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A.L.Baranovski

DESIGN OF SELF-OSCILLATING SYSTEM WITH CONTROLLED CHAOTIC BEHAVIOR

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The development of knowledge about the nonlinear phenomena and complex systems naturally has come to a consideration a general problems of the theory of control by oscillations and chaos and inverse problems of chaotic dynamics of complex systems closely connected with it. Recently that was expressed in growing interest to such problems from both the theorists and the applied researchers.

This work is just thought to the author to demonstrate practical appropriateness in theoretical consideration of inverse problems for breakable dynamic systems. An example of a third order system on general theoretical base [1] is used for this purpose.

Let the required stochastic solution of a synthesized dynamic system look like the sum rectangular and cosine impulse processes, i.e.

$$x(t) = z_1(t) + z_2(t),$$

$$z_1(t) = \sum_{k=1}^{\infty} A_1^{(k)} \Theta_{(k-1)T,kT}(t), \qquad (1)$$

$$z_2(t) = \sum_{k=1}^{\infty} A_2^{(k)} \cos\frac{\pi}{T}(t-(k-1)T)\Theta_{(k-1)T,kT}(t)$$

Being coordinated with the results stated in [1], we establish a distribution of roots of the characteristic polynomial $D(\tilde{\lambda}): \tilde{\lambda}_1 = 0, \tilde{\lambda}_2 = j\frac{\pi}{T}, \tilde{\lambda}_3 = -j\frac{\pi}{T}$ (k = 3).

At known $\tilde{\lambda}_i$, i=1,2, using the Vietta formulas [1, formula (11)] constant coefficients $\gamma_0 = 0, \gamma_1 = \frac{\pi}{T}, \gamma_2 = 0$ are uniquely defined. These coefficients identify a system of differential equations with impulse action [1, formula (18)]. Stochastic decision of the system is x (t) of kind (1) at the following initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$, $\ddot{x}(0) = \ddot{x}_0$.

However, the synthesized system [1, formula (18)] is not autonomous that as is known, complicates its practical realization. Therefore we shall put a problem of design of the breakable dynamic system simulating the required random process x (t) (1).

Let us write the nonautonomous equation in the form:

$$\dot{\bar{x}} = F \cdot \bar{x} + \sum_{k=1}^{\infty} \bar{\delta}_k \delta(t-kT). \qquad (2)$$

- The initial conditions define an arbitrary constant in the general decision $x(t) = c_1 + c_2 \cos\gamma_1 t + c_3 \sin\gamma_1 t$ of the homogeneous linear differential equation $\dot{\bar{x}} = F\bar{x}$ obtained from (2) as $\left.\sum_{k=1}^{\infty} \bar{\delta}_k \delta(t-kT)\right|_{t=0} = 0$.

