ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E6 - 4414

V.A.Karnaukhov

DELAYED PROTONS AND GROSS-STRUCTURE OF β-DECAY

ЛАБОРАТОРИЯ ЯДЕРНЫХ РЕАКЦИЙ

1969

E6 - 4414

V.A.Karnaukhov

DELAYED PROTONS AND GROSS-STRUCTURE OF β-DECAY

Submitted to Yadernaya Fizika

1. Introduction

Delayed protons are produced as a result of positon-decay and K-capture to proton-unstable excited states of daughter nuclei. An averaged proton spectrum for ^{111}Te has been analysed in ref./1/. It has been shown that a spectrum shape is strongly affected by the dependence of an averaged squared matrix element of beta-transition $\langle\mu^2\rangle$ on the excitation energy of a daughter nucleus. Thus the spectrum of delayed protons is sensitive to gross structure of beta-decay and may be used for its investigation. This paper contains the analysis of the proton spectra for ^{111}Te from this point of view. The data obtained in measuring $\beta^+ - p$ -coincidences with ^{111}Te are used/2/.

2. Gross-Structure of Beta-Decay

By this term the dependence of the beta-decay strength or of the averaged squared matrix element $\langle\mu^2\rangle$ on the transition energy is meant. For the given isotope the transition energy varies in the decay into the different excited states of daughter nucleus.

It should be expected that for the allowed Gamow-Teller transitions $\langle\mu^2\rangle$ reduces proportionally to level spacing D with increasing excitation energy of daughter nucleus, that is $\langle\mu^2\rangle \approx D$ [x/]. This proceeds from the following. Low-lying nuclear states are known to be relatively simple, with one configuration being frequently predominant. With the increase of the excitation energy states became more complicated. A number of configurations with comparative weight included increases. In other words, due to the residual interaction simple configurations are distributed among many real levels of the nucleus. In this connection, matrix element of any single-particle process related to a certain simple state is also distributed among the levels in the region of this simple configuration. The distribution width depends on a strength of the residual interaction. Beta-decay is a single-particle process. For the given initial nucleus it chooses definite simple configurations in the final states which are produced by the allowed Gamow-Teller transition of one of nucleons. Matrix element μ_0^2 conforming to the β-transition into the pure simple configuration is spread over real nuclear levels: $\mu_0^2 = \sum_i \mu_i^2$. The more the level density, the less the squared matrix element which accounts for one level on the average: $\langle\mu^2\rangle \approx D$. Note that $\langle\mu^2\rangle / D$ is analogous by its structure to a strength function which is used in the nuclear reaction theory. That is proportional to the amount of a simple configuration per unit energy. The strength function has to have a maximum at energies of unperturbed simple configurations. However, if the distribution half-width is

---

[x/] The author is grateful to Prof. B.Mottelson who has pointed in 1966 to the validity of this relation.

larger than the distance between simple states the strength function should be smoothed and one could believe that $\langle\mu^2\rangle/D$ = const. The problem of simple state distribution among the levels of complicated structure is a general one. It is related to any single-particle process but it has not been studied yet. The detailed information on this problem is given in a review by P.Axel[3].

The general behaviour of a strength function of the beta-decay has been considered for the last few years by Japanese theoreticians S.Fujii, J.Fujita, Y.Futami, K.Ikeda[4] and M.Yamada[5]. In the papers of the first authors the gross structure of beta-decay has been considered in the pairing model with the account of $n-p$ correlations and $\log ft$ for some transitions have been calculated. In Yamada's paper[5] a theory is developed which describes averaged matrix elements without taking into account properties of individual states. There is no doubt that the account of the gross structure is of importance for understanding main features concerning the β -decay probability. It follows from these papers that the assumption $\langle\mu^2\rangle/D$ = const. is valid only for a relatively small change of the transition energy. In a general case a strength function has a shape of giant resonance with a peak corresponding to transitions into the region of an isobaric-analogous state.

In the case of decay of the Fermi type this assumption proceeds from the fact of the existence of such states. Indeed, the operator of the transfer of a nucleus into the analogous state coincides with the allowed Fermi transition operator ($T_\pm$). As a result of action of this operator on the initial nucleus wave function Ψ_i the collective state $(\Psi_f)_F = T_\pm \Psi_i$ is formed. This is nothing other than the isobaric analogue of the initial nucleus. Such states are studied well via nuclear reactions. The width of analogous states does not exceed 300 keV even for the heaviest nuclei. Therefore the strength of the Fermi transition is concentrated in the narrow region near the analogous state.

In the case of the Gamow-Teller decay the situation is not so clear. However the decay strength is also assumed to be concentrated in the transition $\Psi_i \to (\Psi_f)_{G-T} = Y_{\pm} \Psi_i$, where $Y_{\pm}$ denotes an operator of the allowed Gamow-Teller transition. This transforms protons into neutrons (or on the contrary) keeping the same orbital but changing spin direction. As a result the collective state appears consisting of neutron-proton particle-hole pairs with a spin 1^+. Since $(\Psi_f)_{G-T}$ is not an exact eigen state of a nucleus the strength of the transition $\Psi_i \to (\Psi_f)_{G-T}$ is distributed among many real levels.

