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БИБЛИОТЕКА

1. Introduction

Observed[1-3] delayed proton spectra show a pronounced single-peaked gross structure, i.e., the average intensity of protons increases initially with energy and then decreases. In certain cases one observes some local resonances with a half-width of 0.5 MeV to 1.0 MeV, which seem to be associated with resonances in the beta-decay strength function. For instance, in the case of ^{109}Te, ref.[4], such a resonance is ascribed to the effect of three-quasiparticle states $pg^{-1}_{9/2}\ ng_{7/2}\ nd_{5/2}$ being populated strongly in beta transitions.

However, high energy resolution measurements reveal the fine structure in delayed proton spectra. The proton transitions between certain states are resolved for nuclei with $Z < 30$. As Z increases, the level density increases and hence individual transitions become unresolved, however, intensity fluctuations remain. The magnitude of these fluctuations is dependent on the variations in the strength of individual transitions and on the detector energy resolution relative to the level spacing. Therefore, from the study of the fine structure, it may be possible to obtain some information about the level density in the excitation energy range of 3 to 7 MeV for nuclei far from the beta-stability line. In an earlier work[2] a rather crude attempt was made to statistically analyze intensity fluctuations observed in the ^{111}Te delayed proton spectrum. The distribution of the relative intensities of certain proton peaks in the proton spectrum was compared with the χ^2 distribution to determine the effective number of degrees of freedom related to the average level density.

Hansen[5] suggests a more fruitful method of analyzing the fluctuations, which is based upon the approach developed by Egelstaff[6] for total and partial neutron cross sections. A formula was derived [5] to express the variance of proton intensity as a function of energy in terms of the level density and the average characteristics of the β^+-transition and proton emission.

In the present paper we also take advantage of Egelstaff's concepts [6]. We have derived an expression for the variation of relative proton intensity (in the proton spectrum) which is similar to the formula of ref. [5]. However, there is a distinction which results in a factor of 2 difference in that part of the spectrum where the average proton width $<\Gamma_p>$ is much less than the total width $<\Gamma>$. The calculated variation of the relative proton intensity and that observed for ^{111}Te and ^{109}Te are compared.

2. Theory

2.1. Resolved proton lines

The condition $\Gamma \lesssim \delta E \ll D$ applies to this case, where δE is the energy resolution of the detector and D is the average level spacing. The intensity of the delayed protons associated with the decay of the ν-th state with spin i to the final state f at an energy E_f is equal to

$$Y_{if\nu} = g_{ji}\, F(Z, Q_0 - B_p - E_p \frac{A}{A-1} - E_f)\, \mu^2_{ji\nu} \cdot (\Gamma_p/\Gamma)_{if\nu} \qquad (1)$$

Here g_{ji} is the statistical weight factor for the β^+-transition from the j -spin state to the i-spin state; $F(Z, Q)$ is the known statistical rate function for the sum of positron decay and electron capture, $\mu^2_{ji\nu}$ is the square of the beta decay matrix element, Q_0 is the total K-capture energy, B_p is the proton binding energy in the daughter nucleus, and E_p is the kinetic energy of the proton. Differences in the structure of the states involved lead to the dispersion of the quantities $\mu^2_{ji\nu}$ and $(\Gamma_p/\Gamma)_{if\nu}$ with respect to their average values even with a slight change in excitation energy. Consequently, the quantity $Y_{if\nu}$ fluctuates with changing value of ν. As is commonly done, we take the variance of the quantity $Y_{if\nu}/<Y_{if\nu}>$ as a measure of the fluctuations ϕ_{if}

$$\phi_{if} = \sigma^2 (Y_{if\nu}/<Y_{if\nu}>) = \sigma^2(Y_{if\nu})/<Y_{if\nu}>^2 . \qquad (2)$$

We assume that the beta decay and proton emission are statistically independent processes*. If we then employ the expression for the dispersion of the product of statistically independent quantities, we obtain

$$\phi_{if}(E_p)=\frac{\sigma^2(\mu_{ji}^2)}{<\mu_{ji}^2>^2}+\frac{\sigma^2[(\Gamma_p/\Gamma)_{if}]}{<(\Gamma_p/\Gamma)_{if}>^2}+\frac{\sigma^2(\mu_{ji}^2)}{<\mu_{ji}^2>^2}\cdot\frac{\sigma^2[(\Gamma_p/\Gamma)_{if}]}{<(\Gamma_p/\Gamma)_{if}>^2}\cdot \tag{3}$$

Index ν is omitted in this formulae for convinience.

