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GENERALIZED DOPPLER EFFECT  
IN SPACES WITH A TRANSPORT ALONG PATHS

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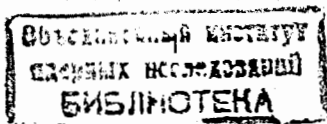
## 1. INTRODUCTION

The present paper continues the began in [1,2] applications of the theory of transports along paths (in fibre bundles) [3] to the mechanics of material point particles. Here is studied a phenomenon consisting in the comparison of the (relative) energies of a material (massive or massless) point particle with respect to two other arbitrary moving point particles (observers). An evident special case of this problem is the well-known Doppler effect [4,5]. When the mentioned transport along paths is linear and, in fact, only paths without self-intersections are taken into account the above problem was investigated in [6]. Here we closely follow [6] without supposing these restrictions.

Sect. 2 contains the strict formulation of the problem of the present work and the derivation of the main results, which in Sect. 3 are applied to the general and special relativity. Sect. 4 closes the paper with some comments.

Below the needed for the following mathematical background is summarized.

All considerations in the present work are made in a (real) differentiable manifold  $M$  [7] whose tangent bundle  $(T(M), \pi, M)$  is endowed with a transport along paths  $I$  [3]. Here  $T(M) := \bigcup_{x \in M} T_x(M)$ ,  $T_x(M)$  being the tangent to  $M$  space at  $x \in M$  and  $\pi: T(M) \rightarrow M$  is such that  $\pi(V) := x$  for  $\forall V \in T_x(M)$ . Besides, the tangent bundle  $(T(M), \pi, M)$  is supposed to be equipped also with a real bundle metric  $g$ , i.e. [8] a map  $g: x \rightarrow g_x$ ,  $x \in M$ , where the maps  $g_x: T_x(M) \otimes T_x(M) \rightarrow \mathbb{R}$  are bilinear, nondegenerate and symmetric. For brevity the defined by  $g$  scalar products of  $X, Y \in T_x(M)$ ,  $x \in M$  will be denoted by a dot  $(\cdot)$ ,



i.e.  $X \cdot Y = g(X, Y)$ . The scalar square of  $X$  will be written as  $(X)^2$  for it has to be distinguished from the second component  $X^2$  of  $X$  in some local basis (in a case when  $\dim(M) > 1$ ). As  $g$  is not supposed to be positively defined,  $(X)^2$  can take any real values.

By  $J$  and  $\gamma: J \rightarrow M$  are denoted, respectively, an arbitrary real interval and a path in  $M$ . If  $\gamma$  is of class  $C^1$ , its tangent vector is written as  $\dot{\gamma}$ .

The transport along paths  $I$  and the bundle metric are supposed to be consistent, i.e.  $I$  preserves the defined by  $g$  scalar products of the vectors (see [3], eq. (2.9)):

$$A \cdot B = (I_{s \rightarrow t}^\gamma A) \cdot (I_{s \rightarrow t}^\gamma B), \quad A, B \in T_{\gamma(s)}(M), \quad s, t \in J,$$

where  $I_{s \rightarrow t}^\gamma$  is the transport along  $\gamma$  from  $s$  to  $t$  [3].

For details concerning transports along paths the reader is referred to [3] and for the ones about relative mechanical quantities (such as velocity, momentum and energy) - to [1].

## 2. STATEMENT OF THE PROBLEM AND GENERAL RESULTS

Let a material object (a point particle which may be massless as well as massive) be moving in  $M$  along the path  $\gamma: J \rightarrow M$  (its world line) which is parameterized with  $r \in J$ . Let  $\gamma$  intersects the paths  $x_a: J_a \rightarrow M$ ,  $a=1,2$ , representing the world lines of the particles 1 and 2, which we call observers; i.e. for some  $r_a \in J$  and  $s_a^0 \in J_a$ , we have  $\gamma(r_a) = x_a(s_a^0)$ ,  $a=1,2$ . If it is necessary, the parameters  $r \in J$  and  $s_a \in J_a$ ,  $a=1,2$  will be considered as proper times of the corresponding particles. As a special case of this construction we can point the case when the material object is emitted from the first particle and/or is detected from the second one, or vice versa. In particular, if the considered material object is a

photon, then the last situation realizes the classical Doppler effect [4,5].