Fig.1 from ref./4c/ shows the result of consideration of the β -decay gross structure. According to calculations a maximum of the strength function is located at the energy corresponding to the decay into analogous state in the case of spherical nuclei. For deformed nuclei a maximum is displaced by 2 MeV towards smaller energies. The half-width of a giant resonance of the Gamow-Teller strength function is essentially broader than that for the Fermi type decay. And so beta transitions between low-lying levels are practically of the Gamow-Teller type. Basing of ref./4/ one can write down the following expression of the Gamow-Teller strength function for spherical nuclei

$$\langle \mu^2 \rangle / D \sim \frac{1}{(Q \pm \Delta E_c)^2 + \frac{\gamma^2}{4}} , \qquad (1)$$

where Q is transition energy, ΔE_c is Coulomb displacement of the isobaric analogue, ($\Delta E_c = 1.44\ Z.A^{-1/3} - 1.1$ MeV), γ is width of a giant resonance (a few MeV). In the case of β^- -decay ΔE_c is taken with a plus and for β^+ -decay with a minus. $E \pm Q$ equals the energy distance between one of the decay partners and the isobaric analogous state. Study of the high-energy beta-decay into excited states of a daughter nucleus may serve as a method of investigating the Gamow-Teller strength function. It is clear that the higher β -decay energy the larger part of the curve can be studied. Delayed proton emitter provides favourable conditions for such investigations.

The next section deals with the averaged proton spectra of the ^{111}Te isotope in order to obtain some information on the strength function.

It should be remarked that similar possibilities are presented by the investigation of delayed neutrons emitted by fission fragments. However, in all the papers on the interpretation of delayed neutron data the assumption $\langle\mu^2\rangle$=const. has been used so far /6,7/. There have been no attempts made to get any information concerning real change in $\langle\mu^2\rangle$ with the excitation energy.

3. Averaged Spectrum of the ^{111}Te Delayed Protons

Isotope ^{111}Te (19.5 sec) was obtained in the reaction ^{102}Pd(^{12}C, 3n) /1/. Fig.2 presents the proton spectrum measured by a semiconductor detector. An illustrative scheme of the ^{111}Te decay is shown in Fig.3. A spin and a parity of ^{111}Te are likely to be equal to $5/2^+$ (known nuclei with N = 59 and even Z have spin $5/2^+$). The ^{111}Te isotope undergoes β -decay and K -capture passing to various states of ^{111}Sb . Levels with spins $3/2^+$, $5/2^+$, $7/2^+$ will be populated by the allowed Gamow-Teller transition. The binding energy of a proton (B_p) in ^{111}Sb equals 2 MeV approx. (see below). That means that levels of ^{111}Sb above 2 MeV are proton unstable ones. However, at a small excess of the excitation energy E^* over B_p radiation width exceeds that of a proton because of a low penetrability of Coulomb barrier.

This is a reason that the proton spectrum starts actually with the energy $E_p \approx 2$ MeV which corresponds to $E^* \approx 4$ MeV. The rise of counts in the region less than 2 MeV is caused by the beta-background (Fig.2). With the increase of the excitation energy the ratio of the proton width to the radiation one enhances. This causes the intensity rise (on the average) within the proton energy range of 2-3 MeV. Further on the envelope of proton spectrum goes down because of reducing level population in the beta-transition.

The averaged shape of the proton spectrum is described[1] as follows:

$$\frac{dN(E_p)}{dE_p} = \sum_{ij} \frac{\Gamma_{pi}^{(j)}}{\Gamma^{(j)}} \frac{\langle\mu^2\rangle}{D(E^*,j)} F_{\beta^+,k}(Q_0 - E^*) \qquad (2)$$

Q_0 is the total energy of K-capture, E^* is the excitation energy of an intermediate nucleus in the state with a spin j; $F_{\beta^+,k}$ is the sum probability of β^+-decay and K-capture normalized to the matrix element squared; $\Gamma_{pi}^{(j)}$ is the mean proton width for the decay into i-state of the final nucleus, $\Gamma^{(j)}$ is the mean total width. The proton spectrum is contributed mainly by the p-decay of $3/2^+$ and $5/2^+$ levels to the ground state of ^{110}Sn. The emission of protons from $7/2^+$ states is noticeably slowed down due to centrifugal barrier ($\ell_p = 4$). The estimation indicates that about 2% of events only are associated with the proton decay to the first excitation state of ^{110}Sn (2+; 1.2 MeV).

Calculations of proton and total widths were performed by statistical formulae in the previous paper [2]. The relative proton width $\frac{\Gamma_p}{\Gamma}$ rises steeply at the proton energy change from 2 up to 3 MeV and then approaches the saturation. The proton binding energy is a parameter for $\frac{\Gamma_p}{\Gamma}$. The function $F_{\beta^+,k}(Q)$ was calculated by nomographs for allowed β^+ -decay and K -capture. Partial transitions energy Q is equal to $Q_0 - E^* = Q_0 - B_p - E_p$. Therefore $F_{\beta,k}$ can be treated as a function of E_p with a $Q_0 - B_p$ as a parameter. Note that a $Q_0 - B_p$ is a total energy of β^+-p -decay.

In the previous paper of our group the averaged proton spectrum of ^{111}Te has been analysed as a function of two unknown parameters Q_0 and B_p. It has been found from the measurement of spectrum of protons in coincidence with positrons that $Q_0 - B_p = (5.07+0.07)$ MeV for the nuclei $^{111}Te - ^{111}Sb$. This is used in the further analysis with B_p as the only parameter.

