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DIFFERENTIATION WITH RESPECT TO PARAMETERS OF THE EQUATIONS OF CHARGED PARTICLE MOVEMENT IN THE MAGNETIC FIELD

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The movement of the charged particle in the magnetic field is described with ordinary differential equations of the second order as follows[1]:

$$x''=\frac{aq}{p}(1+x'^2+y'^2)^{1/2}[B_x x'y'-B_y(1+x'^2)+B_z y']=\phi(x,y,z,x',y'),$$
$$y''=\frac{aq}{p}(1+x'^2+y'^2)^{1/2}[B_x(1+y'^2)-B\,x'_y y'-B_z x']=f(x,y,z,x',y'), \qquad (1)$$

where a= 0.2998, q= ±1,±2,...- particle charge, p- its momentum, B_x,B_y,B_z- vector components of the magnetic field $\vec{B}$ at the point (x,y,z), $x'= \dot{x}= x/dz$, $x''= d^2x/dz^2$.

The solution of equations (1) can be obtained by numerical integration with Runge-Kutt method[2] at the initial conditions:

$$z_1,\ x_1= x(z_1),\ y_1= y(z_1),\ x'_1= x'(z_1),\ y'_1= y'(z_1).$$

At the (i+1)-th step

$$\begin{aligned}
z_{i+1}&= z_i+ h= z_1+ hi,\\
x_{i+1}&= x_i+ x'h+ 1/6(m^i_1+ m^i_2+ m^i_3)h^2,\\
y_{i+1}&= y_i+ y'_i h+ 1/6(k^i_1+ k^i_2+ k^i_3)h^2,\\
x'_{i+1}&= x'_i+ 1/6(m^i_1+ 2m^i_2+ 2m^i_3+ m^i_4)h,\\
y'_{i+1}&= y'_i+ 1/6(k^i_1+ 2k^i_2+ 2k^i_3+ k^i_4)h,
\end{aligned} \qquad (2)$$

where h= Δz- integrating step,

$$\begin{aligned}
m^i_1&= \phi_1(x_i,y_i,z_i,x'_i,y'_i),\quad k^i_1= f_1(x_i,y_i,z_i,x'_i,y'_i),\\
m^i_2&= \phi_2(\chi_i,\ae_i,\zeta_i,\chi'_i,\ae'_i),\quad k^i_2= f_2(\chi_i,\ae_i,\zeta_i,\chi'_i,\ae'_i),\\
m^i_3&= \phi_3(o_i,\omega_i,\zeta_i,o'_i,\omega'_i),\quad k^i_3= f_3(o_i,\omega_i,\zeta_i,o'_i,\omega'_i),\\
m^i_4&= \phi_4(\gamma_i,\upsilon_i,\xi_i,\gamma'_i,\upsilon'_i),\quad k^i_4= f_4(\gamma_i,\upsilon_i,\xi_i,\gamma'_i,\upsilon'_i),
\end{aligned} \qquad (3)$$

$$\begin{aligned}
&\chi_i=\chi_i(x_i,\zeta_i,x'_i)=x_i+x'_i\tfrac{h}{2},\ \chi'_i=\chi'_i(x'_i,\zeta_i,m^i_1)=x'_i+m^i_1\tfrac{h}{2},\ \zeta_i=z_i+\tfrac{h}{2},\\
&\ae_i=\ae_i(y_i,\zeta_i,y'_i)=y_i+y'_i\tfrac{h}{2},\ \ae'_i=\ae'_i(y'_i,\zeta_i,k^i_1)=y'_i+k^i_1\tfrac{h}{2},\ \zeta_i=z_i+\tfrac{h}{2},\\
&o_i=o_i(x_i,\zeta_i,x'_i,m^i_1)=x_i+x'_i\tfrac{h}{2}+m^i_1\tfrac{h^2}{4},\ o'_i=o'_i(x'_i,\zeta_i,m^i_2)=x'_i+m^i_2\tfrac{h}{2},\ \zeta_i=z_i+\tfrac{h}{2},\\
&\omega_i=\omega_i(y_i,\zeta_i,y'_i,k^i_1)=y_i+y'_i\tfrac{h}{2}+k^i_1\tfrac{h^2}{4},\ \omega'_i=\omega'_i(y'_i,\zeta_i,k^i_2)=y'_i+k^i_2\tfrac{h}{2},\ \zeta_i=z_i+\tfrac{h}{2},\\
&\gamma_i=\gamma_i(x_i,\xi_i,x'_i,m^i_2)=x_i+x'_i h+m^i_2\tfrac{h^2}{2},\ \gamma'_i=\gamma'_i(x'_i,\xi_i,m^i_3)=x'_i+m^i_3 h,\ \xi_i=z_i+h,\\
&\upsilon_i=\upsilon_i(y_i,\xi_i,y'_i,k^i_2)=y_i+y'_i h+k^i_2\tfrac{h^2}{2},\ \upsilon'_i=\upsilon'_i(y',\xi_i,k^i_3)=y'_i+k^i_3 h,\ \xi_i=z_i+h.
\end{aligned} \qquad (4)$$