We put the following problem. On the basis of the introduced in [1] concepts we wish to compare the relative energies of the material object with respect to the observers 1 and 2 at the points  $\gamma(r_1) = x_1(s_1^0)$  and  $\gamma(r_2) = x_2(s_2^0)$  respectively.

Let  $p(r)$  be the momentum of the material object (the observed particle) at  $\gamma(r)$  and  $V_a(s_a) = \dot{x}_a(s_a)$ ,  $a=1,2$  be the velocities of the observers. For brevity we let

$$p_a := p(r_a), \quad V_a := V_a(s_a^0), \quad a=1,2. \quad (1)$$

In accordance with the definition of a relative energy (see [1], sect. 4) we have to compare the relative energies

$$E_1 := \varepsilon((V_1)^2) p_1 \cdot V_1 \quad \text{and} \quad E_2 := \varepsilon((V_2)^2) p_2 \cdot V_2, \quad (2)$$

of the observed particle with respect to the particles 1 and 2 at the points  $\gamma(r_1) = x_1(s_1^0)$  and  $\gamma(r_2) = x_2(s_2^0)$  respectively. Here  $\varepsilon(\lambda) := +1$  for  $\lambda > 0$  and  $\varepsilon(\lambda) := -1$  for  $\lambda \leq 0$ .

We introduce the quantities:

$$(V_2)_1 := I_{r_2 \rightarrow r_1}^\gamma V_2, \quad (3)$$

$$\Delta p(r_1, r_2; \gamma) := p(r_2) - I_{r_1 \rightarrow r_2}^\gamma p(r_1) = p_2 - I_{r_1 \rightarrow r_2}^\gamma p_1. \quad (4)$$

The quantity (4) is defined analogously to the relative momentum (cf. [1], sect. 3), but it defines along  $\gamma$  with the help of the transport along paths  $I$  the change of the momentum of the observed particle when it moves from  $\gamma(r_1)$  to  $\gamma(r_2)$ .

Now we shall prove that the quantity

$$\Delta E_{21} := \varepsilon((V_2)^2) \Delta p(r_1, r_2; \gamma) \cdot V_2 \quad (5)$$

is closely connected with the change of the energy of the material object along  $\gamma$  with respect to the point  $\gamma(r_2) = x_2(s_2^0)$ , at which its

world line intersects the world line of the second particle, when the object moves from  $\gamma(r_1)$  to  $\gamma(r_2)$  along  $\gamma$ . In fact, using (1), (4) and (5), we get

$$\Delta E_{21} = \varepsilon((V_2)^2)(p_2 - I_{r_1 \rightarrow r_2}^{\gamma} p_1) \cdot V_2 = E(r_2, r_2; \gamma) - E(r_1, r_2; \gamma). \quad (6)$$

Here

$$E(r, r_2; \gamma) = \varepsilon((V_2)^2)(I_{r \rightarrow r_2}^{\gamma} p(r)) \cdot V_2, \quad (7)$$

as a consequence of the considerations of [1], is defined along  $\gamma$  by means of the transport along paths relative energy of the observed particle when it is situated at  $\gamma(r)$ ;  $r \in [r', r'']$  with respect to the second particle when it is situated at  $\gamma(r_2) = x_2(s_2^0)$ .

From one hand, putting  $r=r_1$  in (7) and (due to the consistency between the metric and the transport along paths) applying to the both multiplies  $I_{r_2 \rightarrow r_1}^{\gamma}$  and, from the other hand, letting  $r=r_2$  in (7) and using (2), we, respectively, get

$$E(r_1, r_2; \gamma) = \varepsilon((V_2)^2) p_1 \cdot (V_2)_1, \quad E(r_2, r_2; \gamma) = E_2. \quad (8)$$