Fig.4 shows hypotheses applied in the calculations concerning relative trend of $\langle\mu^2\rangle/D$ as a function of a daughter nucleus excitation function. The proton energy related to the excitation energy as $E_p = E^* - B_p$ is plotted along the abscissa axis. The first curve corresponds to the assumption $\langle\mu\rangle$=const. In this case $\langle\mu^2\rangle/D$ goes up with the energy E_p because of the level spacing decrease. The assumption $\langle\mu^2\rangle$ =const. is evidently wrong but it is widely applied in papers on delayed neutrons. The next variant $\langle\mu^2\rangle/D$ =const. corresponds to a uniform spread of β- transition strength. The third curve was obtained with the account of the existence of a giant resonance in the β -decay strength function. It was calculated from (1). As noticed before, the value $\Delta E_c - Q$ provides a distance between one of the partners in the β - transition and an isobaric analogue of another. In our case it is the distance between the ground state of ^{111}Te and the isobaric analogue of the proton active staic of a daughter nucleus. It equals $\Delta E_c - (Q_0 - B_p) + E_p$. Coulomb difference ΔE_c was taken to be equal to 14.5 MeV. Thus the curve (I, F, F) in Fig.4 was calculated from the relation

$$\langle\mu^2\rangle / D \sim \frac{1}{(9,43 + E_p)^2}. \qquad (3)$$

The half-width of a giant resonance γ is omitted in the denominator. This changes the relative trand of a strength function insignificantly if $\gamma \lessapprox 8$ MeV.

Figs. 5,6 present the comparison of the calculated and the experimental averaged proton spectra. The interval of averaging is 300 keV. Only a shape of the averaged spectrum was calculated, the fit being perforemd by the least square method.

It proceeds from Fig.5 first of all that the assumption $\langle\mu^2\rangle =$ = const. is in disagreement with the data. The difference between theoretical and experimental spectra is far beyond the error limits. Two other assumptions about the strength function succeed in a quite satisfactory description of the experiment, however optimal values of the parameter B_p differ by 0.15 MeV. If the mean value is taken as the most probable one we obtain for the proton binding energy $B_p = (1.83 \pm 0.20)$ MeV. At first sight the calculations agree with the experiment equally well in both cases. It is natural , since the only tail of the strength function resonance curve falls into the energy region of interest. However, one can find the difference at more careful consideration. Fig.7 shows the relative deviation of the experimental points from the theoretical curve. For the resonance shape of a strength function there is a smaller dispersion of the points on the average . The quadratic form (sum of deviations squared multiplied by the statistical weights) for this case is twice as small as for $\langle\mu^2\rangle/D$ = const. We did not include the first point into that comparison, since there is a possibility of a large error in the theoretical value due to unaccuracy in $\frac{\Gamma_p}{\Gamma}$ calculating for subbarrier protons. Two last points were rejected too for the theoretical values are extremely sensitive to the $Q_0 - B_p$. Thus the assumption of reducing $\langle\mu^2\rangle/D$ with the increase of the daughter nucleus excitation energy improves the agreement between the calculations and experimental data.

Let us consider reasons for the experimental points dispersion with respect to zero line in Fig.7. The dispersion exceeds the permissible one due to statistical errors. Two last points deviation of 30-35% is possibly connected with the over-estimating the value $Q_0 - B_p$ = 5.07 MeV taken in calculations. It can be seen from Fig.7 that the position of these two points changes considerably if we take $Q_0 - B_p$ = 5.00 MeV which is within the accuracy of determinating that energy. Another noticeable deviation (≈ 20%) is observed in the region of 4 MeV.

Two reasons of an additional dispersion are possible:

a) Statistical fluctuations of a number of levels accounted for the averaging interval;
b) The existence of local maxima of the strength function at the energies corresponding to simple configurations well populated in the β-decay.

Let us consider these points.

