10)			0,247(10)	0,226(40)
28	^{48}Ca	0,101(17)	0,152(5)	0,062(20)	0,240(20)	0,220(60)
	^{50}Ti	0,166(11)	0,269(7)	0,228(45)		
	^{52}Cr	0,224(5)	0,293(8)	0,189(37)	0,250(6)	0,065(27)
50	^{86}Kr	0,145(6)	0,149(4)	0,008(16)	0,18(2)	0,070(60)
	^{88}Sr	0,117(3)	0,128(10)	0,019(22)	0,120(19)	0,005(30)
	^{90}Zr	0,091(4)	0,12(2)	0,041(30)	0,103(4)	0,017(11)
	^{92}Mo	0,106(3)	0,14(2)	0,060(40)	0,151(2)	0,079(22)
82	^{136}Xe	0,086(19)	0,120(10)	0,064(40)	0,11(2)	0,100(70)
	^{138}Ba	0,092(2)	0,124(8)	0,063(21)	0,15(2)	0,110(60)
	^{140}Ce	0,101(1)	0,127(7)	0,056(18)	0,124(2)	0,049(8)
	^{142}Nd	0,093(1)	0,123(10)	0,063(20)	0,131(4)	0,083(15)
	^{144}Sm	0,088(1)	0,127(10)	0,086(20)	0,125(10)	0,084(20)
126	^{206}Hg	0,040(10)	0,069(1)	0,063(21)		
	^{208}Pb	0,023(3)	0,054(3)	0,065(10)	0,050(10)	0,045(15)

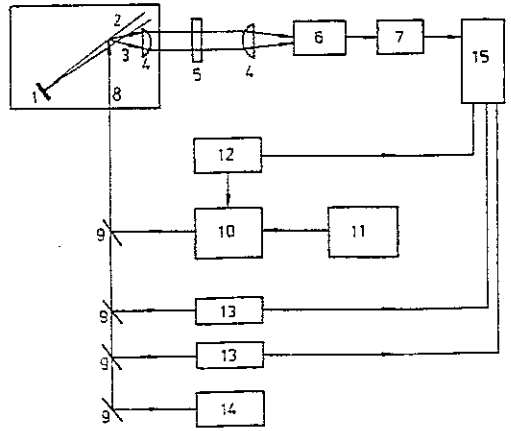

Fig. 1. The block diagram of the experimental set-up: 1- sample, 2 – atomic beam, 3 - resonance scattered radiation, 4 - focusing lenses, 5 – interference filter, 6 – photo-multiplier, 7 - pulsing laser, 8 - radiation of scanning frequency, 9 - mirror, 10 – dye laser, 11 – argon-ion laser, 12 - block of scanning, 13 – Fabri-Pero interferometer, 14 – wave-length meter, 15 - personal computer.

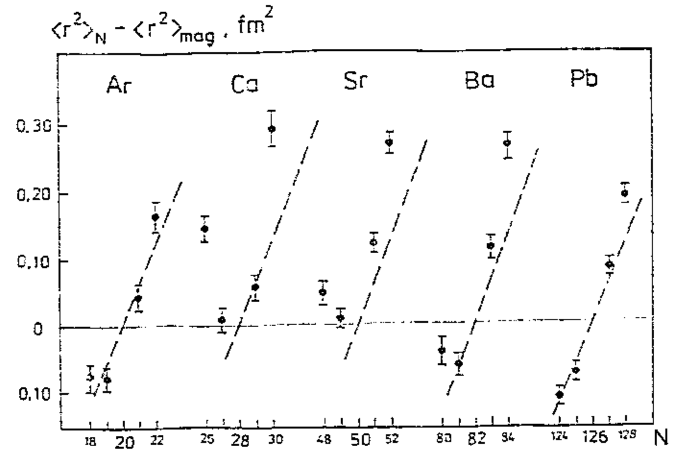

Fig. 2. The dependence of the MSCR differences of nuclei with the closed neutron shell and distinguished from them on one or two neutrons on the neutron number in the nuclei. The dotted line are the droplet model calculations.

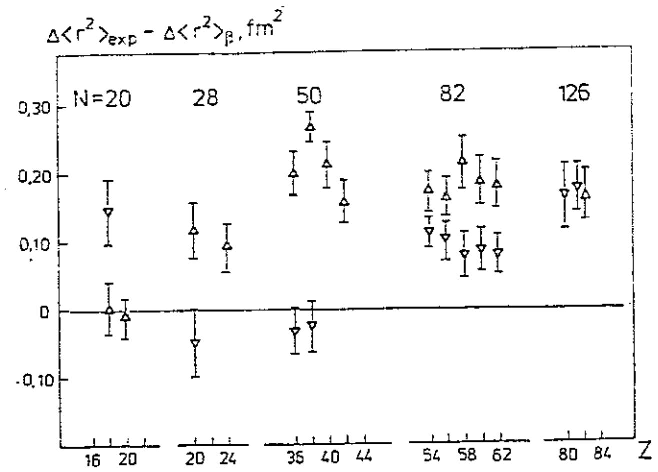

Fig. 3. The same, as in Fig. 2 dependence, but on atomic number of an element Z at N = 82.

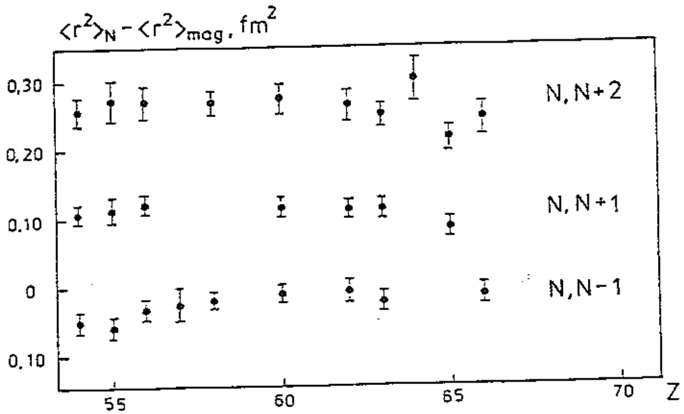

Fig. 4. The dependence of experimental values of the MSCR differences after the deduction of the correction on the change quadrupole deformation of a nucleus on atomic number Z.

Δ – $\Delta\langle r^2\rangle_V^{N,N-2}$

∇ – $\Delta\langle r^2\rangle_V^{N,N+2}$.

All considered cases of the MSCR change are concerned to the nuclei near the valley of β-stability. For the nuclei far from this valley there can be other character of the MSCR change at the crossing of the closed neutron shell. The indication on it can be served a chain of the neutron-rich Na isotopes, in which the sharp increase of MSCR was observed at the transition from the nucleus ^{30}Na (N = 19) to ^{31}Na (N = 20) [20]. Such jump of MSCR can mean, that the Na nuclei at the access of the magic neutron number (N = 20) become not spherical, as in the all considered above cases, but strongly deformed. Therefore it is very interesing to measure the charge radii of the nuclei with the magic neutron number, but situated far from the valley of β-stability.

In the conclusion it is possible to note, that the charge radii reflect a lot of details of the nuclear structure: the deformations of the various orders, the nucleon configuration, the degree of the neutron shell filling. Therefore their measurement is the valuable source of the information about these nuclear characteristics.

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Received by Publishing Department
on June 23, 1999.