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COMPUTATION OF COHOMOLOGY OF LIE SUPERALGEBRAS OF HAMILTONIAN VECTOR FIELDS

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Вычисление когомологий супералгебр Ли гамильтоновых векторных полей

Мы представляем результаты вычисления когомологий некоторых (супер) алгебр Ли гамильтоновых векторных полей и родственных алгебр. К настоящему времени полные кольца когомологий этих алгебр неизвестны даже для векторных полей в малых размерностях. Частичные «экспериментальные» результаты могут дать некоторые подсказки для полного решения рассматриваемых задач. Вычисления были проведены с помощью недавно написанной программы на языке C.

Некоторые из приведенных результатов являются новыми.

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Kornyak V.V. E5-99-95
Computation of Cohomology of Lie Superalgebras of Hamiltonian Vector Fields

We present the results of computation of cohomology for some Lie (super) algebras of Hamiltonian vector fields and related algebras. At present, the full cohomology rings for these algebras are not known even for the low dimensional vector fields. The partial «experimental» results may give some hints for solution of the whole problem. The computations have been carried out with the help of recently written program in C language. Some of the presented results are new.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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1 Introduction and Basic Definitions

There are many applications of the Lie (super)algebra cohomology in mathematics: characteristic classes of foliations; invariant differential operators; MacDonald-type combinatorial identities, etc. (see [1] for details). Moreover, the cohomology is widely used in mathematical and theoretical physics: construction of the central extensions and deformations for Lie superalgebras; construction of supergravity equations for N-extended Minkowski superspaces and search for possible models for these superspaces; study of stability for nonholonomic systems like ballbearings, gyroscopes, electro-mechanical devices, waves in plasma, etc.; description of an analogue of the curvature tensor for nonlinear nonholonomic constraints [2]; new methods for the study of integrability of dynamical systems; construction of the so-called *higher order Lie algebras* [3] which allow in turn to construct the *Nambu mechanics* generalizing the ordinary Hamiltonian mechanics [4]; construction of possible invariant effective actions of Wess-Zumino-Witten type and the study of anomalies [5]. In [6] the *cohomological field theories* have been defined. This work is an example of application of physical ideas to problems in pure mathematics, in particular, to enumerative problems of algebraic geometry.

General definitions and properties of cohomology of Lie algebras and superalgebras are described in [1]. Let us recall briefly some basic definitions.

A *Lie superalgebra* is a $\mathbf{Z}_2$-graded algebra over a commutative ring K with a unit:

$$L = L_{\bar{0}} \oplus L_{\bar{1}},\ u \in L_\alpha,\ v \in L_\beta,\ \alpha, \beta \in \mathbf{Z}_2 = \{\bar{0}, \bar{1}\} \Longrightarrow [u, v] \in L_{\alpha+\beta}$$

The elements of $L_{\bar{0}}$ and $L_{\bar{1}}$ are called *even* and *odd*, respectively. We shall assume K is one of the fields $\mathbf{C}$ or $\mathbf{R}$. By definition, the *Lie product* (shortly, *bracket*) $[\cdot, \cdot]$ satisfies the following axioms

$$\begin{aligned} [u, v] &= -(-1)^{p(u)p(v)}[v, u], && \text{skew-symmetry,} \\ [u, [v, w]] &= [[u, v], w] + (-1)^{p(u)p(v)}[v, [u, w]], && \text{Jacobi identity,} \end{aligned}$$

where $p(a)$ is the parity of element $a \in L_{p(a)}$.

A *module* over a Lie superalgebra A is a vector space M (over the same field K) with a mapping $A \times M \to M$, such that $[a_1, a_2]m = a_1(a_2 m) - (-1)^{p(a_1)p(a_2)} a_2(a_1 m)$, where $a_1, a_2 \in A$, $m \in M$. The most important for our purposes are *trivial* (M is an arbitrary vector space, e.g.,$M = K$; $am = 0$), *adjoint* ($M = A$; $am = [a, m]$) and *coadjoint* ($M = A'$; $am = \{a, m\}$ is coadjoint action) modules.