Thus, when the momentum and magnetic field map are known, the trajectory of the charged particle moving will be presented with the collection of (x,y,z)- coordinates for N points.

Very often it is necessary to solve the inverted task: to reconstruct the particle momentum using the given N points of the movement trajectory and magnetic field power at these points. While solving such a task in many cases it is necessary to calculate derivatives using the momentum as a parameter at the given points of the trajectory described with the particle movement equations.

The aim of this work is to get presentations for derivatives with respect to the momentum from the solutions of the movement equations (2). In work[3] the general view of presentations for the derivatives with respect to the parameters for solutions of the ordinary differential second-order equations of the type:

$$\begin{aligned}
x'' &= \phi(p_1,\dots,p_n,x,y,z,x',y'),\\
y'' &= f(p_1,\dots,p_n,x,y,z,x',y'),
\end{aligned} \qquad (5)$$

where $p_1,\dots,p_n$- parameters.

Let us use the results of work[3] for equation (1) supposing it looks as follows:

$$\begin{aligned}
x'' &= \phi(p,x,y,z,x',y'),\\
y'' &= f(p,x,y,z,x',y'),
\end{aligned} \qquad (6)$$

where p- particle momentum is a parameter.

Then we perform differentiation in (2) with respect to parameter p:

$$\begin{aligned}
\frac{\partial x_{i+1}}{\partial p}&= \frac{\partial x_i}{\partial p}+ \frac{\partial x'_i}{\partial p}h+ 1/6\left(\frac{\partial m^i_1}{\partial p}+ \frac{\partial m^i_2}{\partial p}+ \frac{\partial m^i_3}{\partial p}\right)h^2,\\
\frac{\partial y_{i+1}}{\partial p}&= \frac{\partial y_i}{\partial p}+ \frac{\partial y'_i}{\partial p}h+ 1/6\left(\frac{\partial k^i_1}{\partial p}+ \frac{\partial k^i_2}{\partial p}+ \frac{\partial k^i_3}{\partial p}\right)h^2,\\
\frac{\partial x'_{i+1}}{\partial p}&= \frac{\partial x'_i}{\partial p}+ 1/6\left(\frac{\partial m^i_1}{\partial p}+ 2\frac{\partial m^i_2}{\partial p}+ 2\frac{\partial m^i_3}{\partial p}+ \frac{\partial m^i_4}{\partial p}\right)h,\\
\frac{\partial y'_{i+1}}{\partial p}&= \frac{\partial y'_i}{\partial p}+ 1/6\left(\frac{\partial k^i_1}{\partial p}+ 2\frac{\partial k^i_2}{\partial p}+ 2\frac{\partial k^i_3}{\partial p}+ \frac{\partial k^i_4}{\partial p}\right)h.
\end{aligned} \qquad (7)$$

We introduce designations:

$$\alpha_i= \alpha_i(x'_i,y'_i)= (1+ x'^2_i+ y'^2_i)^{1/2}, \quad c= aq/p,$$

$$\beta^i_{xj}= \frac{\partial m^i_j}{\partial B^i_x}= c\alpha_i x'_i y'_i,\ \beta^i_{yj}= \frac{\partial m^i_j}{\partial B^i_y}= -c\alpha_i(1+ x_i^2),\ \beta^i_{zj}= \frac{\partial m^i_j}{\partial B^i_z}= c\alpha_i y'_i,$$

$$\mathfrak{B}^i_{xj}=\frac{\partial k^i_j}{\partial B^i_x}=c\alpha_i(1+y_i'^{\,2}),\quad \mathfrak{B}^i_{yj}=\frac{\partial k^i_j}{\partial B^i_y}=-c\alpha_i x_i' y_i',\quad \mathfrak{B}^i_{zj}=\frac{\partial k^i_j}{\partial B^i_z}=-c\alpha_i x_i', \tag{8}$$