4. Fine Structure of Delayed Proton Spectrum

The ground state of ^{111}Te has an odd neutron in $d\,5/2$ state. With the most probability the Gamow-Teller transition occurs between the states being partners of the spin-orbit splitting. It may be expected that in the β-decay of ^{111}Te three-quasiparticle states $[\pi d_{5/2}, \nu d_{3/2}]_{1^+} \nu d_{5/2}$ and $[\pi g^{-1}_{9/2}, \nu g_{7/2}]_{1^+} \nu d_{5/2}$ would be populated well /8/. The former is produced by the transition of a d 5/2 proton into a d 3/2 neutron with the formation of a pair with a spin 1^+; the latter is produced by the transition of a proton from a g 9/2 closed shell into a g 7/2 neutron with the formation of a pair with a spin 1^+. In both cases an odd neutron remains on its place. Both states are triplet, two components ($3/2^+$ and $5/2^+$) can participate in a proton decay. According to ref. /9/ one can expect that the first state is located at the excitation energy of about 3 MeV, the second state at the energy of about 5 MeV. These states will be distributed among the neighbouring levels of a nucleus in a certain energy range. Using the Cameron-Gilbert semiempirical method one can estimate for ^{111}Sb that a summary density of levels with the spin values $3/2^+$ and $5/2^+$ varies from 6.10^2 up to $2.6.10^3$ MeV^{-1} at the excitation energy changing from 4.3 MeV (E_p = 2.5 MeV) up to 5.3 MeV (E_p = 3.5 MeV). However, it is not clear beforehand whether all these levels take part in the proton spectrum formation. To answer this question a spectrum taken with a good resolution (Fig.2) has been analysed. This spectrum was decomposed into groups with a half-width of 45 keV in the range from 2.2 up to 3.6 MeV. This group intensity distribution after normalizing to the averaged spectrum was determined. It is shown in Fig.8. For comparison there are also shown χ^2-distributions for a number of degrees of freedom n = 4,8,16. Similar analysis is performed when studying nuclear reactions. It is known, for example, that partial widths Γ_n and Γ_γ of neutron resonances in the case of transitions into definite states obey χ^2-distribution with n = 1. If the decay results in several nuclear states, the number of degrees of freedom in the width distribution increases respectively. We conclude from Fig.8 that relative intensities in the spectrum can be described by χ^2 -distribution with $n = 9 \pm 3$. Accuracy of n -value determination is rather low, since only 30 groups are involved in the analysis. The average number of ^{111}Sb levels corresponding to a single peak in the proton spectrum has not to exceed the obtained n -value. If one takes the calculated density of levels with spins $3/2^+$ and $5/2^+$, it should be expected that a number of degrees of freedom in χ^2 -distribution is equal to 60. It is possible that not all the levels populated in the β -decay equally participate in the delayed proton spectrum formation. One can believe that the proton width is determined by contribution of one proton component into a wave function of a given excited state of ^{111}Sb. Near the Fermi surface there are two proton states with suitable spin values: the ground state - $d\ 5/2$ and the excited state - $d\ 3/2$. The $d\ 3/2$ excitation energy is likely to be about 3 MeV. A larger impurity of $d\ 3/2$ state should be expected in comparison to $d\ 5/2$ state for the levels in the energy range from 4 up to 6 MeV. Therefore it is possible that the delayed protons are emitted largely from $3/2^+$ states. The estimated average number of $3/2^+$ levels corresponding to $\Delta E = 45$ keV in the energy interval of interest is equal to 26. This figure exceeds the experimental number of channels "n" though the difference is not so large.

It follows from the obtained n -value that the dispersion of points in Fig.7 might be associated with the statistical fluctuation of the number of proton-unstable levels populated in the β -decay. Indeed, if the channel number $n = 9 \pm 3$ corresponds to $\Delta E = 45$ keV, then for the averaging interval of $\Delta E_p = 300$ keV n -value will be about 60. For that case standard deviation equal to 18% should be expected.

5. Delayed Neutrons

In analysing the data on delayed neutrons an assumption $\langle\mu^2\rangle$ = const. is traditionally applied. There is no doubt at the moment that this condition is incorrect for the region of high level density. It has been already noted in our previous papers /1,2/. As the first approximation the constant value of a strength function of β^--transition should be used. An assumption of the resonance shape of a strength function results in a weak increasing $\langle\mu^2\rangle/D$ with the increase of the residual nucleus excitation energy. The difference as compared with the case of β^+-decay is associated with a part of an isobaric analogue in a daughter nucleus which is approached with the increase of the excitation energy.[x]

The use of condition $\langle\mu^2\rangle/D$ = const. will reduce the calculated probability of delayed neutron emission as compared to the case of $\langle\mu^2\rangle$ = const. The calculated shape of neutron spectrum will be changed as well, the mean energy becoming lower. Some discrepancies in the interpretation of the data on delayed neutrons are likely to be cleared up if more real assumptions of $\langle\mu^2\rangle/D$ are employed.

6. Conclusions

1. It was shown that the delayed proton emitters may be used for investigation of the averaged probability of β-decay into excited states of a daughter nucleus.

2. The averaged shape of ^{111}Te proton spectrum was calculated on the assumptions that $\langle\mu^2\rangle$ = const., $\langle\mu^2\rangle/D$ = const. and for a resonance shape of the Gamow-Teller strength function. It

x/ We consider here only nuclei with $N > Z$.

was demonstrated that the assumption $\langle\mu^2\rangle$ =const. does not fit the experimental data. Two other assumptions provide a satisfactory agreement between theoretical and experimental results.

3. It is concluded from the analysis of a fine structure of the proton spectrum that a number of ^{111}Sb levels participating in the proton spectrum formation is about 6 times smaller than that calculated according to statistical formulas.

4. In the analysis of the experimental data on delayed neutrons the condition $\langle\mu^2\rangle/D$ =const. should be used instead of $\langle\mu^2\rangle$ = =const.

The author wish to thank Academician G.N.Flerov for his interest in this work, Prof. K.E.Alexander, Doctors D.D.Bogdanov, L.Petrov and G.M.Ter-Akopyan for useful discussions.

References

1. D.D.Bogdanov, S.Daroczy, V.A.Karnaukhov, L.A.Petrov, G.M.Ter-Akopyan. Yadern. Fizika, 6, 893 (1967).
2. J.Bacse, D.D.Bogdanov, S.Daroczy, V.A.Karnaukhov, L.A.Petrov. Yadern. Fizika, 7, 1153 (1968).
3. P.Axel. Proc. of Intern. Symp. Nucl. Struct., Dubna (1968).
4. K.Ikeda, S.Fujii, J.Fujita. Phys. Lett., 3, 271 (1963).
   J.Fujita, K.Ikeda. Nucl. Phys., 67, 145 (1965).
   J.Fujita, K.Ikeda, Y.Futami. Proc. of Intern. Conf. on Nucl. Structure, Tokyo, 1967, Suppl. to J. Phys. Soc. Japan, 24, 437 (1968).
5. M.Yamad. Bull. Sci. Eng. Res. Lab. Waseda University, 31-32, 146 (1965).
6. G.P.Keepin, J.Nucl. Energy, 7, 13 (1958).