Let us denote $c_1 = \tilde{A}_1^{(1)}$, $c_2 = \tilde{A}_2^{(1)}$, $c_3 = \tilde{A}_3^{(1)}$. According to the accepted description (1) in the required stochastic decision x (t) the impulses of the sine form are away. So, it is necessary to put $\dot{x}_0 = 0$ in order to make $\tilde{A}_3^{(1)}$ be equal to zero. The movement on a trajectory $x(t) = \tilde{A}_1^{(1)} + \tilde{A}_2^{(1)} \cos\gamma_1 t$ comes true according to (2) up to a moment t=T-0 when the *δ-action* moves system (2) from a condition $\bar{x}(T-0)$ to x (T + 0) on a rule [1, formula (15)], i.e. x (T + 0) = $\tilde{A}_1^{(1)} - \tilde{A}_2^{(1)}$, $\dot{x}(T+0) = \dot{x}(T-0) = 0$, $\ddot{x}(T+0) = \gamma_1^2 \tilde{A}_2^{(1)} + \delta_1$. New initial conditions define coefficients $\tilde{A}_2^{(1)}$, $\tilde{A}_2^{(2)}$ in continuous decision: $x(t) = \tilde{A}_1^{(2)} + \tilde{A}_2^{(2)} \cos\gamma_1 t$ of system (2) at t> T and t < 2T. And so on solution x (t) is restored. Let us consider its k-th fragment $x(t) = \tilde{A}_1^{(k)} + \tilde{A}_2^{(k)} \cos\gamma_1 t = x_k(t), (k-1)T<t<kT$. (It is clear that $\tilde{A}_1^{(k)} = A_1^{(k)}$, $\tilde{A}_2^{(k)} = A_2^{(k)}(-1)^{k-1}$). Denoting $x(kT) = \eta_{k-1}$, we calculate values: $x_k(kT) = \tilde{A}_1^{(k)} + (-1)^{k-1}\tilde{A}_2^{(k)} = \eta_{k-1}$. The amplitudes are expressed as follows: $\tilde{A}_1^{(k)} = (\eta_k + \eta_{k-1})/2$, $\tilde{A}_2^{(k)} = (-1)^k(\eta_k - \eta_{k-1})/2$. Let the right end η_k is some function of left one: $\eta_k = \varphi(\eta_{k-1})$ where φ is the map with nonsingular probabilistic measure. Denoting $\psi(\eta) = \frac{1}{2}(\varphi(\eta) + \eta)$ it is easy to find that $\tilde{A}_1^{(k)} = \psi(\eta_{k-1})$ and the value of impulse action, for example, at a moment t=kT: $\delta_k = \ddot{x}_{k+1}(kT+0) - \ddot{x}_k(kT-0) =$

$$= (-1)^k \gamma_1^2 (\tilde{A}_2^{(k)} - A_2^{(k+1)}) = \gamma_1^2 (\tilde{A}_1^{(k+1)} - A_1^{(k)}) = \gamma_1^2 (\psi(x) - \psi(x(t-T)))\Big|_{t=kT}.$$

All fixed moments t=kT (k=1,2,...) of impacts in the system are characterized by a common property: $\dot{x}(kT) = 0, \forall k$. Thus the system (2) can be rewritten in a more compact form:

$$\ddot{x} + \gamma_1^2 x = \gamma_1^2 \delta(\dot{x})(\psi(x) - \psi(x(t-T))).$$

Let us integrate the equation from 0 up to $t \in (kT, (k+1)T)$. The left hand side will be rewritten as $\ddot{x} + \gamma_1^2 x$, the right hand one as

$\gamma_1^2 \sum_{i=1}^{k} (\psi(x(iT)) - \psi(x(i-1)T)) = \gamma_1^2 (\psi(x(kT) - \psi(x(0)))$. The value $\psi(x(kT))$ can be represented in the form of integral $\int_{t-T}^{t} \delta(\dot{x})\psi(x)dt$, whose meaning on an interval kT < t < (k + 1) T remains constant and equal to the (k+1)-th rectangular amplitude. Therefore the final expression of the synthesized auto oscillatory system is as follows

$$\ddot{x} + \gamma_1^2 x = \gamma_1^2 (y - \psi(x_0))$$

$$y = \int_{t-T+0}^{t} \delta(\dot{x})\psi(x)dt. \qquad (3)$$

On a phase plane of variables $(x, \dot{x})$ the solution of the equation (3) looks like an nonclosed curve made of arches of half-circles (see fig. 1) with centers in points with coordinates $(\psi(\eta_k), 0)$.

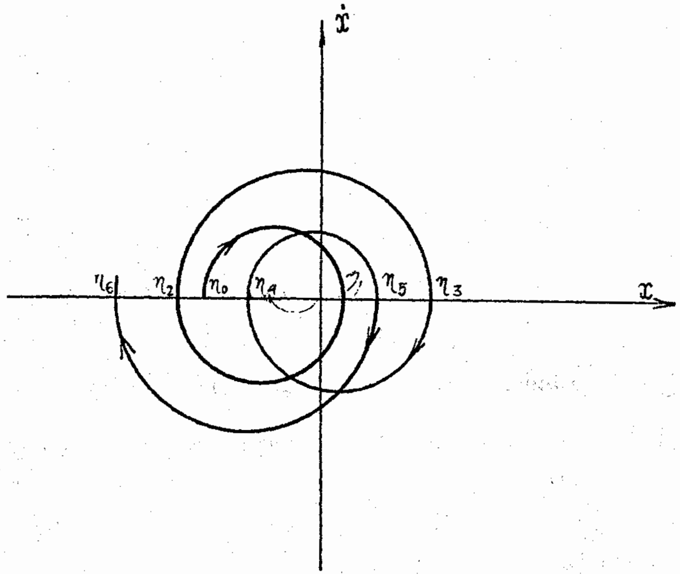

Fig.1.

