ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E5-95-213

I.L.Bogolubsky, A.A.Bogolubskaya

STRING-LIKE SOLITONS IN GAUGED MODELS OF ANISOTROPIC HEISENBERG ANTIFERROMAGNET

Submitted to «Physical Review Letters»

1995

1. Conception of spontaneous symmetry breaking of the Lagrangian is one of the most fruitful ideas of the contemporary theoretical physics. The mechanism of such breaking within the model describing the Higgs scalar field $\varphi_a(x)$ with selfinteraction of the form $V(\varphi) = A^2(\varphi_b\varphi_b - B^2)^2, a, b = 1, 2, ...N$ was studied by Goldstone [1], and in [2] interaction of the complex Higgs field with the gauge Maxwell field was considered (abelian-Higgs (AH) model). The well-known Ginzburg-Landau model of superconductivity [3] can be considered as a nonrelativistic analog of the AH model. Extended solutions to these models in (2+1) dimensions (namely, Nielsen-Olesen strings [4] and Abrikosov vortices [5], respectively), found in stationary ($\partial/\partial t = 0$) case are identical. These solutions (we shall name them ANO strings for brevity) and their numerous analogs, which describe localized energy distributions, are widely discussed in condensed matter physics, cosmology, particle physics [6,7]. Notice however that ANO strings cannot be refered to as solitons because neither Higgs field nor Maxwell one do not attain unique asymptotic value at $|\mathrm{x}| = \infty$.

2. In the present paper we study $2D$ soliton solutions to Lorentz-invariant models supporting non-Goldstone mechanism of symmetry breaking. Consider a unit isovector field $s_a(x)$ having Lagrangian density

$$\mathcal{L} = (\partial_\mu s_a)^2 - V(\mathrm{s}), \quad s_a s_a = 1, \quad V(\mathrm{s}) = 1 - s_3^2,$$

$$\mu = 0, 1, ..., D, \qquad a = 1, 2, 3. \tag{1}$$

(we shall call it the A3-field). It is easily seen that the Lagrangian (1) possesses $U(1) \times Z(2)$ internal symmetry; its vacuum manifold comprises two points on the S^2 sphere: $s_3 = 1$ and $s_3 = -1$ and possesses discrete $Z(2)$ symmetry.

At $D = 1$ the model (1), which proves to be completely integrable generalization of the sine-Gordon equation [8], possesses kink and antikink solutions, which break $Z(2)$ symmetry of the vacuum manifold. Futhermore, it can be shown that $Z(2)$ symmetry is also broken on nonstationary topological solitons of the model (1) at $D = 2, 3$.

Lagrangian (1) can be derived when describing in continuous approximation easy-axis Heisenberg antiferromagnets [9] and ferroelectrics [10] with easy-axis anisotropy; thus, the pattern of the symmetry breaking under discussion is realized in condensed matter physics. Hopefully, the investigation of solitons in $(D+1)$-dimensional field theory models, comprising the A3-field and/or its generalizations will give advantageous results in particle physics as well, in particular, in electroweak interaction theory.

3. Consider $2D$ Lorentz-invariant model which describes minimal interaction of the A3-field with gauge Maxwell field $A_\mu(x)$ ("MA3 model"):

$$\mathcal{L} = (\bar{\mathcal{D}}_\mu \bar{S}^b)(\mathcal{D}_\mu S^b) - V(\mathbf{S}) - \frac{1}{4}F_{\mu\nu}^2, \tag{2}$$

$$\bar{\mathcal{D}}_\mu = \partial_\mu + ieA_\mu, \quad \mathcal{D}_\mu = \partial_\mu - ieA_\mu,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad V(\mathbf{S}) = \beta[1 - s_3^2],$$

$$\bar{S}^b = (s_1 - is_2, s_3), \quad S^b = (s_1 + is_2, s_3), \quad b = 1,2,$$

$$\bar{S}^b S^b = s_1^2 + s_2^2 + s_3^2 = 1,$$

where β, e are coupling constants.

