ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E6 - 7419

Pham Zuy Hien

ON SPIN ASSIGNMENT FOR SPONTANEOUSLY FISSIONING ISOMERS

1973

ЛАБОРАТОРИЯ ЯДЕРНЫХ РЕАКЦИЙ

E6 - 7419

Pham Zuy Hien

ON SPIN ASSIGNMENT FOR SPONTANEOUSLY FISSIONING ISOMERS

Submitted *to Nuclear Physics*

1. Introduction

The determination of the quantum characteristics of spontaneously fissioning isomers is of great importance for the deeper understanding of the fission barrier structure for transuranic elements. Since spontaneous fission is as yet the only observable decay of these isomers, this problem cannot presently be solved using conventional methods based on α , β and γ -spectroscopy. The authors of ref. /1/ made an attempt to study the anisotropy of the angular distribution of spontaneous fission fragments from a nucleus oriented in reactions with charged particles. The theoretical analysis of ref. /2/ shows that this method provides a real possibility of determining the quantum characteristics of isomeric states, in spite of some uncertainties due to the choice of the level density parameters in the second potential well and the evaluation of the hyperfine interaction between nuclear momenta and the electron shell.

Recently in a number of nuclei, e.g., in ^{236}Pu , ^{237}Pu , ^{238}Pu, etc., a couple of s.f. isomers in each have been observed in the second potential well /3,4/. The ratio of the production cross sections for a nucleus in the first and second isomeric states in the second potential well and the dependence of this ratio on the momentum added to the nucleus are functions of the spins of the isomeric states. Therefore, the measurement of the isomer ratio permits the estimation of the s.f. isomer spin. However it is, as a rule, difficult to obtain the exact spin values using this method, especially in odd nuclei. For instance, the measurements of the isomer ratio for ^{237}Pu (ref. /3/) in reactions with deuterons and γ-particles have led to a variety of possible spin values for the two isomeric states.

It is natural to expect that a combination of the two methods (measurements of the isomer ratio and fission fragment angular distribution) will permit a more unambiguous spin assignment for isomeric states. This combined method is expected to be especially efficient for even-even isotopes. In this case the spin of the low-lying isomeric level is usually equal to zero ($I = K = 0$) and the fragment angular distribution corresponding to this particular level should be isotropic. The spin of the higher-lying state should only be determined, which is a two-quasiparticle excited state in the second potential well ($I = K \neq 0$). In the present paper the general scheme for calculating the population for the two-quasiparticle isomer is developed. This scheme may form the basis for calculating both the isomer ratio and fission fragment angular distribution. A numerical calculation has been performed for $^{238mf}Pu$. A comparison of the calculated results with experimental data /4,5/ permits both spin assignment and determination of the gamma to s.f. branching ratio for the two-quasiparticle s.f. isomer.

2. A Model for the Population of a Two-Quasiparticle Spontaneously Fissioning Isomer

We consider a reaction of the (α, xn) type, leading to the formation of a couple of s.f. isomers in the second potential well of an even-even isotope. The problem consists in calculating the relative probability β_K for two-quasiparticle isomer formation and its spin orientation relative to the incident beam direction. The spin orientation is described by some distribution function f_M for the nucleus over the spin projection M ($M = -I, \ldots, +I$). The model of the population of isomeric states is shown in Fig. 1. The dashed lines show the competing fission processes. Another possibility of populating isomeric sta-

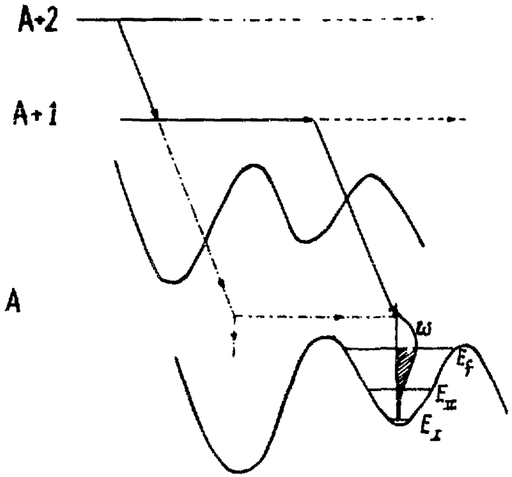

Fig. 1. A model for the population of isomeric states in the second potential well of an even-even nucleus.