A *cochain complex* is a sequence of linear spaces C^k with linear mappings d^k

$$0 \to C^0 \xrightarrow{d^0} \cdots \xrightarrow{d^{k-2}} C^{k-1} \xrightarrow{d^{k-1}} C^k \xrightarrow{d^k} C^{k+1} \xrightarrow{d^{k+1}} \cdots, \tag{1}$$

where the linear space $C^k = C^k(A; M)$ is a super skew-symmetric k-linear mapping $A \times \cdots \times A \to M$, $C^0 = M$ by definition. The super skew-symmetry means symmetry w.r.t. transpositions of odd adjacent elements of A and antisymmetry for all other transpositions of adjacent elements. Elements of C^k are called *cochains*.

The linear mapping d^k (or, briefly, d) is called the *differential* and satisfies the following property: $d^k \circ d^{k-1} = 0$ (or $d^2 = 0$).

The cochains mapped into zero by the differential are called the *cocycles*, i.e., the space of cocycles is

$$Z^k = \mathrm{Ker}\ d^k = \{C^k \mid dC^k = 0\}.$$

The cochains which can be represented as differentials of other cochains are called the *coboundaries*, i.e., the space of coboundaries is

$$B^k = \mathrm{Im}\ d^{k-1} = \{C^k \mid C^k = dC^{k-1}\}.$$

Any coboundary is obviously a cocycle.

The non-trivial cocycles, i.e., those which are not coboundaries, form the *cohomology*. In other words, the cohomology is the quotient space

$$H^k(A; M) = Z^k/B^k.$$

The explicit form of the differential for a Lie superalgebra is

$$\begin{aligned}
dC(e_0, \ldots, e_q; O_{q+1}, \ldots, O_k) \ &= \\
\sum_{i<j}^{q}(-1)^j C(e_0, \ldots, e_{i-1}, [e_i, e_j], \ldots, \widehat{e}_j, \ldots, e_q; O_{q+1}, \ldots, O_k) &+ \\
(-1)^{q+1} \sum_{i=0}^{q} \sum_{j=q+1}^{k} C(e_0, \ldots, e_{i-1}, [e_i, O_j], \ldots, e_q; O_{q+1}, \ldots, \widehat{O}_j, \ldots, O_k) &+ \\
(-1)^{i+1} \sum_{i=q+1}^{k-1} \sum_{j=q+2}^{k} C(e_0, \ldots, e_q; O_{q+1}, \ldots, O_{i-1}, [O_i, O_j], \ldots, \widehat{O}_j, \ldots, O_k)& \\
+ \sum_{i=0}^{q}(-1)^{i+1} e_i C(e_0, \ldots, \widehat{e}_i, \ldots, e_q; O_{q+1}, \ldots, O_k)& \\
+ (-1)^q \sum_{i=q+1}^{k} O_i C(e_0, \ldots, e_q; O_{q+1}, \ldots, \widehat{O}_i, \ldots, O_k)&.
\end{aligned}$$

Here e_i and O_i are even and odd elements of the algebra, respectively, and the hat "^" marks the omitted elements.

Here are some properties and statements we use in the sequel.

An algebra and a module are called *graded* if they can be presented as sums of homogeneous components in a way compatible with the algebra bracket and the action of the algebra on the module:

$$A = \oplus_{g \in G} A_g,\ M = \oplus_{g \in G} M_g,\ [A_{g_1}, A_{g_2}] \subset A_{g_1+g_2},\ A_{g_1} M_{g_2} \subset M_{g_1+g_2},$$

where G is some abelian (semi)group. We assume $G = \mathbf{Z}$ in this paper. To avoid confusion, we use in the sequel the terms *grade* and *degree* for element of G and number of cochain arguments, respectively. The grading in the algebra and module induces a grading on cochains and, hence, in the cohomology:

$$C^*(A; M) = \oplus_{g \in G} C^*_g(A; M), \quad H^*(A; M) = \oplus_{g \in G} H^*_g(A; M).$$

This property allows one to compute the cohomology separately for different homogeneous components; this is especially useful when the homogeneous components are finite–dimensional.