So, from (6), we find

$$E_2 = \Delta E_{21} + \varepsilon((V_2)^2) p_1 \cdot (V_2)_1. \quad (9)$$

Further, supposing  $(V_1)^2 \neq 0$ , which is interpreted as a movement of the first observer with a velocity different from (less than) the one of light in vacuum, we shall express  $p_1 \cdot (V_2)_1$  through  $E_1$ . Representing  $(V_2)_1$  in the form  $(V_2)_1 = (V_2)_1^{\parallel} + (V_2)_1^{\perp}$ , where the longitudinal  $(V_2)_1^{\parallel}$  and the transversal  $(V_2)_1^{\perp}$  components with respect to  $V_1$  are given by

$$(V_2)_1^{\parallel} := V_1 (V_1 \cdot (V_2)_1) / (V_1)^2 \quad ((V_2)_1^{\parallel} \cdot V_1 = (V_2)_1 \cdot V_1), \quad (10a)$$

$$(V_2)_1^{\perp} := (V_2)_1 - (V_2)_1^{\parallel} \quad ((V_2)_1^{\perp} \cdot V_1 = (V_2)_1^{\perp} \cdot (V_2)_1^{\parallel} = 0), \quad (10b)$$

we obtain:

$$\begin{aligned} p_1 \cdot (V_2)_1 &= p_1 \cdot (V_2)_1^{\perp} + p_1 \cdot (V_2)_1^{\parallel} = p_1 \cdot (V_2)_1^{\perp} + \\ &+ (p_1 \cdot V_1)(V_1 \cdot (V_2)_1) / (V_1)^2 = p_1 \cdot (V_2)_1^{\perp} + \\ &+ \varepsilon((V_1)^2) E_1 [((V_2)_1)^2 - ((V_2)_1^{\perp})^2] / (V_1)^2]^{1/2}, \end{aligned} \quad (11)$$

where we have used (2) and the equality  $(V_1 \cdot (V_2)_1) / (V_1)^2 = [((V_2)_1^{\parallel})^2 / (V_1)^2]^{1/2} = [((V_2)_1)^2 - ((V_2)_1^{\perp})^2] / (V_1)^2]^{1/2}$  (the square root sign is uniquely defined by  $(V_1 \cdot (V_2)_1) / (V_1)^2|_{(V_2)_1 = V_1} = +1$ ), which follows from  $((V_2)_1^{\parallel})^2 = (V_1 \cdot (V_2)_1)^2 / (V_1)^2$  and  $((V_2)_1^{\perp})^2 = ((V_2)_1)^2 - ((V_2)_1^{\parallel})^2$ .

Substituting (11) into (9), we get:

$$\begin{aligned} E_2 = \Delta E_{21} + \varepsilon((V_1)^2) \varepsilon((V_2)^2) E_1 [((V_2)_1)^2 - ((V_2)_1^{\perp})^2] / (V_1)^2]^{1/2} + \\ + \varepsilon((V_2)^2) p_1 \cdot (V_2)_1^{\perp}. \end{aligned} \quad (12)$$

This formula is the answer of the stated above problem and it expresses the "generalized" Doppler's effect for the considered process.

Now we will put the last term of (12) into a slightly different form (cf. [4]).

Let the vector  $N_1 \in T_{\gamma(r_1)}(M)$  be defined in the following way. If  $p_1$  and  $V_1$  are not collinear, then  $N_1$  is coplanar with them, i.e.  $N_1 = aV_1 + bp_1$ , for some  $a, b \in \mathbb{R}$ , and satisfies the conditions

$$N_1 \cdot V_1 = 0, \quad (13a)$$

$$(N_1)^2 = N_1 \cdot N_1 = \varepsilon((p_1)^2 - (E_1)^2 / (V_1)^2), \quad (13b)$$

$$N_1 \cdot p_1 < 0. \quad (13c)$$

In this case  $N_1$  is uniquely defined and its connection with  $p_1$  and

$V_1$  may be written, for example, as

$$p_1 = V_1(V_1 \cdot p_1)/(V_1^2) - N_1 \varepsilon((p_1)^2 - (E_1)^2/(V_1^2)) | (p_1)^2 - (E_1)^2/(V_1^2) |^{1/2}, \quad (14a)$$