tes, shown by a dot-dashed line, contributes very little and therefore, is neglected in the calculation. The curve ω in Fig. 1 shows the probability for the A nucleus production in the second potential well, as a function of excitation energy. The competing fission of this nucleus is taken into account by introducing some effective barrier E_f /4,6/ so that only the shaded part under this curve populates the isomeric states. Since E_f is typically noticeably smaller than the first barrier height, the influence of the transition from the second potential well back to the first potential well after the A nucleus production can be neglected. The part of the curve lying between E_I and E_{II} populates only the lower isomer. A contribution from this part is usually negligibly small. Thus, only the states falling into the interval between E_{II} and E_f populate both the high-lying and low-lying isomers.

It is noteworthy that this simplified model may prove uncapable of describing well the population process of isomeric states at incident particle energies too close to the threshold of the reaction leading to a quasiparticle isomer. In this instant one can hardly neglect the contribution from the $E_I \div E_{II}$ part.

Prior to determining the relative probabilities for these isomers to be populated, we note that the effective barrier E_f lies only 0.5 MeV above the upper limit of the region of two-quasiparticle states in the second potential well. Since it may be assumed that the spin projection onto the symmetry axis of the nucleus K_J in the energy region $(E_{II} \div E_f)$ is a good quantum number, the selection rule with respect to K_J is of great importance to the γ-transitions populating the isomeric states. This selection rule leads to the fact that among the states following the evaporation of the last neutron, only those with large K_J values populate the two-quasiparticle isomer, while the states with small K_J values disintegrate directly into the ground state of the second potential well or via the β-and γ-bands lying somehow below the region of two-quasiparticle states. Like in ref. [7] we assume the isomer ratio to be equal to the ratio of probabilities for states with $K_J \geq K$ and $K_J < K$ to be formed in the region $(E_{II} \div E_f)$. It should be noted that this assumption in the isomer ratio calculation is in good agreement with the experimental results on the two-quasiparticle isomer in ^{174}Hf (ref. [7]).

To calculate spin orientation for the two-quasiparticle state, a more detailed description of the character of γ-transitions in the region of two-quasiparticle states is required. The decay of states with $J, K_J > K$ to a two-quasiparticle state proceeds either through the intermediate quasiparticle bands $(\Delta J \neq 0, \Delta K_J \neq 0)$ or the rotational levels of one and the same band $(\Delta J \neq 0, \Delta K_J = 0)$.

On the basis of the results of an analysis made in refs. [8,9] one can describe the character of these transitions in the following way:

1. Each of the $|J, K_J\rangle (K_J > K)$ states produced following neutron evaporation decays to a two-quasiparticle isomer by the only cascade of γ -transitions.

2. The number of γ -transitions in each cascade is considered to be the minimum possible one.

3. As to character of these transitions, two variants will be considered. a) Only E1 transitions with band changes $(\Delta K_J = 1, \Delta J = 1)$ occur in the first stage of the cascade. This process lasts until the quantum number K_J becomes equal to the projection of spin K of the two-quasiparticle isomer. Afterwards the rotation E2 transitions $(\Delta K_J = 0, \Delta J = 2)$ within the band corresponding to the isomer take place. b) All the γ -transitions are dipole (E1 and M1). In this case the order of sequence of E1 and M1 does not influence the calculated results, which follows from eq. (11).

3. Determination of Isomer Ratio and Spin Orientation for a Two-Quasiparticle Isomer

Nuclear states in different stages of the reaction are defined by energy E, spin J and spin projection onto the beam direction M_J. As has been mentioned above, states in the second potential well of the nucleus A are also characterized by spin projection onto the nuclear symmetry axis K_J. For determining the isomer ratio and spin orientation for a two-quasiparticle isomer one should calculate successively production probabilities for different nuclear states after each of reaction stages. The dependence of compound-nucleus production probabilities on spin J_c and the spin projection onto the beam direction M_{J_c} has the following form

$$P_{J_c M_{J_c}} = \begin{cases} C\sigma(J_c)/(2I_0+1), & |M_{J_c}| \le I_0 \\ 0, & |M_{J_c}| > I_0, \end{cases} \qquad (1)$$

where C is a normalized factor, I_0 is the target nucleus spin. The compound nucleus cross section σ_c is determined by the formula

$$\sigma(J_c) = \pi \lambda\!\!\!^{-2} \frac{2J_c+1}{2I_0+1} \sum_{\ell=|J_c-I_0|}^{J_c+I_0} T_\ell(\epsilon_\alpha). \qquad (2)$$