If there is an element $a_0 \in A$, such that eigenvectors of the operator $a \mapsto [a_0, a]$ form a (topological) basis of algebra A, then $H^*(A) \simeq H^*_0(A)$. In other words, all the non-trivial cocycles of the cohomology in the trivial module lie in the zero grade component. The element a_0 is called an *internal grading element*. If also eigenvectors of the operator $m \mapsto a_0 m$ form a topological basis of module M, then the same statement holds for the cohomology in the module M: $H^*(A; M) \simeq H^*_0(A; M)$.

In the case of trivial module, the exterior multiplication of cochains provides the cohomology with a structure of graded ring. There are also another multiplicative structures in cohomology, but we shall not use them in this work.

2 Outline of Algorithm and Its Implementation

To compute the cohomology one needs to solve the equation

$$dC^k = 0, \qquad (2)$$

and throw away those solutions of (2) which can be expressed in the form

$$C^k = dC^{k-1}.$$

In some exceptional cases it is possible to solve equation (2) in closed form. Often, in the case of Lie superalgebras of vector fields, determining equation (2) is a system of linear homogeneous functional equations with integer arguments. Unfortunately there is no general method for solving such systems in closed form. Hence, we need to carry out the corresponding computation "numerically". There are several packages for computing cohomology of Lie algebras and superalgebras written in *Reduce* [8], [9] and *Mathematica* [2]. Some new results were obtained completely or partially with the help of these packages. However, these packages, being based on general purpose computer algebra systems, appeared to be too inefficient for large real problems. In view of this, we wrote the program in C language [10].

The C code, of total length near 14000 lines, contains about 300 functions realizing top level algorithms, simplification of indexed objects, working with Grassmannian objects, exterior calculus, linear algebra, substitutions, list processing, input and output, etc. As internal structures we use 8 types of lists for different objects. We represent Grassmann monomials by integer numbers using one-to-one correspondence between (binary codes of) non-negative integers and Grassmann monomials.

This representation allows one efficiently to implement the operations with Grassmann monomials by means of the basic computer commands.

The program performs sequentially the following steps:

1. *Reading input information.*

2. *Constructing a basis* for the algebra. The basis can be read from the input file; otherwise the program constructs it from the definition of the algebra. Non-trivial computations at this step arise only in the case of divergence-free algebras. The basis elements of such algebras should satisfy some conditions. In fact, we should construct the basis elements of a subspace given by a system of linear equations. The task is thereby reduced to some problem of linear algebra combined with shifts of indices. For example, among the divergence-free conditions for the *special Buttin* algebra SB(3) there are the following two equations
$$ia_{ijk;UV} - (k+1)a_{i-1,j,k+1;VW} = 0,$$
$$ia_{ijk;UW} + (j+1)a_{i-1,j+1,k;VW} = 0.$$
Here $a_{ijk;UV},\ldots$ are coefficients at the monomials $p^iq^jr^kUV,\ldots$ in the generating function; p,q,r and U,V,W are even and odd variables, respectively. First of all, we have to shift indices j and k in the second equation to reduce the last terms of both equations to the same multiindices. Then, using some simple tricks of linear algebra, we can easily construct the corresponding basis element
$$E_{ijk} = (k+1)p^iq^jr^kUV - jp^iq^{j-1}r^{k+1}UW + ip^{i+1}q^jr^{k+1}VW.$$

3. *Constructing the commutator table* for the algebra (if this table has not been read from the input file).

4. *Creating the general form* of expressions for coboundaries and determining equations for cocycles.

5. *Transition to a particular grade* in general expressions. At this step expressions for coboundaries take the form $\mathbf{x} = \mathbf{bt}$, equations for cocycles take the form $\mathbf{Zx} = \mathbf{0}$, where vector $\mathbf{x}$ corresponds to C^k, parameter vector $\mathbf{t}$ corresponds to C^{k-1}, matrices $\mathbf{Z}, \mathbf{b}$ correspond to the differential d. All these vector spaces are finite-dimensional for any particular grade.