Lagrangian (2) is easily transformed to the form

$$\mathcal{L} = (\partial_\mu s_a)^2 - V(s_a) - \frac{1}{4}F_{\mu\nu}^2 + 2eA_\mu(s_2\partial_\mu s_1 - s_1\partial_\mu s_2) + e^2 A_\mu A_\mu. \tag{3}$$

Note that due to the interaction with the A3-field described by (2), the gauge field becomes massive, and this is the exact result contrary to the case of the Higgs model in which mass of the vector field A_μ is obtained when expanding the Lagrangian density of the model in series in the vicinity of its vacuum manifold.

We begin studing the localized solutions of the (2+1)-dimensional model (3) in the simplest stationary case. Use the hedgehog ansatz for the A3-field

$$s_1 = \frac{x}{R}\sin\theta(R), \quad s_2 = \frac{y}{R}\sin\theta(R), \quad s_3 = \cos\theta(R), \quad R^2 = x^2 + y^2, \tag{4}$$

and look for the vector field solution in the form

$$A_0 = 0, \quad A_1 = A_x = -A_t(R)\frac{y}{R}, \quad A_2 = A_y = A_t(R)\frac{x}{R}. \tag{5}$$

Introducing

$$a(R) = A_t(R)R, \tag{6}$$

and then going over to variables $\alpha(r), r$, given by

$$a = \alpha e^{-1}, \quad R = re^{-1}, \tag{7}$$

we get for stationary Hamiltonian density $\mathcal{H}(r)$:

$$e^{-2}\mathcal{H}(r) = \left(\frac{d\theta}{dr}\right)^2 + \sin^2\theta\left(p + \frac{1}{r} - \frac{2\alpha}{r^2}\right) + \frac{1}{2}\left(\frac{1}{r}\frac{d\alpha}{dr}\right)^2 + \frac{\alpha^2}{r^2}, \tag{8}$$

$$p = \frac{\beta}{e^2}. \tag{9}$$

Calculating $\delta\mathcal{H}/\delta\theta$ and $\delta\mathcal{H}/\delta\alpha$ and setting them equal to zero, we get the set of equations for $\theta(r)$ and $\alpha(r)$:

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} + \sin\theta\cos\theta\left(\frac{2\alpha - 1}{r^2} - p\right) = 0. \tag{10}$$

$$\frac{d^2\alpha}{dr^2} - \frac{1}{r}\frac{d\alpha}{dr} - 2\alpha + 2\sin^2\theta = 0. \tag{11}$$

We shall look for solutions of Eqs. (10),(11) under following boundary conditions:

$$\theta(0) = \pi, \quad \theta(\infty) = 0, \tag{12}$$

$$\alpha(0) = 0, \quad \alpha(\infty) = 0. \tag{13}$$

Notice that Eqs.(4),(12) define the class of mappings $R^2_{comp} \to S^2$, such that $Q_t = 1$, where Q_t is the topological index ("winding number") of localized distributions $s_a(x)$, described by the mappings of this class.

Taking into account boundary conditions at $r = 0$ and expanding Eqs. (10), (11) into series at $r \to 0$, we find

$$\theta(r) = \pi - (C_1 + C_2)r + o(r), \tag{14}$$

$$\alpha(r) = r^2(C_1^2 - \frac{1}{4}C_2^2 r^2) + o(r^4). \tag{15}$$

Choosing appropriate C_1 and C_2 values we find such solutions to Eqs. (10), (11), which satisfy boundary conditions $\theta(\infty) = 0, \alpha(\infty) = 0$ ("shooting method"). Thus we get solution of the boundary value problem (10)-(13).