The denominators of all the terms in eq. (3) are the squares of the mathematically expected values of the corresponding quantities for excitation energy $E=B_p+E_p\frac{A}{A-1}+E_f$. One may expect that the squares of beta matrix elements, like the partial widths for particle emission, obey the Porter-Thomas distribution /7/. Then the first term in eq. (3) is equal to 2, consequently

$$\bar{\phi}_{if}=2+3\,\frac{\sigma^2[(\Gamma_p/\Gamma)_{if}]}{<(\Gamma_p/\Gamma)_{if}>^2}\,. \tag{4}$$

Some experimental support of this assumption is provided by fig. 1 which displays the distribution function for the squares of the matrix elements for the known allowed beta transitions (the data from ref. /8/ are used). The experimental data for β^- and β^+decay are close to the χ^2-distribution with the number of degrees of freedom equal to unity. Of course, the matrix elements should be expected to show a systematic behaviour with variation of A, Z and Q values. It means that one should analyse the data over a rather narrow range of A, Z and Q values to make a correct conclusion about the beta-matrix element distribution. However, this is impractical because of poor statistics. In addition to fig. 1, we have considered the data for $70 \leq A \leq 110$; the distribution of the matrix elements squared has also turned out to be close to the Porter-Thomas one.

It is reasonable to assume that the proton width Γ_p, like the neutron width Γ_n, obeys the Porter-Thomas distribution. The distribution of the relative proton widths $(\Gamma_p/\Gamma)_{if}$ depends on the value

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* The problem of correlation between μ^2 and Γ_p/Γ needs a special study.

of $\langle\Gamma_p\rangle_{if} / \langle\Gamma_\gamma\rangle_{if}$. In the case where $\langle\Gamma_\gamma\rangle \gg \langle\Gamma_p\rangle$, the total width actually coincides with Γ_γ. It means that the distribution of $(\Gamma_p/\Gamma)_{if}$ is practically the same as that of $(\Gamma_p)_{if}$, since the dispersion of $\Gamma_\gamma/\langle\Gamma_\gamma\rangle$ is small. In this case the second term in (4) is equal to 6. Now we consider the case where $\langle\Gamma_\gamma\rangle \ll \langle\Gamma_p\rangle$, and the total width is approximately equal to the sum of partial proton widths $\Gamma_i \simeq \sum_f(\Gamma_p)_{if}$. If the transition to the ground state is predominant, the second term in (4) will be small. If the different final states are populated in p-decay with comparable probabilities, the situation becomes more complicated. In the transitions to the collective excited states one can expect a strong correlation between the partial proton widths. This will result in a small difference between the value of $(\Gamma_p)/\sum_f(\Gamma_p)_{if}$ and its average, and therefore the second term in (4) will be close to zero. However, if the correlation between the proton partial widths is weak, the second term in (4) may appear to be rather large also in the case of $\langle\Gamma_p\rangle \gg \langle\Gamma_\gamma\rangle$. Thus, the value of ϕ_{if} decreases monotonically (for the limit case from 8 to 2) with increasing proton energy.

The difference between the formula for the fluctuation (3) and the analogous one in ref. [5] consists in the presence in the former of the third cross term equal to the sum of the two first terms if $\langle\Gamma_p\rangle_{if} \ll \langle\Gamma_\gamma\rangle_{if}$.

Figure 2 shows the dependence of $\sigma^2(\Gamma_p/\Gamma)/\langle\Gamma_p/\Gamma\rangle^2$ upon $\langle\Gamma_p\rangle/\langle\Gamma_\gamma\rangle$ for the case where only one partial proton width is predominant. Such a situation exists for the delayed proton emitters ^{111}Te and ^{109}Te. At least 90% of the spectra are associated with the p-decay into the ground state of the final nucleus. The curve in fig. 2 is calculated with the assumption of the Porter-Thomas distribution for Γ_p and of the Gaussian one with negligible dispersion for Γ_γ.