Along with the realization of random process x (t) of kind (1) (the possible realization is shown on fig. 2) system (3) forms y (t) - sequence of adjoining to each other rectangular impulses of fixed duration T but with random amplitudes distributed on the law $p_A(x) = \int \delta(\psi(z) - x) p_\eta(z) dz$ (on fig. 2 y (t) is shown by a dotted line). Obviously, $p_{y(t)}(x,t) = p_A(x), \forall t$.

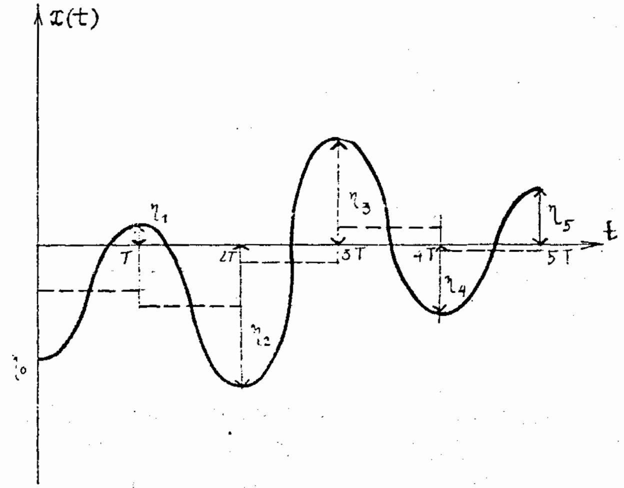

Fig.2.

The stationary probability density of the stochastic solution obtained by observation of indefinitely long realization x (t) with the use of the Rozenblatt and Parsen estimation [2] takes the following form:

$$p_{x(t)}(y) = \frac{1}{\pi} \int_{\{z<y<\varphi(z)\}} \frac{p_\eta(z)dz}{\sqrt{-(y-\varphi(z))(y-z)}} + \frac{1}{\pi} \int_{\{\varphi(z)<y<z\}} \frac{p_\eta(z)dz}{\sqrt{-(y-\varphi(z))(y-z)}} \quad (4)$$

Thus, nominating the φ it is possible analytically to define a probability density of the solution x (t). For example,

$$\varphi(\eta) = \begin{cases} 2\eta + 1, & -1 < \eta < 0 \\ 2\eta - 1, & 0 < \eta < 1. \end{cases} \quad (5)$$

The map φ belongs to a class of piece-wise linear maps with uniform probabilistic measure. Therefore, the iterations η_k uniformly cover (-1,1),i.e. $p_\eta(x)=1/2$, $|x|<1$. . Substituting these data in (4), we obtain

$p_{x(t)}(z) = \frac{1}{2\pi\sqrt{2}}\left(\frac{3\pi}{2} - \arcsin\frac{3|z|-1}{|z|+1}\right)$, $|z| < 1$ (fig. 3).

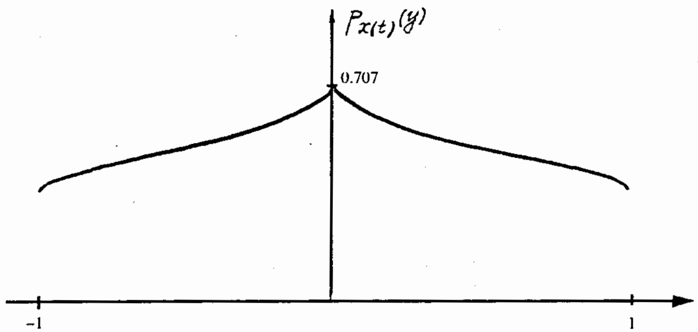

Fig.3.