Numerical studies of these solutions with $Q_t = 1$ have been accomplished for various values of the dimensionless parameter p, given by (9); the most detailed computations have been made on the interval $0 < p < 2$. Soliton solutions are plotted in Fig.1 for various p. It can be seen that the characteristic width of the soliton and maximum values of $\alpha(r)$ and $A_t(R)$ functions decrease with the growth of p. Energy density $\mathcal{H}(r)$ for $Q_t = 1$ solitons has a peak at $r = 0$ and monotonously decreases with the increase of r. The dependence of the net energy $E = 2\pi \int \mathcal{H}(r) r dr$ on p value is presented in Table 1; note that at $p \approx 0.3$ soliton energy $E = E_0 = 8\pi$ (E_0 is the energy value of Belavin-Polyakov localized solutions in $D = 2$ isotropic Heisenberg magnet [11]).

p	E
0.03	21.6437
0.10	23.8876
0.20	24.8351
0.26	25.0589
0.30	25.1328
0.33	25.1479
0.40	25.1600
0.50	25.1833
1.00	25.2449

Table 1

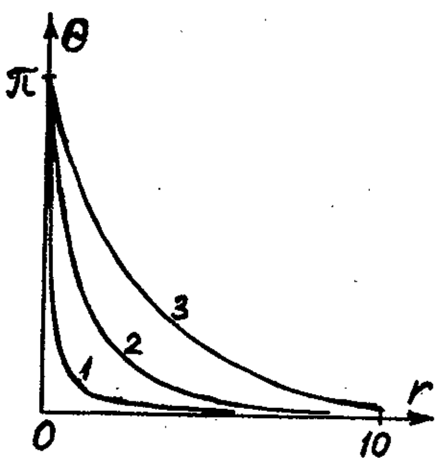

Fig.1a Radial functions $\theta(r)$ of solitons with $Q_t = 1$ in the MA3 model; $1 - p = 0.3$, $2 - p = 0.1$, $3 - p = 0.03$.

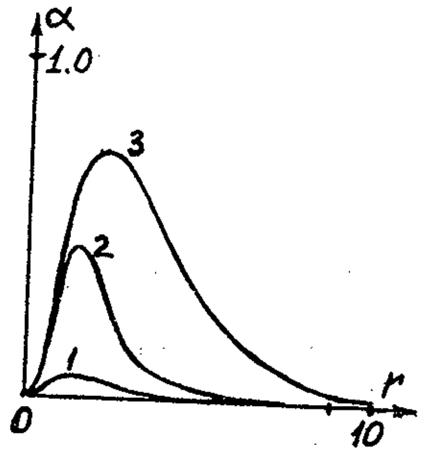

Fig.1b Radial functions $\alpha(r)$ of solitons with $Q_t = 1$ in the MA3 model; $1 - p = 0.3$, $2 - p = 0.1$, $3 - p = 0.03$.

4. Compare soliton solutions found above in the $D = 2$ MA3 model with ANO strings in the AH model. Both solutions represent field energy lumps, which are exponentially localized in space, both solutions describe distributions of the scalar (with respect to Lorentz transformations) field (A3-field or Higgs one) with nonzero topological indices.

Nevertheless, there exist essential distinctions between string-like MA3 solitons and ANO strings, namely:

1) the A3-field and the Maxwell field constituting MA3 solitons approach unique asymptotic values at $\mathbf{x} \to \infty$, which are independent of the direction of $\mathbf{x}$, namely $s_a(\infty) = (0, 0, 1), \mathbf{A}(\infty) = 0$ (see (12b),(13b)). The latter equality means that

2) the magnetic flux is equal to zero for MA3 solitons, $\int \mathbf{B} d\mathbf{S} = 0$,

3) it is easily seen that magnetic field in $D = 2$ MA3 solitons, $B(r) = -(d\alpha/dr)/r$, changes its sign when r increasing.

Statements 1),2),3) are not valid for ANO strings [4-6].

5. Consider another gauged model of the anisotropic Heisenberg antiferromagnet, which describe interaction of the A3-field with the Chern-Simons (CS) gauge field ("CSA3 model"). The Lagrangian of

the CSA3 model is defined by equations, which are obtained from (2),(3) when the Maxwell term $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2$ is replaced in them by the CS term, $\mathcal{L}_{CS} = \epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \quad \mu,\nu,\lambda = 0,1,2$. An essential difference between this CSA3 model and $SU(2)$ symmetric CP^1CS model introduced in [12] and investigated numerically in [13], is the anisotropy of the chiral A3-field, possessing $U(1) \times Z(2)$ symmetry. Adding of such an anisotropy to CP^1CS model changes crucially its properties making emergence of exponentially localized solutions possible.