7. A.C.Papas, T.Tunaal. Arkiv för Fysik., 36, N 51 , 445 (1966).
8. K.Alexander. Preprint P6-3785, Dubna (1968).
9. L.S.Kisslinger, R.A.Sorensen. Mat. Fys. Medd. Dan. Vid. Selsk., 32, 9 (1960).
10. A.Gilbert, A.G.W.Cameron. Can. J. of Physics, 43, 1446 (1965).

Received by Publishing Department
on April 14, 1969.

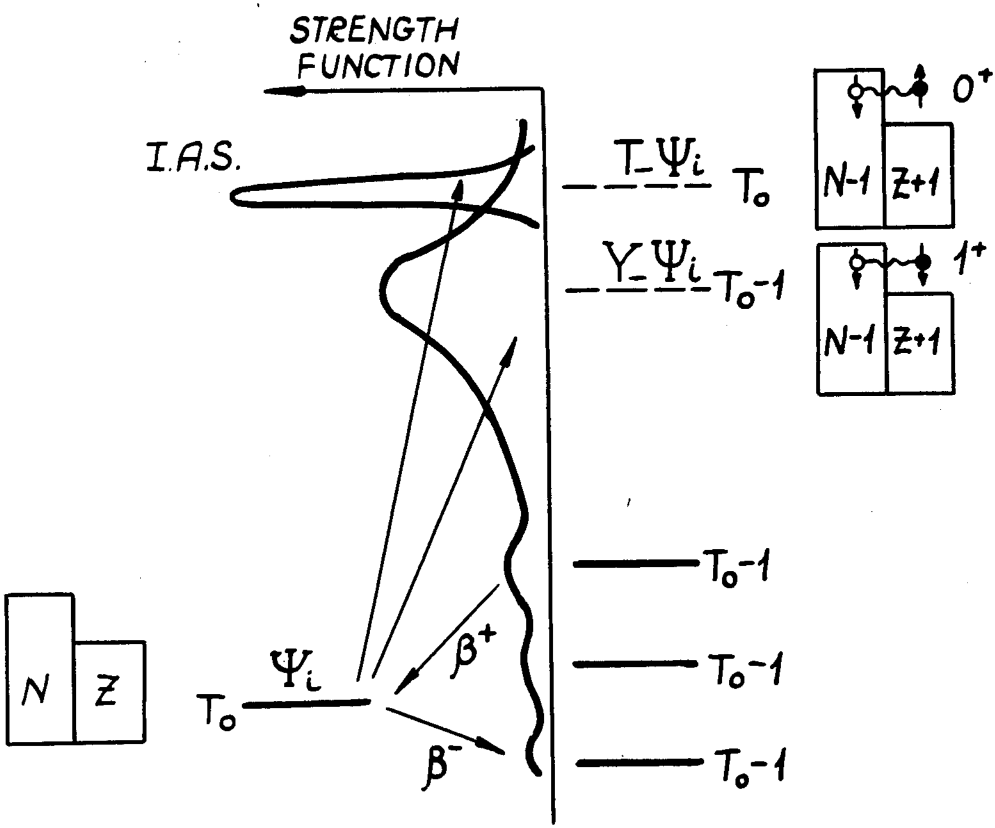

Fig.1. Gross-structure of β -decay strength function /4/.

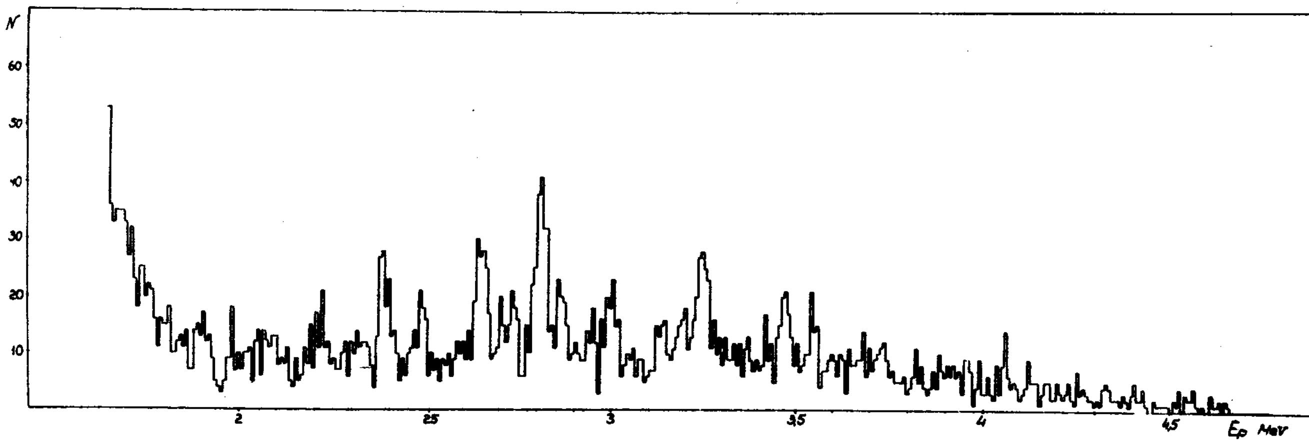

Fig.2. Delayed proton spectrum of ^{111}Te .