2.2. Fluctuations in proton intensity summed over an energy interval $\Delta E_p \gg D, \delta E$.

This case is entirely analogous to that considered by Egelstaff [6] in an analysis of the average values of neutron cross sections. The proton intensity in the energy interval from E_p to $E_p + \Delta E_p$ is equal to

$$I(E_p)\Delta E_p = \sum_{if}\sum_{\nu} Y_{if\nu} = \sum_{if} g_{ji} \; F[\sum_{\nu}(\mu^2 \frac{\Gamma_p}{\Gamma})_{if\nu}]. \qquad (5)$$

With change in energy, the value of $I(E_p)$ will fluctuate around its average value $\langle I(E_p)\rangle$ due to fluctuations in the intensities of individual lines and in the level density.

It is shown by Egelstaff /6/ that the contribution of the level density to the variance in average neutron cross section is rather small. A similar situation characterizes the delayed proton emission. Indeed, the level spacing d obeys the Wigner distribution and consequently $\sigma^2(d)/\langle d\rangle^2 = 0.27$. This quantity is substantially less than ϕ_{if}, i.e., the intensity fluctuations in the proton spectrum are mainly due to the variances of μ_{if}^2 and $(\Gamma_p/\Gamma)_{if}$, as has already been noted in ref. /5/. If we make use of the known rules for the variance of the linear combination of statistically independent quantities and neglect level spacing dispersion, we obtain

$$\Phi = \sigma^2\left(\frac{I(E_p)}{\langle I(E_p)\rangle}\right) = \sum_{if} \frac{\langle I_{if}(E_p)\rangle^2}{\langle I(E_p)\rangle^2} \cdot \frac{D_{if}}{\Delta E_p} \cdot \phi_{if} \quad . \tag{6}$$

Here $\langle I_{if}(E_p)\rangle$ is the average intensity of the $i \to f$ transition, and D_{if} is the average spacing between the i-spin levels at an energy $B_p + E_p \frac{A}{A-1} + E_f$. The quantity $\langle I_{if}(E_p)\rangle/\langle I(E_p)\rangle$ is essentially the weight factor for the contribution to the proton spectrum from the $i \to f$ transition; $\Delta E_p / D_{if}$ is equal to the average number of $i \to f$ transitions falling into the energy interval ΔE_p.

2.3. Unresolved proton lines $(\delta E \gg D \gg \Gamma)$

This particular condition would apply to proton emitters in the region of $Z > 50$. In this case the proton spectrum is formed by summing up the contributions from all lines into each channel (with a width of ΔE), the weight factor being determined by the detector response function which is assumed to be a Gaussian distribution with the halfwidth δE. Thus in eq. (6) there appears the weight factor taking account of the contribution from the ν-th proton line, and the summation over ν is made over the entire spectrum.

If in finding the variance of $I(E_p)\Delta E_p$ one goes from the summation over ν to integration, one obtains (in suggestion of $\Delta E_p \ll \delta E$)

$$\Phi(E_p) = \langle I(E_p)\rangle^{-2} \sum_{if} \langle I_{if}(E_p)\rangle^2 \frac{D_{if}}{1{,}5\,\delta E} \cdot \phi_{if} \quad . \tag{7}$$

The structure of this formula is the same as (12) of ref.[5], but the factors ϕ_{if} are different. The value of $1{,}5\delta E/D_{if}$ is equal to the effective number of the $i \to f$ transitions contributing to the spectrum interval of E_p to $E_p + \Delta E_p$.

Equation (7) is substantially simplified when in the proton decay one state of the final nucleus is predominantly populated. This situation occurs for the delayed proton emitters ^{109}Te and ^{111}Te. In this case we can restrict ourselves with good accuracy to the consideration of the proton transition to the ground state of the ^{108}Sn and ^{110}Sn nuclei, and hence eq. (7) is rewritten as follows

$$\Phi(E_p) = (1{,}5\delta E)^{-1} \left(\sum_i g_{ji} S_i\right)^{-2} \sum_i g_{ji}^2 S_i^2 D_i \phi_i . \qquad (8)$$

When summed up, i takes on the values 3/2, 5/2 and 7/2, since the spin of the ^{109}Te and ^{111}Te is taken to be equal to $5/2^+$, refs.[1,4]. In deriving (8) one assumes that the beta strength function is the same for all values of i; $S_i = \langle \Gamma_{pi}/\Gamma_i \rangle / \langle \Gamma_{p3/2}/\Gamma_{3/2} \rangle$.

3. Comparison of Calculations and Experimental Data for ^{109}Te and ^{111}Te

To perform calculations using formula (8), one should know $g_{5/2,i}$, S_i, D_i and ϕ_i. Following paper [3] we take $g_{5/2,i} = \frac{2i+1}{18}$.