where  $|\lambda| := \lambda \varepsilon(\lambda)$  is the absolute value of  $\lambda \in \mathbb{R}$ . (Note that  $(p_1)^2 - (E_1)^2/(V_1^2) = 0$  if and only if  $p_1$  and  $V_1$  are collinear; see (1).) If  $p_1$  and  $V_1$  are collinear, i.e.  $p_1 = \lambda V_1$  for some  $\lambda \in \mathbb{R}$  (which, because of  $(V_1)^2 \neq 0$ , is given by  $\lambda = p_1 \cdot V_1 / (V_1^2)$ ), then we put  $N_1 = 0$ :

$$N_1 = 0 \text{ for } p_1 = \lambda V_1. \quad (14b)$$

(Note that due to (14a) from  $N_1 = 0$  follows  $p_1 = \lambda V_1$ , which is equivalent to  $(p_1)^2 - (E_1)^2/(V_1^2) = 0$ .)

Hence the defined by (14) vector  $N_1 \in T_{\gamma(r_1)}(M)$  is orthogonal to a path  $x_1$  at the point  $\gamma(r_1) = x_1(s_1^0)$  ( $N_1 \cdot V_1 = 0$ ) and if  $p_1$  and  $V_1$  are not collinear, then it is a unit vector "pointing from  $x_1$  to  $x_2$ " ( $(N_1)^2 = \varepsilon((p_1)^2 - (E_1)^2/(V_1^2))$ ).

By definition the recession speed  $u_{21}$  of the particle 2 from the particle 1 is the projection of  $(V_2)_1$  on  $N_1$  (see [4]), i.e.

$$u_{21} = (V_2)_1 \cdot N_1 = (V_2)_1^\perp \cdot N_1. \quad (15)$$

If  $p_1$  and  $V_1$  are collinear, then  $u_{21} = 0$ .

From (14), (15) and (10b), we obtain

$$p_1 \cdot (V_2)_1^\perp = -u_{21} \varepsilon((p_1)^2 - (E_1)^2/(V_1^2)) | (p_1)^2 - (E_1)^2/(V_1^2) |^{1/2}. \quad (16)$$

Substituting this expression into (12), we find the sought connection between the relative energies  $E_1$  and  $E_2$  in the form

$$E_2 = \Delta E_{21} + \varepsilon((V_1)^2) \varepsilon((V_2)^2) E_1 [(((V_2)_1)^2 - ((V_2)_1^\perp)^2)/(V_1^2)]^{1/2} - \varepsilon((V_2)^2) \varepsilon((p_1)^2 - (E_1)^2/(V_1^2)) u_{21} | (p_1)^2 - (E_1)^2/(V_1^2) |^{1/2}. \quad (17)$$

Namely this formula gives the Doppler's effect in terms of par-

ticles and their energies in the considered process, which can be called a "generalized Doppler effect". The corresponding to it "red shift" [4] is given by the equality

$$(E_2 - E_1)/E_2 = 1 - \{\Delta E_{21}/E_1 + \varepsilon((V_1)^2) \varepsilon((V_2)^2) [(((V_2)_1)^2 - ((V_2)_1^\perp)^2)/(V_1^2)]^{1/2} - \varepsilon((V_2)^2) \varepsilon((p_1)^2 - (E_1)^2/(V_1^2)) \times \varepsilon(E_1) u_{21} | (p_1)^2 - (E_1)^2/(V_1^2) |^{1/2}\}^{-1}. \quad (18)$$

### 3. EXAMPLES: GENERAL AND SPECIAL RELATIVITY

In this section are considered the applications of the obtained in Sect. 2 general results to the cases of the Doppler's effect in general relativity and the "generalized Doppler effect" (for constant velocities) in special relativity.

The Doppler effect in general relativity investigated, e.g. in [4], ch. III, section 7, where the condition (13c) is not stated explicitly but it is used in the calculation, consists in the following. At the point  $x_2(s_2^0)$  a proton is emitted which moves along the isotropic geodesic path  $\gamma$  to the point  $x_1(s_1^0)$ , where it is detected. The problem is to be compared with the detection and emission energies  $E_1$  and  $E_2$ , respectively.