6. *Computing the quotient space* $H^k(A;M) = Z^k/B^k$. Here cocycle subspace Z^k is given by relations $\mathbf{Zx} = \mathbf{0}$, and coboundary subspace B^k is given parametrically by $\mathbf{x} = \mathbf{bt}$.

   Substeps:

   (a) Eliminate $\mathbf{t}$ from $\mathbf{x} = \mathbf{bt}$ to get equations $\mathbf{Bx} = \mathbf{0}$

   (b) Reduce both relations $\mathbf{Bx} = \mathbf{0}$ and $\mathbf{Zx} = \mathbf{0}$ to the canonical form by Gauss elimination. If rank$\mathbf{B}$ = rank$\mathbf{Z}$, then there is no non-trivial cocycle; otherwise go to Substep (c).

   (c) Set $\mathbf{Bx} = \mathbf{y}$ and substitute these relations into $\mathbf{Zx} = \mathbf{0}$ to get relations $\mathbf{Ay} = \mathbf{0}$. The *parametric* (non-leading) y's of the last relations are non-trivial cocycles; that is, they form a basis of the cohomology.

   In fact, the above procedure is based on the relation for quotient spaces
$$Z/B = \frac{Y/B}{Y/Z},$$
   where Y is an artificially introduced space combining the above x's and y's.

7. *Output the non-trivial cocycles.*

3 Hamiltonian Vector Fields and Related Algebras

To define the formal vector fields on the supermanifold of the superdimension $(2n|m)$ we consider the sets of even $p_1,\ldots,p_n,q_1,\ldots,q_n$; and odd (called also *Grassmann*) $U_1,\ldots,U_m$ variables, and formal power series $f,g,\ldots$, in these variables. These power series are called *generating functions,* because the vector fields considered in this work can be expressed in terms of the derivatives of $f,g,\ldots$ The (super)commutator of vector fields induces the bracket on generating functions. The Lie superalgebra of *Poisson* vector fields $Po(2n|m)$ is a set of generating functions with the bracket
$$\{f,g\} = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} - \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}\right) - (-1)^{p(f)}\sum_{k=1}^{m}\frac{\partial f}{\partial U_k}\frac{\partial g}{\partial U_k}.$$
The *Hamiltonian* superalgebra is a quotient algebra of the Poisson algebra with respect to its center:
$$H(2n|m) = Po(2n|m)/Z.$$
Observe that $H(0|m)$ is not simple; it has a simple ideal of codimension 1 denoted by $SH(0|m)$ and called *special Hamiltonian superalgebra.* The algebra $SH(0|m)$ contains the subalgebra $O(m)$ and this fact can be used for analysis of the structure of the cohomology ring $H^*(SH(0|m))$. Note that all the algebras depending only on the odd variables are finite-dimensional. It does not mean however that their cohomologies are finite-dimensional too. All the above algebras are graded due to prescribed grading of the variables. The *standard* grading assumes all variables q_i, p_i, U_i have the grade 1.

4 Computations

In the below tables and formulas we use the small $a, b, c, \ldots$ and capital $A, B, C, \ldots$ letters for even and odd cocycles, respectively. The optional superscript and subscript indicate the cochain degree and grade, correspondingly. The letters without indices denote the genuine cocycles, i.e., generating elements of the cohomology ring. Empty position in the tables means the absence of non-trivial cocycles in the given degree and grade. The columns containing only trivial cocycles are omitted. We use the notations p, q for even and U_i for odd variables of the vector field generating functions.

4.1 Special Hamiltonian Superalgebra

Table 1: $H^n_g(\mathrm{SH}(0|4))$

$n \backslash g$	-6	-4	-2	0	2	4	6
1							
2			a	b	c		
3				f			
4		a^2	ab	b^2	bc	c^2	
5			af	bf	cf		
6	a^3	a^2b	ab^2	b^3	ac^2	bc^2	c^3