In order to obtain the S_i-values, we have used the proton and gamma widths calculated earlier[4]. It should be noted that $S_{7/2}$ does not exceed 0.015, i.e., the $7/2^+$ levels do not substantially change the picture of fluctuations. The average level spacing was calculated following Gilbert and Cameron[9] (fig. 3). The proton binding energies for ^{109}Sb and ^{111}Sb were taken to be equal to 1.3 MeV and 1.8 MeV, respectively[2,4].

For determining the experimental value of the variance of relative intensity in the proton spectrum, one should first find $\langle I(E_p) \rangle$. It seems unreasonable to use for this function the spectrum calculated in the framework of the statistical model of delayed proton emission because of the possible presence of intermediate structures in the beta strength function which makes this calculations somewhat ambiguous. To find $\langle I(E_p) \rangle$, we smoothed the experimental spectrum using the Gaussian weight function

$$\langle I(E_p)\rangle = \langle I(k)\rangle = \frac{1}{\sqrt{2\pi}\,\Delta_k} \sum_{k'} I(k') \exp\left[-\frac{(k-k')^2}{2\Delta_k^2}\right], \qquad (9)$$

where k represents the channel number. The choice of Δ_k is somewhat arbitrary. As a starting point in our calculations, we assume that the half-width of the weight function (2.36 Δ_k) should considerably exceed the energy resolution but not the half-width of the possible local resonances of the beta strength function and of the proton strength function [4]. The value of Δ_k has been chosen to be equal to 127 keV (which corresponds to a half-width of 300 keV).

Figure 4 shows the ^{109}Te delayed proton spectrum measured in the experiments described in ref. [4] and smoothed by formula (9). In order to separate protons from alpha particles, use has been made of a proportional counter - semiconducting detector telescope. As a result, the energy resolution was moderate, ranging from 85 keV to 60 keV with increasing proton energy in the region of interest. Nevertheless, the spectrum exhibits a pronounced structure which disappears after the smoothing procedure. However, a broad peak remains in the region of 3.5 MeV, which is associated with the resonance in the beta strength function [4].

In the determination of the variance of the relative intensity $I(E_p)/\langle I(E_p)\rangle$ the spectrum was derived into intervals of 250 keV each. For each of these intervals the estimate of the variance was made [10] corresponding to the average energy of the interval. The experimental variance consists of two components: the first one is associated with the fluctuations in the strength of the different partial transitions contributing to the intensity, while the second is due to the counting rate statistics. Here we are interested in the first component. In order to determine this, we have subtracted the variance due to the statistical error of the measurement from the measured values of the variances of the relative intensity.

In obtaining the experimental value of $\Phi(E_p)$ it is assumed that the values of the relative intensities in different channels are statistically independent. However, if the width of a channel is less than the energy resolution, one cannot avoid the correlation of intensities in neighbouring channels. This may affect the experimental value of the variance and thus result in some deviation from that predicted by formula (8).

Figure 5 shows a comparison of the experimental and calculated values of the relative intensity variance for ^{111}Te. The ^{111}Te delayed proton spectra measured with resolutions of 25-30 keV and 50-60 keV are taken from ref. /1/; the calculations are performed for δE of 27 keV and 55 keV. We note that the values calculated by formula (8) for $\delta E = 27$ keV are close to the experimental ones for $E_p > 2.7$ MeV. At lower energies the intensity fluctuations are smaller than expected. This may be due to the contribution of beta background to this part of the spectrum; the beta background increases the average intensity without changing the fluctuating part.

The spectrum measured with a resolution of 50-60 keV exhibits fluctuations that are by (2.7 ± 0.8) times weaker than those of the first spectrum. According to formula (8) with increasing δE from 27 to 55 keV, one might expect a two-fold decrease in the relative variation. A further possible suppression of fluctuations is due to the effect of the correlation between the intensities in neighbouring channels which increases with increasing δE. Although the accuracy in the estimation of this effect is not high, this effect does not essentially influence the results obtained at the resolution of $\delta E = 27$ keV.

After considering the data presented in fig. 5 one can draw the following general conclusion. The calculation describes satisfactorily the fluctuations in the delayed proton spectrum for ^{111}Te. The level density for ^{111}Sb differs from that calculated following Gilbert and Cameron by no more than a factor of 1.5. This deviation actually may be related to the uncertainty in the level density due to the error in the estimation of B_p for ^{111}Sb ref. /2/. In fact, the proton binding energy for ^{111}Sb has been estimated to an accuracy of about 200 keV. Consequently, this is the uncertainty in knowledge of the excitation energy of the state which emits protons. According to fig. 3 a change of 250 keV in energy results in a variation of the level density by a factor of 1.5.