Choosing  $M$  to be the space-time of general relativity with signature  $(-+++)$  and  $I^\gamma$  to be a parallel transport along  $\gamma$ , we find (see section 2 and [1]):  $(V_1)^2 = (V_2)^2 = -c^2$  ( $c$  is the velocity of light in vacuum),  $(p_1)^2 = (p_2)^2 = 0$ ,  $E_1 > 0$ ,  $\Delta p(r_1, r_2; \gamma) = 0$ ,  $\Delta E_{21} = 0$  and  $N_1 = -[p_1 + V_1(V_1 \cdot p_1)]/E_1$ . So, (17) and (18) take respectively the form:

$$E_2 = E_1 [u_{21} + (1 + ((V_2)_1^\perp)^2 / c^2)^{1/2}], \quad (19)$$

$$(E_2 - E_1) / E_2 = 1 - [u_{21} + (1 + ((V_2)_1^\perp)^2 / c^2)^{1/2}]^{-1}. \quad (20)$$

By using a local orthonormal basis this result is derived in [4], ch. III, section 7, where  $E_1$ ,  $E_2$ ,  $u_{21}$  and  $(V_2)_1^\perp$  are denoted as  $E$ ,  $E'$ ,  $u_r$  and  $u^2$  respectively and, besides, there are used units in which  $c=1$ .

In the case of special relativity  $M$  is the Minkowski's space-time and  $I^\gamma$  is also a parallel transport along  $\gamma$ . Let the considered particles be moving with constant 3-velocities  $v$ ,  $v_1$  and  $v_2$  with respect to a given frame of reference, i.e. we have  $\gamma(r) = y + (ct, tv)$  and  $x_a(s_a) = y_a + (ct, tv_a)$ ,  $a=1,2$ , where  $y, y_1, y_2 \in M$  are fixed,  $t$  is the time in that frame,  $r=t$  for  $|v|=c$  and  $r=t(1-v^2/c^2)^{1/2}$  for  $|v|<c$ , and  $s_a = t(1-v_a^2/c^2)^{1/2}$ ,  $a=1,2$  are the corresponding proper times (cf. [2,4,5]). (Of course, we suppose that  $\gamma$  intersects  $x_1$  and  $x_2$ .)

From here we find the momenta of the observed particle as  $p(r) = p_1 = p_2 = \mu c(1, v/c)$ , where  $\mu := E/c$  for  $|v|=c$  ( $E$  is the particle's energy in the given frame) and  $\mu := m(1-v^2/c^2)^{-1/2}$  for  $|v|<c$  ( $m$  is the particle's rest mass),  $v_a = c(1, v/c)(1-v_a^2/c^2)^{-1/2}$ ,  $a=1,2$ ,  $(V_2)_1 = V_2$ ,  $\Delta p(r_1, r_2; \gamma) \equiv 0$  and  $\Delta E_{21} = 0$ . The computation of the remaining quantities concerning the phenomenon under consideration is simple but long; that is why we present here only the final result:

$$E_2 = E_1 \left( \frac{1-v_2^2/c^2}{1-v_1^2/c^2} \right)^{1/2} \cdot \frac{1-v_2 \cdot v/c^2}{1-v_1 \cdot v/c^2}, \quad (21)$$

which also may easily be obtain from the expressions  $E_a = \epsilon((V_a)^\perp)^2 p_a \cdot v_a = \mu c^2(1-v_a \cdot v/c^2)(1-v_a^2/c^2)^{-1/2}$ ,  $a=1,2$ .

In the case of the "usual" Doppler effect [5] we have a photon ( $v=cn$ ,  $n^2=1$ ) emitted with energy  $E_2 = E_0$  and detected with energy

$E_1 = E$ , which according to (21) is

$$E = E_0 \left( \frac{1-v_2^2/c^2}{1-v_1^2/c^2} \right)^{1/2} \cdot \frac{1-v_1 \cdot n/c}{1-v_2 \cdot n/c}. \quad (22)$$

Let us note that when  $v_1=0$ , the formulae (21) and (22) may be derived also as a corollary from the definition of relative energy (see [1], sect. 4). This result is in agreement with the considered in [5] Doppler effect in special relativity (in terms of frequencies; cf. the quantum relation  $E=h\nu$ ,  $h$  being the Plank's constant).