$$\mathfrak{M}^i_{xj}=\frac{\partial m^i_j}{\partial x_i}=\beta^i_{xj}\frac{\partial B^i_x}{\partial x_i}+\beta^i_{yj}\frac{\partial B^i_y}{\partial x_i}+\beta^i_{zj}\frac{\partial B^i_z}{\partial x_i},$$

$$\mathfrak{M}^i_{yj}=\frac{\partial m^i_j}{\partial y_i}=\beta^i_{xj}\frac{\partial B^i_x}{\partial y_i}+\beta^i_{yj}\frac{\partial B^i_y}{\partial y_i}+\beta^i_{zj}\frac{\partial B^i_z}{\partial y_i},$$

$$\mathfrak{K}^i_{xj}=\frac{\partial k^i_j}{\partial x_i}=\mathfrak{B}^i_{xj}\frac{\partial B^i_x}{\partial x_i}+\mathfrak{B}^i_{yj}\frac{\partial B^i_y}{\partial x_i}+\mathfrak{B}^i_{zj}\frac{\partial B^i_z}{\partial x_i}, \tag{9}$$

$$\mathfrak{K}^i_{yj}=\frac{\partial k^i_j}{\partial y_i}=\mathfrak{B}^i_{xj}\frac{\partial B^i_x}{\partial y_i}+\mathfrak{B}^i_{yj}\frac{\partial B^i_y}{\partial y_i}+\mathfrak{B}^i_{zj}\frac{\partial B^i_z}{\partial y_i},$$

$$\mathfrak{L}^i_{\dot{x}j}=\frac{\partial m^i_j}{\partial x_i'}=m^i_j x_i'/\alpha_i^2+c\alpha_i(B^i_x y_i'-2B^i_y x_i'),$$

$$\mathfrak{L}^i_{\dot{y}j}=\frac{\partial m^i_j}{\partial y_i'}=m^i_j y_i'/\alpha_i^2+c\alpha_i(B^i_x x_i'+B^i_z),$$

$$\mathfrak{N}^i_{\dot{x}j}=\frac{\partial k^i_j}{\partial x_i'}=k^i_j x_i'/\alpha_i^2-c\alpha_i(B^i_y y_i'+B^i_z), \tag{10}$$

$$\mathfrak{N}^i_{\dot{y}j}=\frac{\partial k^i_j}{\partial y_i'}=k^i_j y_i'/\alpha_i^2+c\alpha_i(2B^i_x y_i'-B^i_y x_i'),\quad j=1,\dots,4,\quad i=1,\dots,N.$$

Taking into account designations (8)-(10) of the present work and formulae (8) of work/3/, for $\left\{\frac{\partial m^i_j}{\partial p},\frac{\partial k^i_j}{\partial p}\right\}_{j=1}^{4}$ we shall get:

$$\frac{\partial m^i_1}{\partial p}=-\frac{1}{p}m^i_1+\mathfrak{M}^i_{x1}\frac{\partial x_i}{\partial p}+\mathfrak{M}^i_{y1}\frac{\partial y_i}{\partial p}+\mathfrak{L}^i_{\dot{x}1}\frac{\partial x_i'}{\partial p}+\mathfrak{L}^i_{\dot{y}1}\frac{\partial y_i'}{\partial p},$$

$$\frac{\partial m^i_2}{\partial p}=-\frac{1}{p}m^i_2+\mathfrak{M}^i_{x2}\frac{\partial x_i}{\partial p}+\mathfrak{M}^i_{y2}\frac{\partial y_i}{\partial p}+(\mathfrak{L}^i_{\dot{x}2}+\frac{h}{2}\mathfrak{M}^i_{x2})\frac{\partial x_i'}{\partial p}+(\mathfrak{L}^i_{\dot{y}2}+\frac{h}{2}\mathfrak{M}^i_{y2})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h}{2}(\mathfrak{L}^i_{\dot{x}2}\frac{\partial m^i_1}{\partial p}+\mathfrak{L}^i_{\dot{y}2}\frac{\partial k^i_1}{\partial p}),$$