Here $\lambda\!\!\!^-$ is the wave length of α-particles, $T_\ell(\epsilon_\alpha)$ is the penetrability coefficient of α-particles with energy ϵ_α through the target nucleus.

As is generally done in isomer ratio calculations /10/ in calculating the distribution of probabilities of compound nucleus production after neutron evaporation, it is assumed that each evaporated neutron carries off a certain average amount of energy. Then the probability for a residual nucleus to be formed in the $|\bar{E}_2, J_2, M_{J_2}>$ state after evaporation of a neutron from the $|\bar{E}_1, J_1, M_{J_1}>$ state can be calculated by the following formula

$$P_{\bar{E}_2 J_2 M_{J_2}} = N \sum_{J_1 M_{J_1}} \frac{\Omega(\bar{E}_2, J_2)}{\Omega(\bar{E}_1, J_1)} P_{\bar{E}_1 J_1 M_{J_1}} \left(\frac{1}{1+\Gamma f/\Gamma n}\right)_{\bar{E}_1 J_1} \sum_{S=J_2-1/2}^{J_2+1/2} \qquad (3)$$

$$\sum_{\ell=|J_1-S|}^{J_1+S} T_\ell(\bar{\epsilon}_n)\, C^2(J_2 \ell J_1,\ M_{J_2} M_{J_1}-M_{J_2})$$

Here N is some unessential factor, $\bar{E}_1$ and $\bar{E}_2$ are the average excitation energies of the nucleus prior to and after neutron evaporation ($\bar{E}_1 = \bar{E}_2 + \bar{\epsilon}_n + B_n$, where B_n is the neutron binding energy), $T_\ell(\bar{\epsilon}_n)$ is the penetration factor for a neutron with energy ϵ_n, C is the Clebsch-Gordan coefficient, $\Omega(E,J)$ is the nuclear level density parameter, which is of the following form

$$\Omega(E,J) = \Omega(E)\,(2J+1)\,\exp\frac{-(J+1/2)^2}{2\sigma^2}, \qquad (4)$$

where $\sigma^2 = \Theta_{\parallel} T/h^2$, T and $\Theta_{\parallel}$ are the temperature and moment of inertia of the nucleus with respect to the axis parallel to the nuclear symmetry axis. The expression

$\left(\frac{1}{1+\Gamma_f/\Gamma_n}\right)_{E,J}$ in eq. (3) takes into account the spin dependence of the two competing processes, neutron emission and fission. Like in ref. /11/, the quantities $(\Gamma_f/\Gamma_n)_{EJ}$ are calculated using the Fermi gas model for the nuclear level density

$$\left(\frac{\Gamma_f}{\Gamma_n}\right)_{EJ} = \alpha\,\frac{[2a^{1/2}(E-B_f-E_R^f)-1]}{E-B_n-E_R}\exp[2a^{1/2}((E-B_n-E_R)^{1/2}-$$

$$-(E-B_f-E_R^f)^{1/2})]. \tag{5}$$

Here α is a spin-independent factor, a is the level density parameter of the Fermi gas model, E_R and E_R^f are the effective rotational energies of the nucleus after neutron evaporation and at the saddle point, respectively, B_f is the fission barrier height. For the case where the nucleus is in the first potential well (the A+2 nucleus) the first barrier height can be taken for B_f. The effective rotational energies are defined by the equations

$$E_R = \frac{\hbar^2}{2\Theta_\perp} J^2 \tag{6}$$

and

$$E_R^f = \frac{\hbar^2}{2\Theta_\perp^f}\left(J^2-\overline{K_f^2}\right)+\frac{\hbar^2}{2\Theta_\parallel^f}\overline{K_f^2} \tag{7}$$

