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ADAPTIVE PROJECTIVE FILTERS

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1. Introduction

Detecting curves (contours) with gaps and noisy backgrounds is the known classic problem in pattern recognition and digital image processing [2,3]. Track finding is the example of such problem in high energy physics (HEP). For modern experiments in particle physics this problem consist in the getting of parameters of individual tracks for minimum times and has discussed in many papers within the past few years. Various track finding methods have been proposed and employed in experience, e.g. Hough transformation (HT), histogramming, neural network (NN), Kalman filtering(KF), road finding (RF), elastic tracking (ET) [3,4,5,6,7], etc. Each method has some advantages and disadvantages with respect to another [7].

In this paper the new approach to solving of the track finding problem is suggested. The approach is based on the new kind of transformations - discrete projective transformations (DPT) [1]. Using DPT the track finding process can be considered as a model of the linear system [8] with known nonlinear weight functions-projective invariants.

Main features of DPT and the Least Square Fitting (LSF) have been used for creation of the recurrent algorithm, which realizes the whole family of the digital feedback filters - Adaptive Projective Filters (APF). Using the information feedback, APF provides the stability to measurement errors everywhere except of two 'noise' points (*poles*). Estimated by APF parmeters (measurement coordinates on TS) are *unbiased and have the minimal dispersion.* The weight functions are defined by *a cross ratio (CR)* of distances of four collinear points taken in the special form [1]. In geometry the *CR*-functions have the fundamental property and are known as projective invariants [9].

The complication of the track finding problem increases drastically if the number of tracks and background points grows, so as the number of operations is changed by the combinatorial low. APF allows to reduce the number of such operations by means of increasing *the accuracy in prediction* of the point in both *local* and *global* zones, by the efficient rejection of wrong TS-candidates on starting phase and by reducing of calculations per point, etc. The APF apdates the information during the track segment finding and main parameters of TS are determinated at the finish of the process. Obtained parameters of TS and *CR*-functions allow pick out points of the track segment among background points by means of the simple 1D histogramming procedure.

APF can be used as the fast and precise solver for detecting, counting, filtration of a curve and/or linear track segments at several levels of track processing and for vertex finding. In general, APF can be used in adequate control systems for collection, processing and compression of data, including tracking problems for the wide class of detectors.

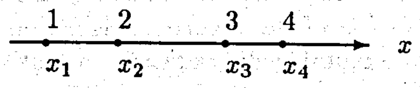
The performance of the DPT method was tested with simulation of linear or/and parabola shape tracks on the image space consisting of 100 x 100 pixels. The algorithm runs in the Turbo Pascal environment.

2. What is DPT?

The main idea and the mathematical formalism are described in this part of the paper.

2.1 The Cross-Ratio Functions of Four Collinear Points

Let us choose on the straight line four arbitrary uncoinciding points



with coordinates x_1, x_2, x_3 and $x_4 (x_i \neq x_j, i \neq j)$, and find the algebraic distances

$$\vec{ij} = x_i - x_j, \vec{ij} = -\vec{ji}, i \neq j.$$

With respect to the first fixed point x_1 we can get six (3!) functions of these distances in accordance with the following rule of the cross-ratio *(CR)*:

$$\frac{\vec{13}}{\vec{24}} : \frac{\vec{23}}{\vec{14}} . \qquad (1)$$

Using (1) we get only three different *CR*-functions $p_i, i = 1, 2, 3$, which are present in the following notation:

$$p_1 = \{1234\} = \{1243\}, p_2 = \{1324\} = \{1342\}, p_3 = \{1423\} = \{1432\},$$

where figures in the braces point out the position of the point in the tetrad.