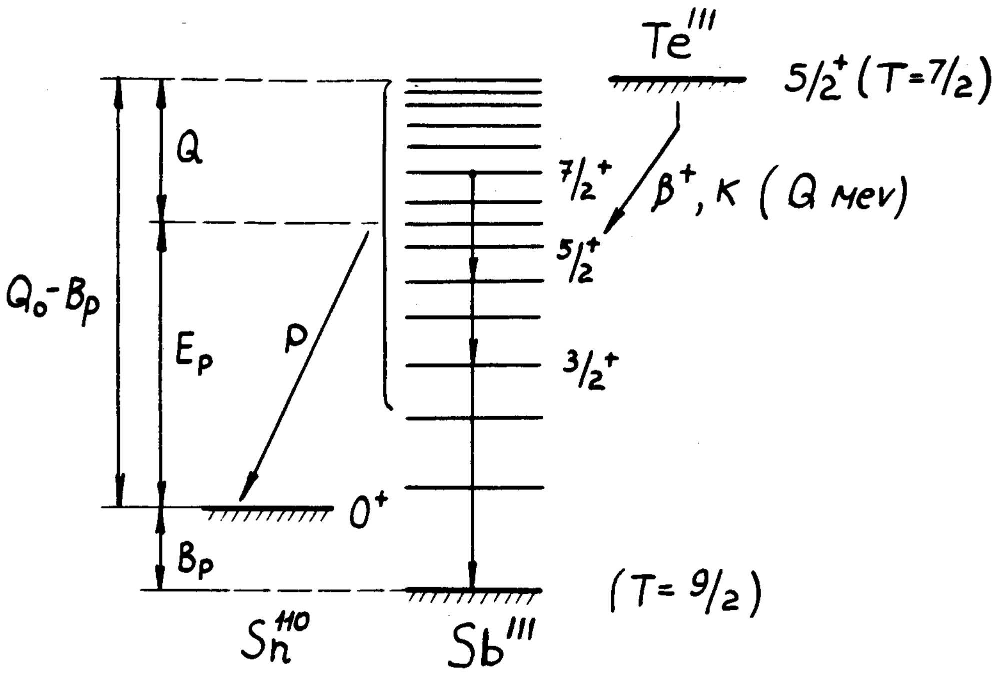

Fig.3. Illustrative decay scheme of ^{111}Te .

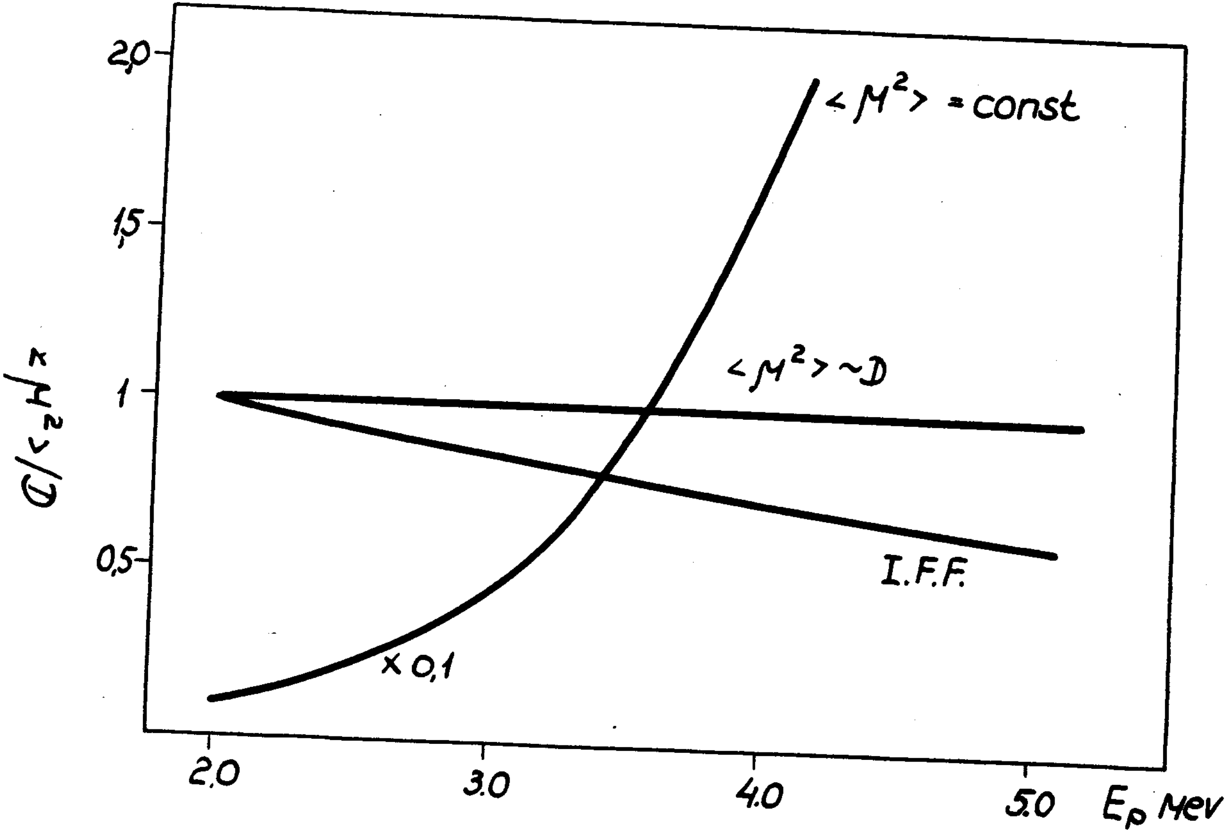

Fig.4. Three assumptions of the strength function $<\mu^2>/D$ used in calculation of the averaged spectra. The daughter nucleus excitation energy E^* is expressed via proton energy: $E^* = B_p + E_p$.