where $\Theta_\perp$ is the moment of inertia of the residual nucleus, $\Theta_\perp^f$ and $\Theta_\parallel^f$ are the moments of inertia of the nucleus at the saddle point, $\overline{K_f^2}$ is the root-mean-square projection of the nuclear spin at the saddle point at the corresponding excitation energy $E-B_f$. The value of $\overline{K^2}$ can be calculated using the known Gaussian distribution over spin projection at the saddle point

$$\overline{K_f^2} = \frac{\sum\limits_{K_J=0}^{J} K_J^2\, e^{-K_J^2/2K_0^2}}{\sum\limits_{K_J=0}^{J} e^{-K_J^2/2K_0^2}} \tag{8}$$

where $\frac{1}{K_0^2} = \frac{\hbar^2}{T}\left(\frac{1}{\Theta^f_{\parallel}} - \frac{1}{\Theta^f_{\perp}}\right)$. The data on the parameter K_0^2 can be taken from measurements of the angular distribution of prompt fission fragments (see, e.g., ref. /12/).

According to the model shown in fig. 1, the A+1 nucleus is produced after evaporation of a neutron in the first potential well. The transition of this nucleus through the first barrier into the second potential well can be described as follows. At the excitation energy typical for the A+1 nucleus, which is of the order of 10 MeV, the states in the first and second potential wells are expected to be mixed to a large extent. Therefore, it may be assumed that the probability of locating the A+1 nucleus in one or the other potential well after neutron evaporation will be proportional to the appropriate level density.

As has been noted above, the quantum number K_J plays an important role at the final stage of isomer state population following the formation of the A nucleus in the energy interval (E_{II}, E_f), and the isomer ratio is determined as that of probabilities for the formation of states with $K_J \geq K$ and $K_J < K$ in this energy interval, i.e.,

$$\beta = \frac{\sum\limits_{J, K_J \geq K} P_{J K_J}}{\sum\limits_{J, K_J < K} P_{J K_J}} = \frac{\sum\limits_{J \geq K} \sum\limits_{J \geq K_J \geq K} P_{J K_J}}{\sum\limits_{J < K} P_{J K_J} + \sum\limits_{J \geq K} \sum\limits_{K_J < K} P_{J K_J}}$$

where $P_{J K_J}$ is the probability for the nucleus to be produced in the state $|J, K_J\rangle$ after evaporation of the last neutron. Since the probability of neutron emission accompanied by the formation of the $|J, K_J\rangle$ state is independent of the quantum number K_J, the value of $P_{J K_J}$ (or $P_{J K_J M_J}$) will be proportional to the density of the state with spin projection K_J at a given spin J. It may be assumed that in the region E_{II}, E_f the Gaussian distribution of the level density over K_J, analogous to the distribution at the saddle point, also holds, i.e.,

$$P_{J K_J M_J} = P_{J M_J} \frac{e^{-K_J^2/2K_{II}^2}}{\sum\limits_{K_J=0}^{J} e^{-K_J^2/2K_{II}^2}} , \qquad (10)$$

where the parameter K_{II}^2 has the same sense as the parameter K_0^2 in formula (8).

The distribution $P_{J K_J M_J}$ is used to calculate the nuclear spin orientation of the two-quasiparticle state. A change in spin orientation by γ-transitions from the $|J' K_J\rangle$ state to the $|J'' K_J'\rangle$ state is described by the following formula

$$f_{M_J''} = A \sum_{M_J'} C^2(J''LJ', M_J'' M_J' - M_J'') f_{M_J'}, \qquad (11)$$

where A is a normalizing factor, L is the γ-transition multipolarity. It is seen from eq. (11) that a change in nuclear spin orientation does not depend on the quantum numbers K_J', K_J'' or transition type (electrical or magnetic). This substantially simplifies the calculation using the model described in the previous section. On the basis of the recurrence formula (11) one can calculate the function that describes spin orientation for a two-quasiparticle isomer. Finally, the angular distribution of fission fragments from this isomeric state can be calculated by a formula from ref. [2]

$$W(\theta) = \sum_{\nu=0}^{2I} A_{2\nu} P_{2\nu}(\cos\theta) \qquad (12)$$

where

$$A_{2\nu} = \sum_{M=-I}^{I} (-1)^{K-M} f_M C(II_{2\nu}, M-M) C(II_{2\nu}, K-K), \qquad (13)$$
$$(K = I)$$