Figure 6 presents the results for ^{109}Te. The calculations using formula (8) with the level density from ref. /9/ predict considerably larger fluctuations than those observed experimentally. To explain the measured magnitude of the fluctuations, one can assume that either the level density of ^{109}Sb exceeds by 4-5 times the calculated one, or the proton binding energy in ^{109}Sb is 2.5 MeV, and not 1.3 MeV. The latter assumption seems less probable /4/. The data obtained for ^{109}Te should be regarded as preliminary. We are unable to eliminate completely the systematic errors due to the insufficient energy resolution, which damp the observed intensity fluctuations. It is desirable to repeat an analysis of the fine structure in the proton spectrum of ^{109}Te after measurements with a higher resolution. We would like to stress the importance of performing experiments which permit the determination of B_p, such as the measurements of the quantities Q_0 and $Q_0 - B_p$ for delayed proton emitters. The problem of determining Q_0 for isotopes with law yield is not easy. However, it is worth being tackled since a precise knowledge of B_p will enable us to obtain more accurate information from the delayed proton spectrum studies.

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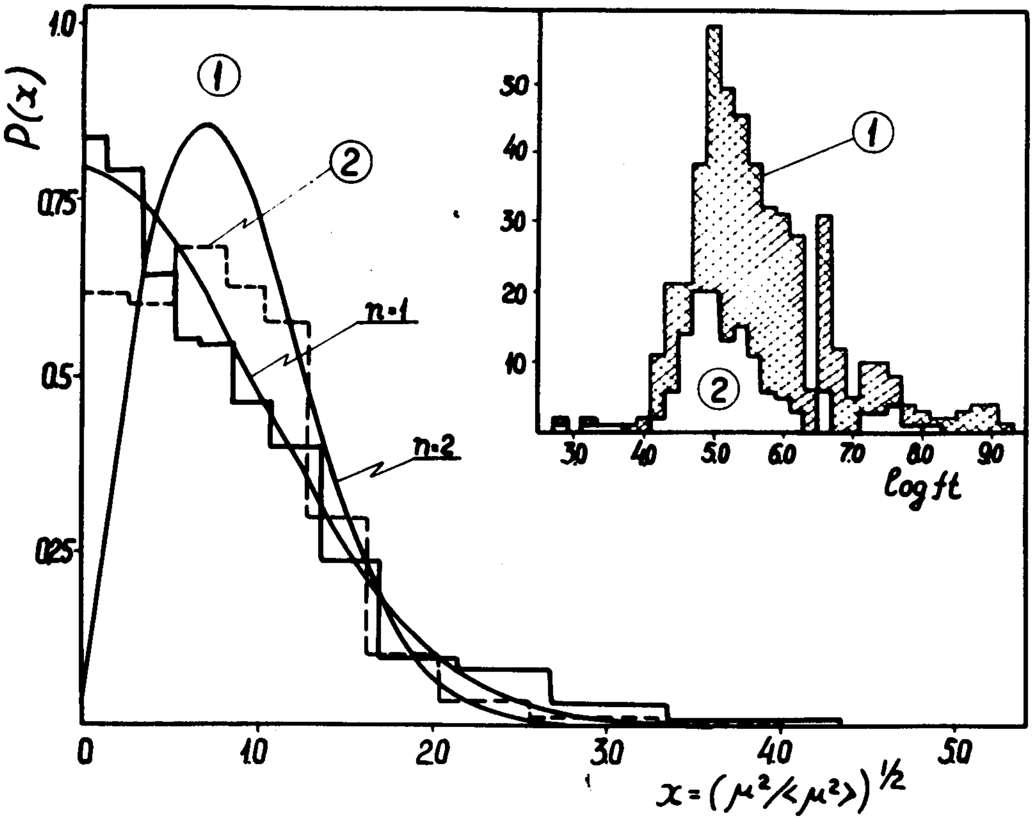

Fig. 1. The distribution of the quantity $(\mu^2/<\mu^2>)^{1/2}$, where μ is a beta matrix element. Histogram 1 includes all allowed beta transitions known before the appearance of ref. /8/ (487 events) and histogram 2 corresponds to β^+-transitions (139 events). Smooth curves show χ^2-distributions with the number of degrees of freedom $n=1$ and 2. The inserted plot shows the initial distributions of $\log Ft$.