Let us define a power spectrum of the stochastic solution x (t) by use [the formula (7), [1]. At the same time let us limit the set of maps φ by ones with an exponential correlation function of values η_k: $K_\eta(j) = M(\eta_k, \eta_{k+j}) = \delta_\eta^2 R_1^j$.

.As $A_1^{(k)} = \frac{1}{2}(\eta_k + \eta_{k-1})$, $A_2^{(k)} = (\eta_{k-1} - \eta_k)/2$, then

$$M(A_1^{(k)} A_2^{(p)}) = \frac{1}{4}(K_\eta(k-p+1) - K_\eta(k-p-1)),$$

$$K_{A_1}(p) = \frac{1}{2}K_\eta(p) + \frac{1}{4}(K_\eta(p-1) + K_\eta(p+1)), \qquad (6)$$

$$K_{A_2}(p) = \frac{1}{2}K_\eta(p) + \frac{1}{4}(K_\eta(p-1) + K_\eta(p+1))$$

Substituting the given ratio in [1, formula (7)], we find

$$S_{x(t)}(\omega) = S_{z_1(t)}(\omega) + S_{z_2(t)}(\omega) - \frac{2T\delta_\eta^2(1-R_1^2)\sin^2\omega T}{(1-2R_1\cos\omega T + R_1^2)((\omega T)^2 - \pi^2)},$$

$$S_{z_1(t)}(\omega) = T\delta_\eta^2 \left|S_{g_1(t)}(\omega T)\right|^2 \left(\frac{(1-R_1)^2 + (1+R_1)^2(\cos\omega T - R_1)}{1-2R_1\cos\omega T + R_1^2} + R_1\right), \qquad (7)$$

$$S_{z_2(t)}(\omega) = T\delta_\eta^2 \left|S_{g_2(t)}(\omega T)\right|^2 \left(\frac{(1-R_1)^2 + (1+R_1)^2(\cos\omega T - R_1)}{1-2R_1\cos\omega T + R_1^2} - R_1\right),$$

where $S_{g_i(t)}$ (w), i=1,2 are Fourier transformations of rectangular and cosine forms of impulses.

Power spectrum of the solution when φ looks like (5) is easily calculated under the formula (7) since R_1=1/2.

By change of map φ is also possible to carry out the power spectrum control. As power spectrum is the function of a unique parameter R_1, the control reduces to purposeful change of R_1.

Let us take the white noise as an example of required random process. Therefore, we concretize a problem as follows: to find such a value of parameter R_1 (and after φ) and a maximal value of frequency ω_{*so} the distance function $\left|S_{x(t)}(\omega_*, R_1) - S_{x(t)}(0)\right|$ is less a given Δ -window of non-uniformity of a spectrum. It follows from the representation that the function of dependence ω_* of R_1 at various Δ on fig. 4, for example, we have optimum $R_1^* \approx -\frac{1}{3}$, $\omega_* \approx \pi$ or $f^* = \frac{1}{2} f_{tact}$.. for a 2 dB window of non-uniformity. Let us tell, for comparison, a band of uniformity of power spectrum for noise of the industrial generator G2-57 is limited by value $f^* = \frac{1}{20} f_{tact}$ *at* Δ =2 dB.