$$\frac{\partial m^i_3}{\partial p}=-\frac{1}{p}m^i_3+\mathfrak{M}^i_{x3}\frac{\partial x_i}{\partial p}+\mathfrak{M}^i_{y3}\frac{\partial y_i}{\partial p}+(\mathfrak{L}^i_{\dot{x}3}+\frac{h}{2}\mathfrak{M}^i_{x3})\frac{\partial x_i'}{\partial p}+(\mathfrak{L}^i_{\dot{y}3}+\frac{h}{2}\mathfrak{M}^i_{y3})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h^2}{4}(\mathfrak{M}^i_{x3}\frac{\partial m^i_1}{\partial p}+\mathfrak{M}^i_{y3}\frac{\partial k^i_1}{\partial p})+\frac{h}{2}(\mathfrak{L}^i_{\dot{x}3}\frac{\partial m^i_2}{\partial p}+\mathfrak{L}^i_{\dot{y}3}\frac{\partial k^i_2}{\partial p}), \tag{11}$$

$$\frac{\partial m^i_4}{\partial p}=-\frac{1}{p}m^i_4+\mathfrak{M}^i_{x4}\frac{\partial x_i}{\partial p}+\mathfrak{M}^i_{y4}\frac{\partial y_i}{\partial p}+(\mathfrak{L}^i_{\dot{x}4}+h\mathfrak{M}^i_{x4})\frac{\partial x_i'}{\partial p}+(\mathfrak{L}^i_{\dot{y}4}+h\mathfrak{M}^i_{y4})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h^2}{2}(\mathfrak{M}_{x4}\frac{\partial m^i_2}{\partial p}+\mathfrak{M}_{y4}\frac{\partial k^i_2}{\partial p})+h(\mathfrak{L}_{\dot{x}4}\frac{\partial m^i_3}{\partial p}+\mathfrak{L}_{\dot{y}4}\frac{\partial k^i_3}{\partial p}),$$

$$\frac{\partial k^i_1}{\partial p}=-\frac{1}{p}k^i_1+\mathfrak{K}^i_{x1}\frac{\partial x_i}{\partial p}+\mathfrak{K}^i_{y1}\frac{\partial y_i}{\partial p}+\mathfrak{N}^i_{\dot{x}1}\frac{\partial x_i'}{\partial p}+\mathfrak{N}^i_{\dot{y}1}\frac{\partial y_i'}{\partial p},$$

$$\frac{\partial k^i_2}{\partial p}=-\frac{1}{p}k^i_2+\mathfrak{K}^i_{x2}\frac{\partial x_i}{\partial p}+\mathfrak{K}^i_{y2}\frac{\partial y_i}{\partial p}+(\mathfrak{N}^i_{\dot{x}2}+\frac{h}{2}\mathfrak{K}^i_{x2})\frac{\partial x_i'}{\partial p}+(\mathfrak{N}^i_{\dot{y}2}+\frac{h}{2}\mathfrak{K}^i_{y2})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h}{2}(\mathfrak{N}^i_{\dot{x}2}\frac{\partial m^i_1}{\partial p}+\mathfrak{N}^i_{\dot{y}2}\frac{\partial k^i_1}{\partial p}), \tag{12}$$

$$\frac{\partial k^i_3}{\partial p}=-\frac{1}{p}k^i_3+\mathfrak{K}^i_{x3}\frac{\partial x_i}{\partial p}+\mathfrak{K}^i_{y3}\frac{\partial y_i}{\partial p}+(\mathfrak{N}^i_{\dot{x}3}+\frac{h}{2}\mathfrak{K}^i_{x3})\frac{\partial x_i'}{\partial p}+(\mathfrak{N}^i_{\dot{y}3}+\frac{h}{2}\mathfrak{K}^i_{y3})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h^2}{4}(\mathfrak{K}^i_{x3}\frac{\partial m^i_1}{\partial p}+\mathfrak{K}^i_{y3}\frac{\partial k^i_1}{\partial p})+\frac{h}{2}(\mathfrak{N}^i_{\dot{x}3}\frac{\partial m^i_2}{\partial p}+\mathfrak{N}^i_{\dot{y}3}\frac{\partial k^i_2}{\partial p}),$$

$$\frac{\partial k^i_4}{\partial p}=-\frac{1}{p}k^i_4+\mathfrak{K}^i_{x4}\frac{\partial x_i}{\partial p}+\mathfrak{K}^i_{y4}\frac{\partial y_i}{\partial p}+(\mathfrak{N}^i_{\dot{x}4}+h\mathfrak{K}^i_{x4})\frac{\partial x_i'}{\partial p}+(\mathfrak{N}^i_{\dot{y}4}+h\mathfrak{K}^i_{y4})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h^2}{2}(\mathfrak{K}^i_{x4}\frac{\partial m^j_2}{\partial p}+\mathfrak{K}^i_{y4}\frac{\partial k^i_2}{\partial p})+h(\mathfrak{N}^i_{\dot{x}4}\frac{\partial m^i_3}{\partial p}+\mathfrak{N}^i_{\dot{y}4}\frac{\partial k^i_3}{\partial p}).$$