4. Numerical Results

The numerical calculations of the fission ratio and the angular distribution of fission fragments have been carried out for the two-quasiparticle isomer ^{238}Pu, produced in a reaction of the type $(\alpha, 2n)$ [4,5]. In these

calculations the level density parameters of the Fermi gas model were taken from the systematics of Gilbert and Cameron /13/, while the fission barrier parameters employed were taken from the paper of Britt et al. /14/. The penetrability coefficients for α -particles and neutrons have been calculated using the optical potential /15,16/. The data on the nuclear moments of inertia were taken from the calculation of ref. /17/. For the parameter K_0^2 corresponding to the K_J distribution at the saddle point, we used the experimental data listed in ref. /12/. The parameter K_{II}^2 (eq. (9)) is varied within reasonable limits near the value of K_0^2 corresponding to the region of two-quasiparticle excitations ($2K_0^2 \approx 25$ ref. /12/) at the saddle point.

5. Calculated Results and Discussion

Figure 2 shows the calculated results for the ^{238}Pu isomer ratio. The numerical results are in good agreement with experimental data /4/ if the two-quasiparticle isomer spin is assumed to be equal to 4 ($I = K = 4$). The experimental results are lower than the calculated ones at the α-particle energy ϵ_α = 24 MeV, which is apparently attributed to the inapplicability of this model to energies close to the reaction threshold (see Section 2). As might be expected, the calculated results are slightly sensitive to the fission barrier parameters, since we deal here with the ratio of cross sections for formation of two isomeric states in the same potential well. The dashed curves in fig. 2 show to what extent the calculated results are sensitive to the parameters describing the nuclear level density. The parameter K_{II}^2 (eq. (10)), corresponding to the distribution of the level density over K_J in the region (E_{II}, E_f), proves most crucial in this respect. However it is seen from fig. 2 that an uncertainty in the choice of values for this parameter does not influence the results of spin assignment for the two-quasiparticle isomer on the basis of the experimental data on isomer ratio.

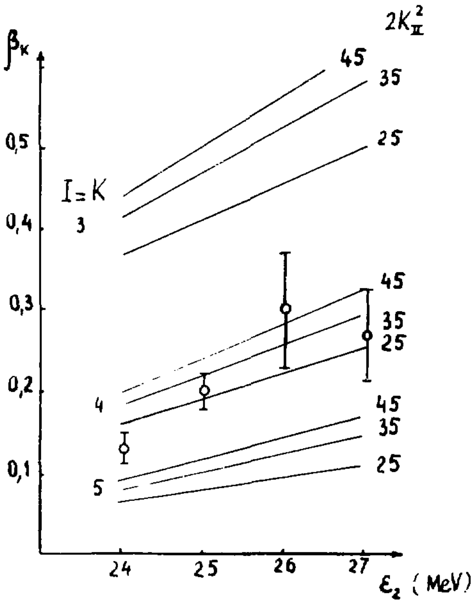

Fig. 2. The energy dependence of the isomer ratio for ^{238}Pu. Circles are experimental data taken from ref. /4/.

The energy dependence of the anisotropy of the angular distribution of fission fragments from the two-quasiparticle isomer is shown in fig. 7. It is seen from this figure to what extent the calculated results are sensitive to variations of the parameter K^2_{II} and to the nature of the γ-transitions populating the two-quasiparticle isomer.