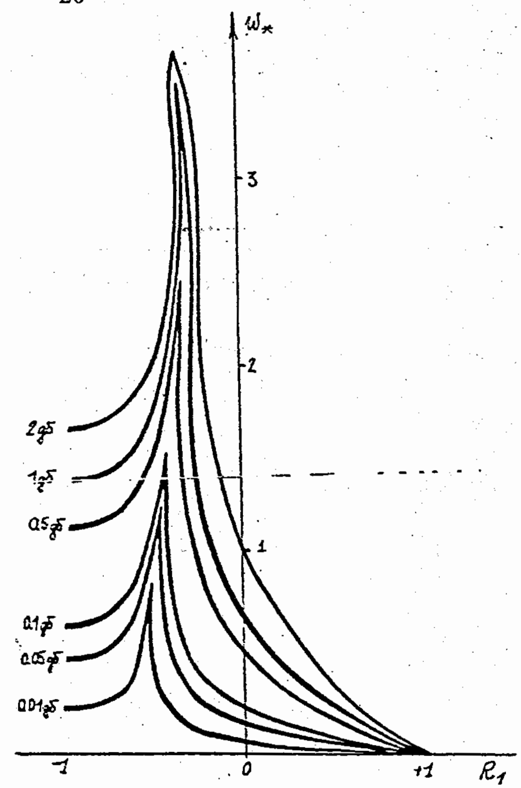

Fig.4

The correlation function $K_\eta(j) = \sigma_\eta^2 \left(-\frac{1}{3}\right)^j$ of the η_k values stipulates the map of the following form:

$$\varphi^*(\eta)=\begin{cases}-3\eta-2, & -1<\eta<-\frac{1}{3}\\ -3\eta, & -\frac{1}{3}\le\eta<\frac{1}{3}\\ -3\eta+2, & \frac{1}{3}\le\eta<1.\end{cases}$$

So, equation (3) including the map φ^* has the stochastic solution x(t) with a flat power spectrum in a maximally wide frequency band (0, $\frac{1}{2}f_{tact.}$).

In course measurements the signals containing simultaneously random and deterministic components with a given attitude are often required. As a rule signals of two generators: generator of noise and generator of sine wave signals should be summarized for this purpose. This problem rather easily can be solved within the framework of the equation (3) by choice of an appropriate map φ, for example, of following kind

$$\varphi_\alpha(\eta)=\begin{cases}2\eta+3-2\alpha, & -1<\eta<-1+\alpha\\ -2\eta-1+2\alpha, & -1+\alpha<\eta<\alpha\\ 2\eta-1-2\alpha, & \alpha<\eta<1.\end{cases}$$

A ratio of deterministic and random components in the spectrum can be controlled by change α .So deterministic component is absent (continuous spectrum) α=0,

spectrum consists of both continuous and discrete parts α=0,5. Let us show it.

Iterations η_k of map φ_α uniformly distributed on (-1,1) and their correlation function look like:

$$K_\eta(j)=\frac{1}{2}\int_{-1}^{1}\varphi_\alpha^{(j)}(x)x\,dx=\frac{1}{2}I_{j-1}(1-2\alpha),\ j\ge 1,\ \varphi_\alpha^{(j)}=\varphi_\alpha\circ\varphi_\alpha\circ...\circ\varphi_\alpha, \qquad (8)$$

where $I_{j-1}(\beta)=-\frac{1}{2}I_{j-2}(\beta)+\frac{1}{2}sign(\dot{\varphi}_\alpha(\beta))I_{j-2}(\varphi_\alpha(\beta)),\ I_0(\beta)=\frac{1}{2}(\beta^2-1),$

β=1-2α.

At α=0,5: $K_\eta(j)=\frac{1}{2}I_{j-1}(0)=-\frac{1}{2}I_{j-2}(0)=...=(-1)^j\cdot\frac{1}{4}$,

$\forall j\ge 1$. The undamped character of the given correlation function is caused by a strict periodicity of alternations of signs ($sign\eta_{k+1}=-sign\eta_k$. Then by calculating (6) and by substituting in (7) we have $S_{x(t)}(\omega)=S_c(\omega)+S_d(\omega)$, where

$$S_c(\omega)=T\sigma_\eta^2\left(\frac{\sin^2\frac{\omega T}{2}}{(\omega T/2)^2}+\frac{(\frac{\omega T}{2})^2\cos^2\frac{\omega T}{2}}{\left(\left(\frac{\pi}{2}\right)^2-\left(\frac{\omega T}{2}\right)^2\right)^2}\right),\ S_d(\omega)=S_c(\omega)\sum_{r=1}^{\infty}d_r\delta(\omega-\frac{2\pi r}{T}).$$

At α=0: $K_\eta(j)=0$, $\forall j\ge1$ and $S_{x(t)}(\omega)=2S_c(\omega)+2T\sigma_\eta^2\frac{\sin^2\omega T}{\pi^2-(\omega T)^2}$ has a continuous component only.