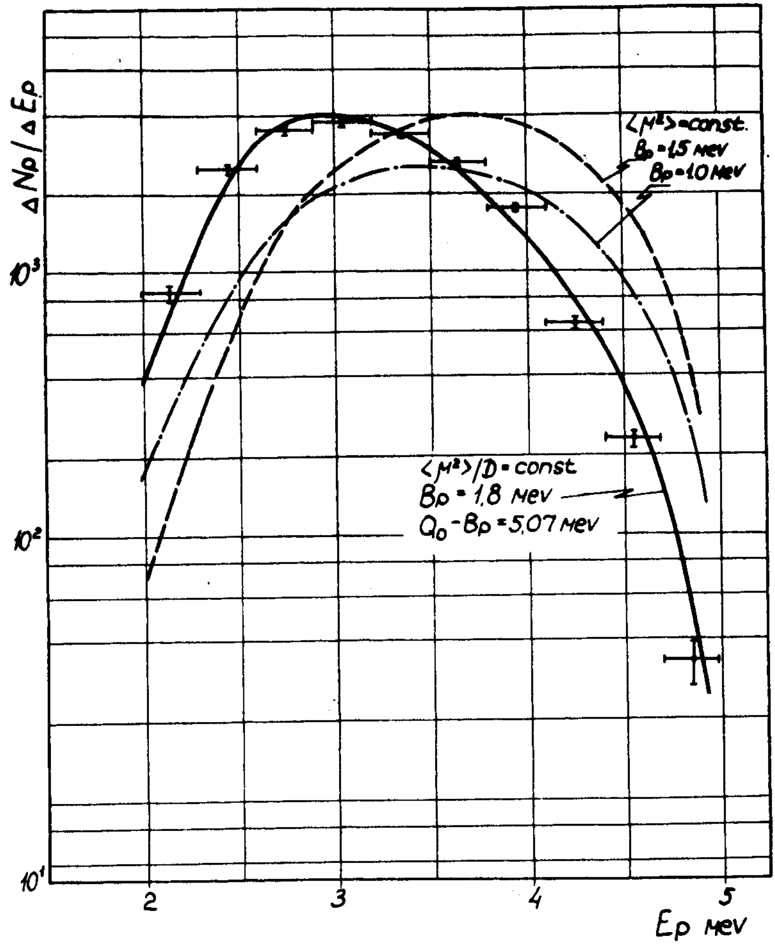

Fig.5. The comparison of the averaged spectrum of delayed protons. of ^{111}Te to the calculated one. The solid line corresponds to the assumption $<\mu^2>/D=$ const., $B_p=1.8$ MeV. The dotted line presents calculation under condition that $<\mu^2>=$ const.

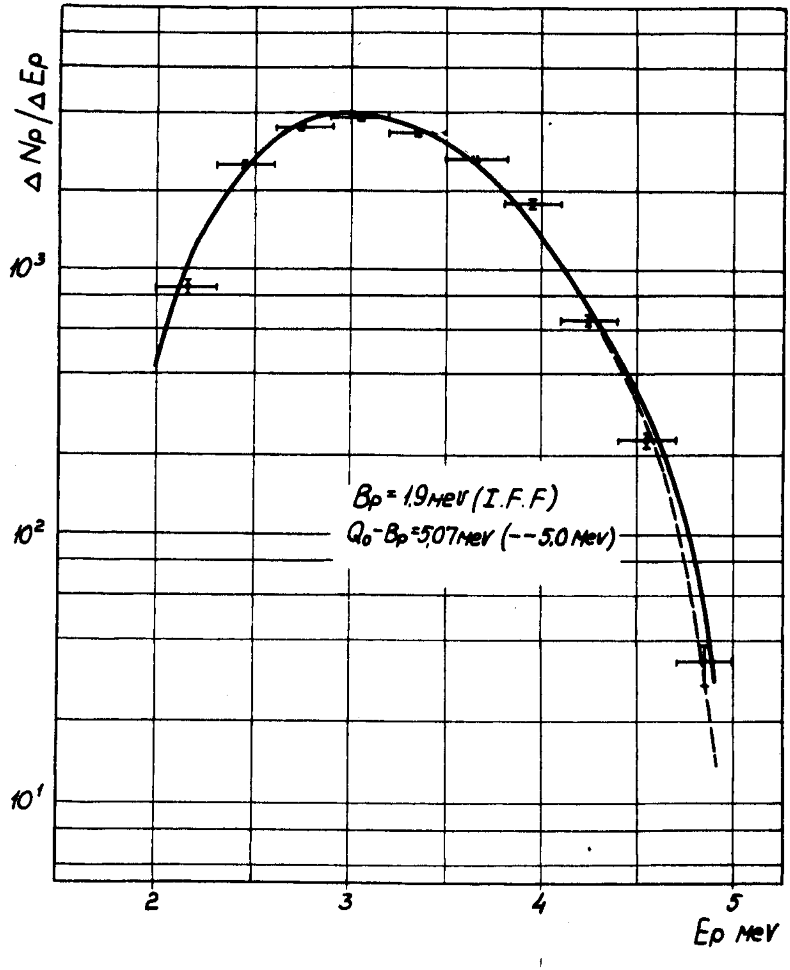

Fig.6. The comparison of the averaged spectrum to the calculated one on the assumption of a giant resonance shape of the Gamow-Teller strength function.