The anisotropy experimental values for ^{238mf}Pu /5/ correspond to the very low spin of the two-quasiparticle isomer, ($I=K \sim 1,2$), and contradict the results based on isomer ratio measurements ($I=K=4$). Under the experimen-

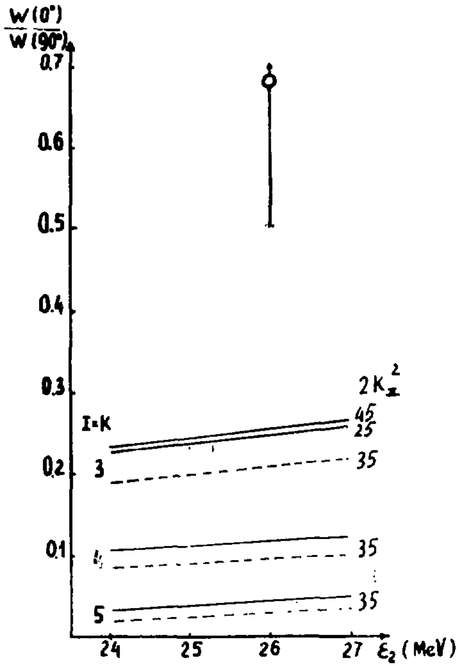

Fig. 3. The anisotropy of the angular distribution of fission fragments from the two-quasiparticle isomer of ^{238}Pu. Experimental values are from ref. /5/ . Solid curves correspond to calculation version b), dashed curves stand for version a).

tal conditions described in ref. [5], the effect of hyperfine nuclear interactions on the angular distribution of fission fragments does not seem to play an essential role. Therefore, this discrepancy is most likely to be associated with the probability for decay of the two-quasiparticle isomer to the ground state of the second potential well, which was neglected in the angular distribution calculation. This decay channel yields the isotropic angular distribution of fission fragments. There is no difficulty in estimating the relative probability Γ_{γ} / Γ of this decay channel. If one assumes that the two-quasiparticle isomer has spin $I=K=4$, the experimental data on the angular distribution [5] suggest $\Gamma_{\gamma} / \Gamma \sim 2/3$.

In conclusion it should be noted that the calculated results presented in figs. 2 and 3 may be helpful for estimating the spins of other two-quasiparticle states produced by reactions of the type $(\alpha, 2n)$.

The author is deeply grateful to Academician G.N.Flerov for his interest in the work and Yu.P.Gangrsky and N.Vilcov for helpful discussions.

References

1. Yu.P.Gangrsky, Nguyen Kong Khan, D.D.Pulatov, Pham Zuy Hien. JINR Communication, P7-6466, Dubna, 1972.
2. Pham Zuy Hien. Yad.Fiz., 17, 489 (1973).
3. P.A.Russo, R.Vandenbosch, M.Mehta, J.R.Tesmer, K.L.Wolf. Phys.Rev., C3, 1595 (1971).
4. S.Limkilde, G.Sletten. Nucl.Phys., A199, 504 (1973).
5. E.Konecny, C.Kozuharov, H.J.Specht, J.Weber. Jahresbericht des Universität München, 1972.
6. R.Vandenbosch. Phys.Rev., C5, 1428 (1972).
7. H.Ejiri, G.B.Hagemann, T.Hammer. Statistical Properties of Nuclei, Albany (1971).
8. T.Hammer, H.Ejiri, G.Hagemann. Nucl. Phys., A202, 321 (1973).
9. S.M.Ferguson, H.Ejiri, I.Halpern. Nucl.Phys., A188, 1 (1972).
10. N.D.Dudey, T.T.Sugihara. Phys.Rev., B139, 896 (1965).
11. G.R.Huizenga, R.Vandenbosch. Nuclear Reactions, vol. II, Amsterdam (1962).

12. D.L.Shpak, Yu.B.Ostapenko, G.N.Smirenkin. Yad.Fiz., 13, 950 (1970).
13. A.Gilbert, A.Cameron. Can.J. of Phys., 43, 1460 (1965).
14. H.C.Britt, S.G.Burnett, B.H.Erkkila, J.E.Lynn, W.E.Stain. Phys.Rev., C4, 1444 (1971).
15. I.R.Huizenga, G.Igo. Nucl.Phys., 29, 462 (1962).
16. G.S.Mani, M.A.Melkanoff, I.Iori. Rapport CEA, No. 2380 (1963).
17. J.Damgaard, H.C.Pauli, V.M.Strutinsky, C.Y.Wong, M.Brack, A.Stenholm-Jensen. Phys. & Chem. of Fission, Vienna (IAEA, Vienna, 1969).

Received by Publishing Department
on August 24, 1973.