Completely the discrete power spectrum of the solution of the equation (3) is possible at small change $\varphi_{0,5}(\eta)$, for example, as shown in fig. 5 by a dotted line. Strictly sine wave oscillations are generated in this case.

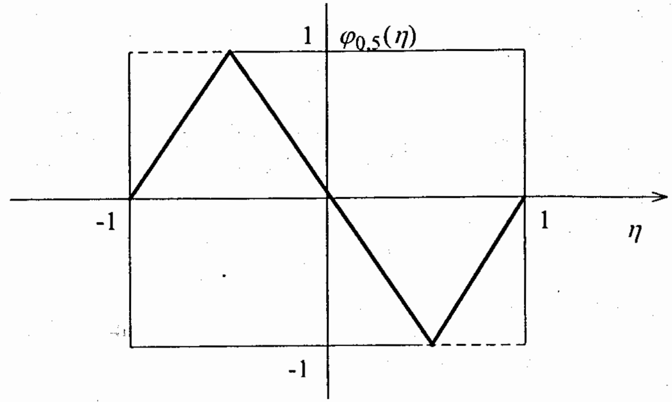

Fig. 5

The synthesized equation (3) can serve as a basis of apparatus realization of the random oscillations generator whose simplest scheme of which is shown on fig. 6. The self-oscillatory system on the inverter and two integrators simulates the left hand part of the differential equation (3) and of pulses former (PF), analog storing device (ASD) and nonlinear converter (NP) simulate the right hand part. The experimental research has confirmed the basic theoretical statements, i.e. an opportunity of a formation of the random signals and stipulated control by their parameters and characteristics.

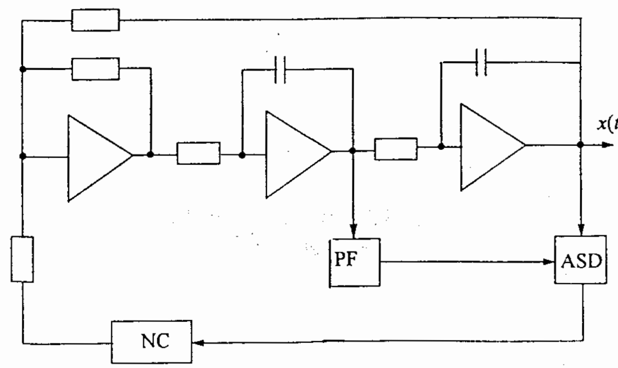

Fig. 6

Conclusion

The self-oscillating system forming a chaotic impulse process is constructed. An expression of the stationary probability density of process as functions of a map prescribing the chaotic character of motions in the system has been analytically obtained. A ratio for a power spectrum of chaos is obtained, control by power spectrum over a wide range: from strictly discrete (case of deterministic harmonic oscillations) up to continuous (a case of advanced chaos in system). has been demonstrated. It was shown that the stipulation of probabilistic measure, at preliminary synthesis of nonlinear map, predetermines a kind of stationary distribution of the system' decision. At the same time the power spectrum of the solution is invariant with respect to a probabilistic measure of the map. It is determined by Fourier images of the impulses forms and correlation function for the map.

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Барановский А.Л.

E5-97-197

Построение автоколебательной системы с управляемым хаотическим поведением

Строится разрывная динамическая система третьего порядка, формирующая хаотические колебания. Даются аналитические выражения их вероятностных характеристик. Предлагается метод управляющего параметра и находится оптимальное значение параметра, при котором обеспечивается равномерность спектральной плотности мощности в максимально широком диапазоне частот.

Дается схемное решение для аппаратурной реализации системы.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Baranovski A.L.

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Design of Self-Oscillating System with Controlled Chaotic Behavior

Third order breakable dynamic system forming chaotic oscillations is built. Analytical expressions for probabilistic characteristics are given. The controlling parameter method is suggested. Optimal value for the parameter is found when the power spectrum is flat in widest frequency range. Simple circuit for apparatus realization is given.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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