In Table 1 the cohomology ring $H^*(\mathrm{SH}(0|4))$ is presented. One can see that this ring is generated by four generators a, b, c, f obeying to the relations $ac - b^2 = 0$ and $f^2 = 0$. The explicit form of the generators given by the computer is

$$
\begin{aligned}
a &= C(U_4, U_4) = C(U_1, U_1) = C(U_2, U_2) = C(U_3, U_3),\\
b &= C(U_4, U_1U_2U_3) = C(U_1, U_2U_3U_4) = C(U_2, U_1U_3U_4) = C(U_3, U_1U_2U_4),\\
c &= C(U_2U_3U_4, U_2U_3U_4) = C(U_1U_2U_3, U_1U_2U_3) = C(U_1U_2U_4, U_1U_2U_4)\\
&\quad = C(U_1U_3U_4, U_1U_3U_4),\\
f &= C(U_1U_4, U_2U_4, U_3U_4) + \frac{1}{2}C(U_1U_4, U_1, U_1U_2U_3) +\\
&\quad \frac{1}{2}C(U_1U_4, U_4, U_2U_3U_4) + \frac{1}{2}C(U_2U_4, U_2, U_1U_2U_3) -\\
&\quad \frac{1}{2}C(U_2U_4, U_4, U_1U_3U_4) + \frac{1}{2}C(U_3U_4, U_3, U_1U_2U_3) +\\
&\quad \frac{1}{2}C(U_3U_4, U_4, U_1U_2U_4) = \ldots
\end{aligned}
$$

We have omitted for brevity the equivalent forms of generator f in the last formula.

Note 1. The cohomology of SH(0|4) has been computed for the first time in [11] by D.Fuchs and D.Leites[1] by hand. We present this example here as a rather short illustration demonstrating many features of cohomology ring structure.

Note 2. An interesting approach based on algebraic geometry was suggested by C. Gruson [12]. Her method enabled her to compute cohomology with trivial coefficients of exceptional simple finite dimensional Lie superalgebras. Though remarkably beautiful, it is not universal: it fails to work in various natural and interesting problems, e.g., for PSL(n|n) = SL(n|n)/Z. It is unclear if Gruson's method works for nontrivial coefficients. Observe that PSL(2|2) = SH(0|4), whose cohomology we presented in Table 1.

The structure of $H^*(\mathrm{SH}(0|\mathrm{m}))$ has some peculiarities at $m = 4$. Computations for $m = 3, 5, 6$ revealed two generators: even 2-cocycle

$$a = a^2_{-2} = C(U_1, U_1) = \ldots = C(U_m, U_m)$$

and 3-cocycle

$$
\begin{array}{lllll}
f = f^3_0 & = & C(U_1U_2, U_1U_3, U_2U_3), & f^2 = 0, & m = 3\\
F = F^3_5 & = & C(U_1U_3U_4U_5, U_2U_3U_4U_5, U_3U_4U_5) = \ldots, & F^2 = 0, & m = 5\\
f = f^3_6 & = & C(U_3U_4U_5U_6, U_4U_5U_6, U_1U_2U_4U_5U_6), & f^2 = 0, & m = 6.
\end{array}
$$

As we checked, there are no other generators in the case $m = 3$ up to 16-cocycles.

It would be interesting to look how extensions of the algebra influence on the structure of its cohomology ring. Tables 2 and 3 present the cohomology structure for the superalgebras H(0|4) and Po(0|4). Our consideration of the multiplicative structure for these cohomologies is very preliminary. In fact, there is a need to write a program for multiplication and comparison of cocycles modulo coboundaries because corresponding computations are rather tedious and error prone.

In Tables 2 and 3 the cocycle a^5_{-2} is a linear combination of af and a^2b, f' is cocycle and α is a 2-cochain. The cocycle $e = C(U_1 \ldots U_m)$ satisfies the relation $e^2 = 0$ for m even and is a free generator for m odd.

[1] D.Leites informed us that they missed the cocycle f which was discovered later by A. Shapovalov with the help of the program written by P.Grozman.