Putting (11),(12) in (7), we get the final expressions:

$$\frac{\partial x_{i+1}}{\partial p}=\frac{\partial x_i}{\partial p}+h\frac{\partial x_i'}{\partial p}-\frac{1}{6p}(m^i_1+m^i_2+m^i_3)h^2+1/6\Big\{\sum_{j=1}^{3}\mathfrak{M}^i_{xj}\frac{\partial x_i}{\partial p}+\sum_{j=1}^{3}\mathfrak{M}^i_{yj}\frac{\partial y_i}{\partial p}+$$
$$+(\sum_{j=1}^{3}\mathfrak{L}^i_{\dot{x}j}+\frac{h}{2}\sum_{j=2}^{3}\mathfrak{M}^i_{yj})\frac{\partial x_i'}{\partial p}+(\sum_{j=1}^{3}\mathfrak{L}^i_{\dot{y}j}+\frac{h}{2}\sum_{j=2}^{3}\mathfrak{M}^i_{yj})\frac{\partial y_i'}{\partial p}+$$
$$+\frac{h}{2}\Big[(\mathfrak{L}^i_{\dot{x}2}+\frac{h}{2}\mathfrak{M}^i_{x3})\frac{\partial m^i_1}{\partial p}+(\mathfrak{L}^i_{\dot{y}2}+\frac{h}{2}\mathfrak{M}^i_{y3})\frac{\partial k^i_1}{\partial p}\Big]+\frac{h}{2}(\mathfrak{L}^i_{\dot{x}3}\frac{\partial m^i_2}{\partial p}+\mathfrak{L}^i_{\dot{y}3}\frac{\partial k^i_2}{\partial p})\Big\}h^2,$$

$$\frac{\partial y_{i+1}}{\partial p}=\frac{\partial y_i}{\partial p}+h\frac{\partial y_i'}{\partial p}-\frac{1}{6p}(k^i_1+k^i_2+k^i_3)h^2+1/6\Big\{\sum_{j=1}^{3}\mathfrak{K}^i_{xj}\frac{\partial x_i}{\partial p}+\sum_{j=1}^{3}\mathfrak{K}^i_{yj}\frac{\partial y_i}{\partial p}+$$
$$+(\sum_{j=1}^{3}\mathfrak{N}^i_{\dot{x}j}+\frac{h}{2}\sum_{j=2}^{3}\mathfrak{K}^i_{yj})\frac{\partial x_i'}{\partial p}+(\sum_{j=1}^{3}\mathfrak{N}^i_{\dot{y}j}+\frac{h}{2}\sum_{j=2}^{3}\mathfrak{K}^i_{yj})\frac{\partial y_i'}{\partial p}+ \tag{13}$$
$$+\frac{h}{2}\Big[(\mathfrak{N}^i_{\dot{x}2}+\frac{h}{2}\mathfrak{K}^i_{x3})\frac{\partial m^i_1}{\partial p}+(\mathfrak{N}^i_{\dot{y}2}+\frac{h}{2}\mathfrak{K}^i_{y3})\frac{\partial k^i_1}{\partial p}\Big]+\frac{h}{2}(\mathfrak{N}^i_{\dot{x}3}\frac{\partial m^i_2}{\partial p}+\mathfrak{N}^i_{\dot{y}3}\frac{\partial k^i_2}{\partial p})\Big\}h^2,$$