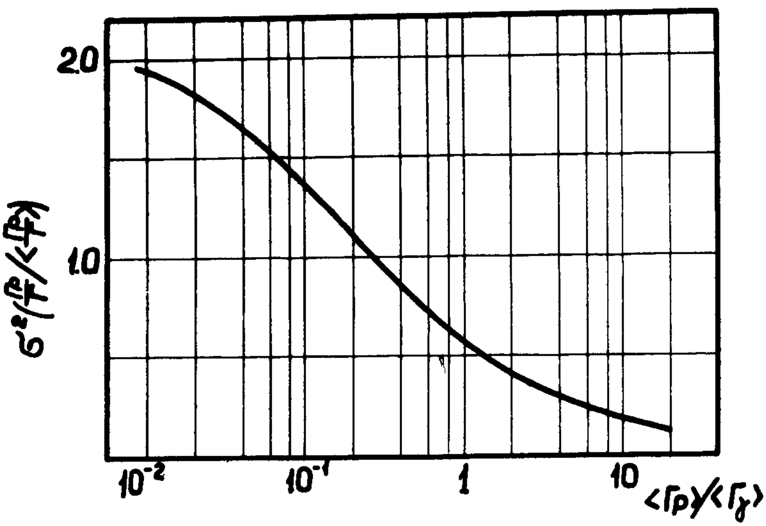

Fig. 2. The relative variance of Γ_p/Γ as a function of $\langle\Gamma_p\rangle/\langle\Gamma_\gamma\rangle$. For Γ_p the Porter-Thomas distribution was used; $\Gamma_\gamma \simeq \langle\Gamma_\gamma\rangle$.

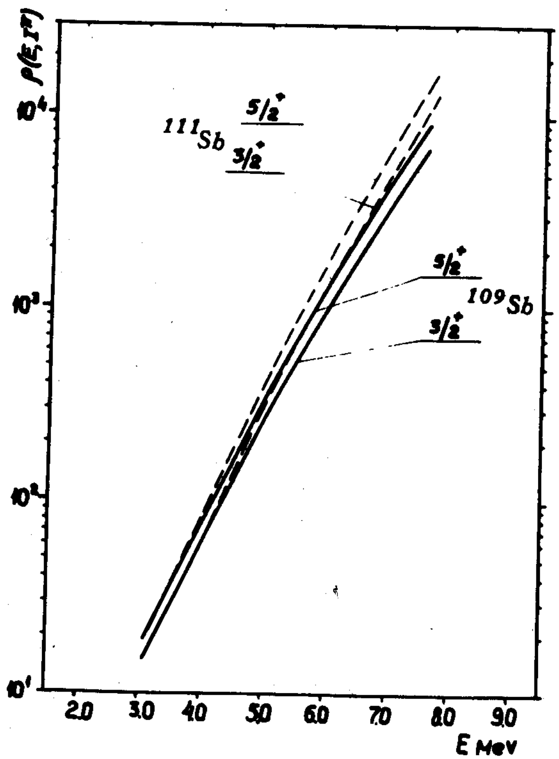

Fig. 3. The density of the $3/2^+$ and $5/2^+$ levels for ^{109}Sb and ^{111}Sb calculated in accordance with ref. /9/.