Table 2: $H_g^n(\mathrm{H}(0|4))$

$n\backslash g$	-6	-4	-2	0	2	4	6
1					e		
2			a				
3				$f = f' + ae$		$r = e\alpha$	
4		a^2			ef		
5			a^5_{-2}				$s = e\alpha^2$
6	a^3					rf	

Table 3: $H_g^n(\mathrm{Po}(0|4))$

$n\backslash g$	0	2	4	6
1		e		
2	b			
3	f		$r = e\alpha$	
4		$ef, h = b\alpha$		
5	bf			$s = e\alpha^2$
6			$rf, k = b\alpha^2$	

4.2 Algebras H(2|0) and Po(2|0)

The case of supermanifold with even variables leads to infinite-dimensional algebras and is much more difficult for the analysis. There are a few results concerning the cohomology of Lie algebra H(2|0). In [13] it has been proved that $\dim H^2(\mathrm{H}(2|0)) \geq 1, \dim \mathrm{H}^5(\mathrm{H}(2|0)) \geq 1, \dim \mathrm{H}^7(\mathrm{H}(2|0)) \geq 2$ and $\dim H^{10}(\mathrm{H}(2|0)) \geq 1$. In [14] the inequality $\dim H^*(\mathrm{H}(2|0)) \geq 112$ has been obtained. The both works were based on the extraction of some easier to handle subcomplex of the full cohomological complex and application of the computer analysis to this subcomplex. Besides, some facts about $\dim H^q(\mathrm{H}(2\mathrm{k}|0))$ for low degrees $q \leq k$ are known [15].

Every 3-valent graph with oriented vertices, or every oriented rational homology 3-sphere, can be associated with a cohomology class of the Lie algebra H(2k|0) for arbitrary k (see [16, 17, 18]). This cohomology class called *graph cohomology* [2] has been used for construction of *Rozansky-Witten invariants* [19].

Some cocycles from $H^*(\mathrm{Po}(2|0))$ (denoted by p_g^n) and $H^*(\mathrm{H}(2|0))$ (denoted by h_g^n) obtained by the program are presented in Table 4. We carried out the computations up to degree 10 and grade 4.

[2]This cohomology class can be calculated via certain complex constructed from finite graphs.

Table 4: $H_g^n(\mathrm{Po}(2|0))$ and $H_g^n(\mathrm{H}(2|0))$

$n\backslash g$	-4	-2	0
2		h^2_{-2}	
3	p^3_{-4}		
4			
5		p^5_{-2}, h^5_{-2}	
6	p^6_{-4}		
7			p^7_0, h^7_0
8		p^8_{-2}	

4.3 Algebras $\widehat{\mathrm{H}}(2|0)$ and $\widehat{\mathrm{Po}}(2|0)$

If we add the grading element G to the algebra then the non-trivial cocycles lie in zero grade only. In this case the space of cochains is finite-dimensional and we can compute the full cohomology. Thus, let's consider the algebras $\widehat{\mathrm{Po}}(2|0) = \mathrm{Po}(2|0) \oplus \mathrm{Span}(\mathrm{G})$ and $\widehat{\mathrm{H}}(2|0) = \mathrm{H}(2|0) \oplus \mathrm{Span}(\mathrm{G})$. The cohomologies $H^*(\widehat{\mathrm{Po}}(2|0))$ and $H^*(\widehat{\mathrm{H}}(2|0))$ are Grassmann algebras of the superdimension $(2|2)$. These algebras are generated by unit[3], two cocycles $a_0^1 = C(G)$ and a_0^7 and contain also 8-cocycle $a_0^1 a_0^7$. For the case of Hamiltonian algebra the explicit form of a_0^7 is

$$a_0^7 = C(q, p, q^2, pq, p^2, q^3, p^3) - 3C(q, p, q^2, pq, p^2, pq^2, p^2 q).$$

In the Poisson case the expression for a_0^7 is much longer.

5 Conclusion

The computation of cohomology is a typical problem with the combinatorial explosion. Nevertheless, some results can be obtained with the help of computer having an efficient enough program. On the other hand, the physicists are interested mainly in the second cohomologies describing the central extensions and deformations. Such cohomologies can be computed rather easily even for large algebras. Some essential possibilities remain for increasing the efficiency of the program. Besides, it would be useful also to write a separate program for investigating the multiplicative structure of cohomology ring.

[3]Taking into account the initial part of cochain complex (1) we can consider 1 formally as a "non-trivial" zero-cocycle.

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