$$\frac{\partial x'_{i+1}}{\partial p}=\frac{\partial x_i'}{\partial p}-\frac{1}{6p}(m^i_1+2m^i_2+2m^i_3+m^i_4)h+1/6\Big\{(\mathfrak{M}^i_{x1}+2\mathfrak{M}^i_{x2}+2\mathfrak{M}^i_{x3}+\mathfrak{M}^i_{x4})\frac{\partial x_i}{\partial p}+$$
$$+(\mathfrak{M}^i_{y1}+2\mathfrak{M}^i_{y2}+2\mathfrak{M}^i_{y3}+\mathfrak{M}^i_{y4})\frac{\partial y_i}{\partial p}+(\mathfrak{L}^i_{\dot{x}1}+2\mathfrak{L}^i_{\dot{x}2}+2\mathfrak{L}^i_{\dot{x}3}+\mathfrak{L}^i_{\dot{x}4}+h\sum_{j=2}^{4}\mathfrak{M}^i_{xj})\frac{\partial x_i'}{\partial p}+$$
$$+(\mathfrak{L}^i_{\dot{y}1}+2\mathfrak{L}^i_{\dot{y}2}+2\mathfrak{L}^i_{\dot{y}3}+\mathfrak{L}^i_{\dot{y}4}+h\sum_{j=2}^{4}\mathfrak{M}^i_{yj})\frac{\partial y_i'}{\partial p}+h\Big\{(\mathfrak{L}^i_{\dot{x}2}+\frac{h}{2}\mathfrak{M}^i_{x3})\frac{\partial m^i_1}{\partial p}+$$
$$+(\mathfrak{L}^i_{\dot{y}2}+\frac{h}{2}\mathfrak{M}^i_{y3})\frac{\partial k^i_1}{\partial p}\Big\}+h\Big\{(\mathfrak{L}^i_{\dot{x}3}+\frac{h}{2}\mathfrak{M}^i_{x4})\frac{\partial m^i_2}{\partial p}+(\mathfrak{L}^i_{\dot{y}3}+\frac{h}{2}\mathfrak{M}^i_{y4})\frac{\partial k^i_2}{\partial p}\Big\}+$$
$$+h(\mathfrak{L}^i_{\dot{x}4}\frac{\partial m^i_3}{\partial p}+\mathfrak{L}^i_{\dot{y}4}\frac{\partial k^i_3}{\partial p})\Big\}h,$$

$$
\begin{aligned}
\frac{\partial y'_{i+1}}{\partial p} = {} & \frac{\partial x'_i}{\partial p} - \frac{1}{6p}(k^i_1 + 2k^i_2 + 2k^i_3 + k^i_4)h + 1/6\Big\{(\mathfrak{R}^i_{x1} + 2\mathfrak{R}^i_{x2} + 2\mathfrak{R}^i_{x3} + \mathfrak{R}^i_{x4})\frac{\partial x_i}{\partial p} + \\
& + (\mathfrak{R}^i_{y1} + 2\mathfrak{R}^i_{y2} + 2\mathfrak{R}^i_{y3} + \mathfrak{R}^i_{y4})\frac{\partial y_i}{\partial p} + (\mathfrak{M}^i_{\dot{x}1} + 2\mathfrak{M}^i_{\dot{x}2} + 2\mathfrak{M}^i_{\dot{x}3} + \mathfrak{M}^i_{\dot{x}4} + h\sum_{j=2}^{4}\mathfrak{R}^i_{xj})\frac{\partial x'_i}{\partial p} + \\
& + (\mathfrak{M}^i_{\dot{y}1} + 2\mathfrak{M}^i_{\dot{y}2} + 2\mathfrak{M}^i_{\dot{y}3} + \mathfrak{M}^i_{\dot{y}4} + h\sum_{j=2}^{4}\mathfrak{R}^i_{yj})\frac{\partial y'_i}{\partial p} + h\Big[(\mathfrak{M}^i_{\dot{x}2} + \frac{h}{2}\mathfrak{R}^i_{x3})\frac{\partial m^i_1}{\partial p} + \\
& + (\mathfrak{M}^i_{\dot{y}2} + \frac{h}{2}\mathfrak{R}^i_{y3})\frac{\partial k^i_1}{\partial p}\Big] + h\Big[(\mathfrak{M}^i_{x3} + \frac{h}{2}\mathfrak{R}^i_{x4})\frac{\partial m^i_2}{\partial p} + (\mathfrak{M}^i_{\dot{y}3} + \frac{h}{2}\mathfrak{R}^i_{y4})\frac{\partial k^i_2}{\partial p}\Big] + \\
& + h(\mathfrak{M}^i_{\dot{x}4}\frac{\partial m^i_3}{\partial p} + \mathfrak{M}^i_{\dot{y}4}\frac{\partial k^i_3}{\partial p})\Big\}h.
\end{aligned}
$$

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