To find soliton solutions of the CSA3 model describing localized distributions of the A3-field with $Q_t = 1$, we again use the ansatz given by Eqs. (4),(5). By using variables (4)-(7), we get the following Eqs. for $\theta(r)$ and $\alpha(r)$:

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} + \sin\theta\cos\theta\left(\frac{2\alpha-1}{r^2} - p\right) = 0, \tag{16}$$

$$\frac{d^2\alpha}{dr^2} - \frac{1}{r}\frac{d\alpha}{dr} - \alpha + \sin^2\theta = 0. \tag{17}$$

Making scaling transformation $R = r/\sqrt{2}$, we get the set of equations (10),(11) with p replaced by $2p$, thus the solitons in the CSA3 model can be easily obtained from solitons found above within the MA3 model.

Note that the same stationary solitons can be found in the nonrelativistic analogs of the MA3 and CSA3 models, in which the A3-field is replaced by the field of the easy-axis Heisenberg ferromagnet, described by the Landau-Lifshitz equation.

6. In conclusion we considered minimal interaction of the chiral A3-field (3-component unit isovector field having easy-axis anisotropy) with the vector gauge fields (Maxwell and Chern-Simons ones) in (2+1)-dimensional space-time and found soliton solutions with unit topological charge of the A3-field within these models. Both the A3-field and the vector fields approach unique asymptotic values at $|\mathbf{x}| \to \infty$. These localized solutions are characterized by zero net magnetic flux.

Part of this investigation was performed during the participation of I.B. in the Programme "Topological Defects" held by Newton Mathematical Institute (Cambridge, UK) in August-September of 1994; kind invitation to take part in the Programme from Profs.T.Kibble and P.Goddard is thankfully aknowledged. We are grateful to I.V.Barashenkov,A.T.Filippov, D.I.Kazakov, T.Kibble, V.B.Priezzhev, A.S. Schwartz, D.V.Shirkov, A.Vilenkin for useful discussions of this study and related subjects. This investigation was supported in part by NSF Grant No. DMS-9418780.

Email: bogoljub@main1.jinr.dubna.su

[1] J.Goldstone, Nuovo Cim. **19**,154 (1961).

[2] P.Higgs, Phys.Lett. **12**,132 (1964).

[3] V.L.Ginzburg, L.D.Landau, ZhETF **20**,1064 (1950).

[4] H.B.Nielsen, P.Olesen. Nucl.Phys. **B61**,45 (1973).

[5] A.A.Abrikosov, ZhETF **32**,1442 (1957).

[6] A.Vilenkin, E.P.S.Shellard, "Cosmic strings and other topological defects" (Cambridge University Press, Cambridge, 1994).

[7] R.Jackiw, S.-Y.Pi, Progr.Theor.Phys.Suppl.**107**,1 (1992).

[8] I.L.Bogolubsky, A.A.Bogolubskaya, submitted for publication.

[9] A.M.Kosevich, B.A.Ivanov and A.S.Kovalev, "Nonlinear magnetization waves.Dynamical and topological solitons" (Naukova Dumka, Kiev, 1983) [in Russian].

[10] J.Pouget, G.A.Maugin, Phys.Rev.**B30**,5306 (1984).

[11] A.A.Belavin, A.M.Polyakov, Pis'ma v ZhETF, **22**, 503 (1975).

[12] I.E.Dzyaloshinskii, A.M.Polyakov, P.B.Wiegmann, Phys.Lett.A**127**, 112 (1988).

[13] M.A.Mehta, J.A.Davis and I.J.R.Aitchison, Phys.Lett. **B281**, 86 (1992).

**Received by Publishing Department
on May 12, 1995.**