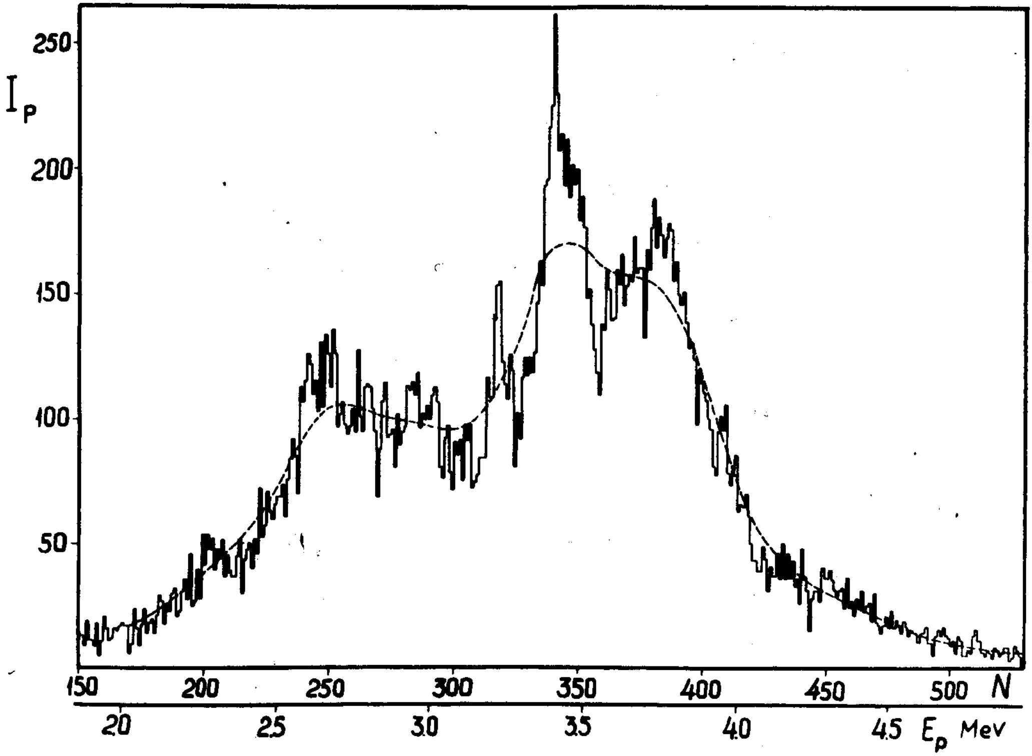

Fig. 4. The ^{109}Te delayed proton spectrum. The smooth curve is obtained from smoothing by a Gaussian function with a half-width of 300 keV.

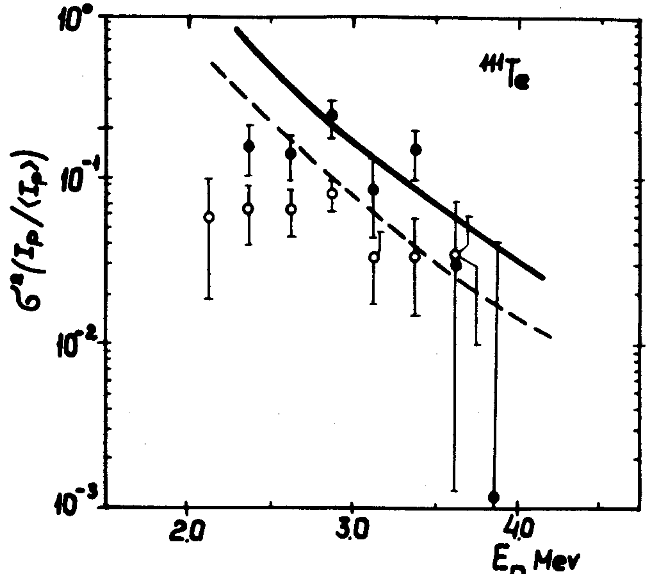

Fig. 5. The variance of relative intensity in the ^{111}Te proton spectrum with the resolution of 27 keV (dots) and 55 keV (open circles). Curves corresponds to the calculation by formula (8).

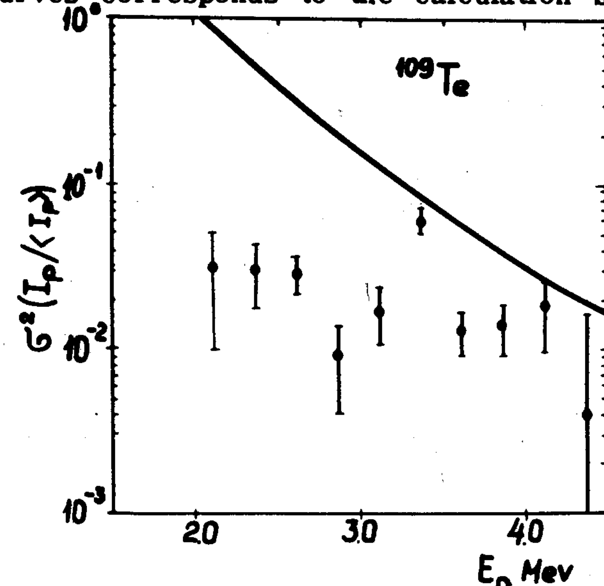

Fig. 6. The variance of relative intensity in the ^{109}Te proton spectrum.