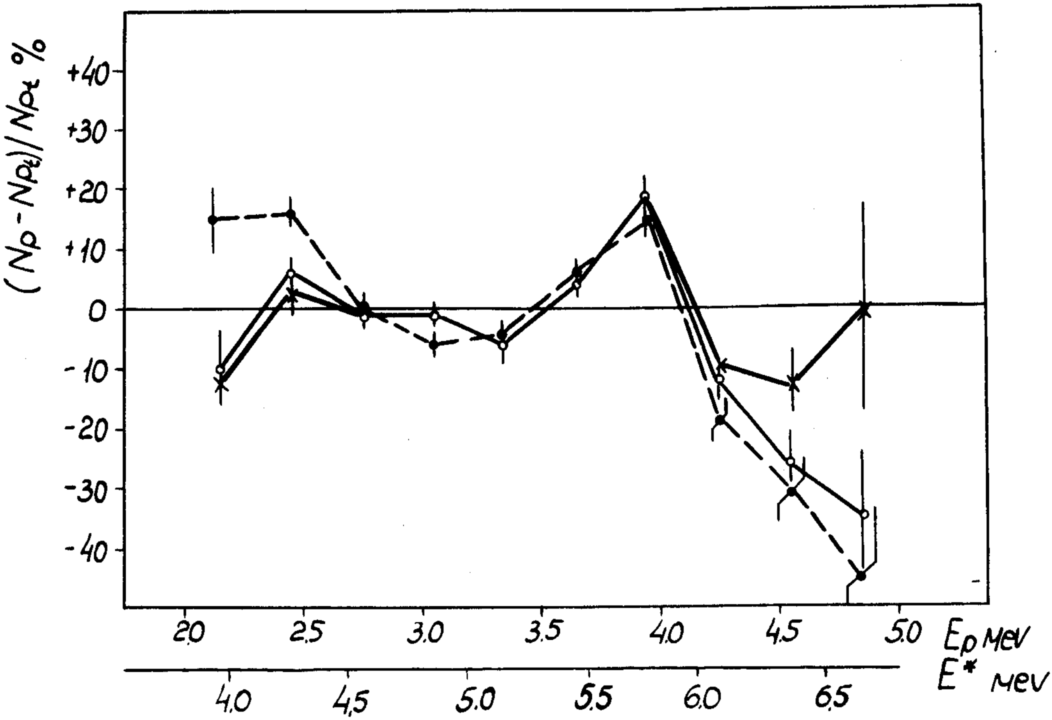

Fig.7. The comparison of the calculated shape of the averaged proton spectrum to the experimental one: ⧫ is the calculation on the assumption that $\langle\mu^2\rangle/D$ = const., $Q_0 - B_p$ = 5.07 MeV; 0 - is the calculation on the assumption of the resonance shape of the strength function, $Q_0 - B_p$ = 5.07 MeV, x -is the same, but $Q_0 - B_p$ = 5.0 MeV.

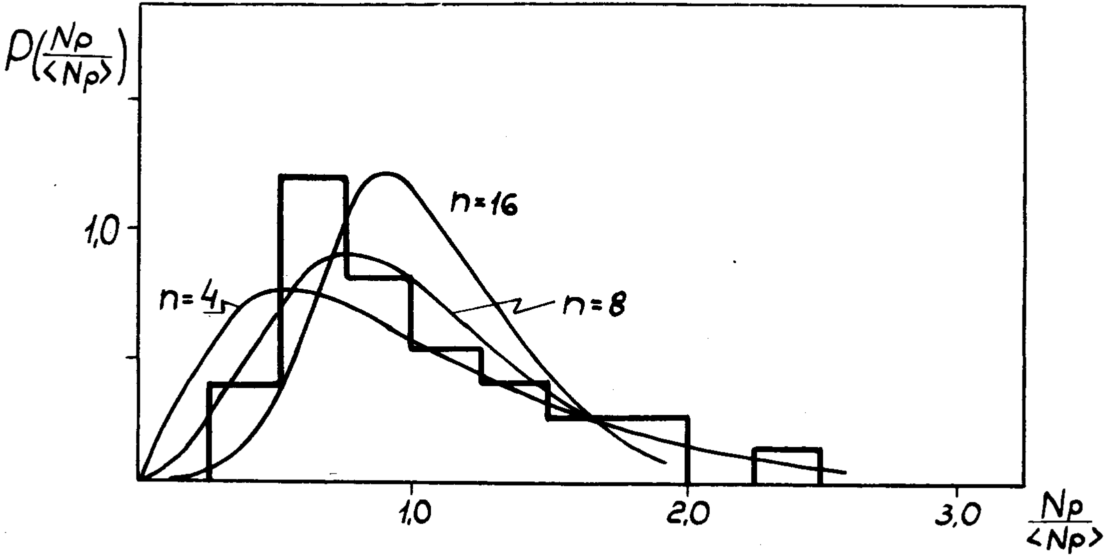

Fig.8. The probability distribution of the relative intensity of groups in the proton spectrum shown in Fig.2. The smooth lines show χ^2 -distribution with the number of degrees of freedom "n" indicated in Fig.