#### 4. CONCLUDING REMARKS

We want to emphasize that the basic result of this work is given by the equality (17) the main difference of which from the usual Doppler effect (e.g. in general relativity; see (19) or [3]) is the existence in it of, generally said, nonvanishing term  $\Delta E_{21}$ , defined by (5). A feature of the equation (17) is its validity for particles with arbitrary, zero or nonzero, masses, i.e. our results do not depend on the mass of the investigated particle.

If  $\Delta E_{21} = 0$ , then eq. (17) takes a form similar to the classical one (in the case of arbitrary mass (cf. [4])). Due to (5) a sufficient condition for this is

$$\Delta p(r_1, r_2; \gamma) = 0, \quad (23)$$

i.e. (see (4))

$$p(r_2) = I_{r_1 \rightarrow r_2}^\gamma p(r_1) \quad (23')$$

which means that the momentum  $p(r_1)$  is  $(I-)$ transported by means of the transport along paths  $I$  from the point  $\gamma(r_1)$  to the point  $\gamma(r_2)$  (cf. [3], eq. (2.4)).

If a point particle is moving along the path  $\gamma: J \rightarrow M$ , then it is natural to call it a *free particle* (with respect to the transport along paths I) if (23') holds for every  $r_1, r_2 \in J$ , i.e. if its momentum is I-transported along its world line  $\gamma$  (cf. [3], definition 2.2). In particular, if  $I^\gamma$  is a parallel transport along  $\gamma$  (generated by a linear connection), then this definition of a free particle coincides with the one in [4], p. 110, given therein as a special case of the geodesic hypothesis. In our case, the corresponding generalization of the geodesic hypothesis states that the world line  $\gamma: J \rightarrow M$  of a free (with respect to I) particle is an I-path (see [2], definition 2.2), i.e.

$$\dot{\gamma}(t) = I_{s \rightarrow t}^\gamma \dot{\gamma}(s), \quad s, t \in J, \quad (24)$$

and besides

$$p(s) = \mu(s; \gamma) \dot{\gamma}(s) \quad (25)$$

for some scalar function  $\mu$  (identified with the particles rest mass if it is not zero).

If the transport along paths is linear (see e.g. [3], eq. (2.8)), then substituting (25) into (23') and comparing the result with (24), we get (cf. [4], p. 110, eq. (9))

$$\mu(s; \gamma) = \text{const.} \quad (26)$$

So, the mass parameter  $\mu$  (the rest mass if it is not zero) of a free particle is constant if the used transport along paths is linear.

It should also be noted, that as for free massive particles with a rest mass  $m$  eq. (25) holds (by definition) for  $\mu(s; \gamma) = m$ , then for linear transports along paths the ("generalized geodesic") hypothesis (24) is a consequence of the condition (23').

At the end we want to mention the equality (cf. (4))

$$\Delta p(r_2, r_1; \gamma) = I_{r_2 \rightarrow r_1}^\gamma \Delta p(r_1, r_2; \gamma) \quad (27)$$

for a linear transport along paths I. Hence, in this case eq. (5) can equivalently (due to the consistency of I and the metric) be written as

$$\Delta E_{21} := \epsilon((V_2)^2) \Delta p(r_2, r_1; \gamma) \cdot (V_2)_1. \quad (28)$$

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Илиев Б.З.

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Обобщенный эффект Доплера в пространствах с переносом вдоль путей.

В данной работе исследуется аналог классического эффекта Доплера для пространств (многообразия) с касательным пространством с параллельным переносом вдоль траекторий.

Полученные результаты применимы как в безмассовом (например, для фотона), так и в случае ненулевой массы.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 1995

Iliev B.Z.

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Generalized Doppler Effect in Spaces with a Transport along Paths

The analog of the classical Doppler effect in spaces (manifolds) is investigated whose tangent bundle is endowed with a transport along paths, which in particular can be parallel. The obtained results are valid irrespective of the particle mass, i.e., they are held for massless particles (e.g., photons) as well as for massive ones.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1995