For all possible dispositions of four points on axis $0x$ the above cross ratio gives us three different functions $p_i(\lambda, L, \tau), i = 1, 2, 3,$ defined by the quadruplet $\{0, \lambda, L, \tau\}$ as follows:

$$p_1(\lambda, L, \tau) = \frac{L\tau}{(\tau - \lambda)(L - \lambda)},$$

$$p_2(\lambda, L, \tau) = \frac{-\lambda\tau}{(\tau - L)(L - \lambda)},$$

$$p_3(\lambda, L, \tau) = \frac{\lambda L}{(\tau - \lambda)(\tau - L)}, \qquad (2)$$

where the parameters λ, L, τ are equal to $x_j - x_1, j = 2, 3, 4$, respectively.

Remark 1. The other definition of functions $p_i(\lambda, L, \tau), i = 1, 2, 3$ as coordinates of the vector $\vec{P} = [p_1, p_2, p_3]^T$, one can obtain through the vector product in the following view [1]:

$$\vec{P} = \frac{\vec{L} \times \vec{S}}{\sum_{i=1}^{3} w_i},$$

where $\vec{L} \times \vec{S} = [w_1, w_2, w_3]^T, \vec{L} = [\lambda, L, \tau]^T$ and $\vec{S} = [\lambda^2, L^2, \tau^2]^T$.

2.2. Some Properties of Weight Functions

Thus we obtain *the system* of three functions $p_i(\lambda, L, \tau), i = 1, 2, 3,$ which have the following principal and significant properties [1]:

1. In the projective geometry, *CR*- functions are known as *projective invariants,* i.e. the value of *CR*-functions is unchangeable with respect to a projective transform. In our case this means that

   $$p_i'(\lambda', L', \tau') = p_i(\lambda, L, \tau), i = 1, 2, 3.$$

   On the projective plane the λ, L and τ coordinates are homogeneous coordinates:

   $$\lambda : L : \tau = \lambda' : L' : \tau',$$

   i.e. the functions $p_i(\lambda, L, \tau)$ are independent of *shift* and *scaling* (a compression or stretching of ΔX_i by a factor $\mu \neq 0$) :

   $$p_i(\mu\lambda, \mu L, \mu\tau) = p_i(\lambda, L, \tau), i = 1, 2, 3.$$

2. *The normalization* of $p_i(\lambda, L, \tau)$:

$$\sum_{i=1}^{3} p_i(\lambda, L, \tau) = 1.$$

3. *The orthogonality* of two vectors $\vec{P}$ and $\vec{\Delta}$:

$$(\vec{P}, \vec{\Delta}) = \sum_{i=1}^{3} p_i(\lambda, L, \tau)\Delta Y_i = 0, \qquad (3)$$

where coordinates Y_i are defined by the equation of *straight line* or *quadratic parabola* (a, b, c - arbitrary real numbers)

$$Y = aX^2 + bX + c$$

in the corresponding coordinate X_i, i.e.

$$\vec{\Delta} = [\Delta Y_1, \Delta Y_2, \Delta Y_3]^T;$$

where $\Delta Y_i = Y_i - Y_0$, $i = 1, 2, 3$.

4. Due to $p_3(\lambda, L\tau) \neq 0$, three *inverse* functions $d_i(\lambda, L, \tau)$ can be obtained from the condition of the normalization:

$$d_1(\lambda, L, \tau) = \frac{-p_1}{p_3} = \frac{-1}{\lambda(L-\lambda)}\tau(\tau - L),$$

$$d_2(\lambda, L, \tau) = \frac{-p_2}{p_3} = \frac{1}{L(L-\lambda)}\tau(\tau - \lambda),$$

$$d_3(\lambda, L, \tau) = \frac{1}{p_3} = \frac{1}{\lambda L}(\tau - \lambda)(\tau - L), \qquad (4)$$

with

$$\sum_{i=1}^{3} d_i(\lambda, L, \tau) = 1.$$

These properties have been used for creation of DPT.

2.3. Discrete Projective Transformations (DPT)

From the orthogonality of $\vec{P}$ and $\vec{\Delta}$ one can get the following equation

$$Y_0 = (\vec{P}, \vec{Y}) = \sum_{i=1}^{3} p_i(\lambda, L, \tau)Y_i, \qquad (5)$$

which is satisfied by an arbitrary set of four uncoinciding points situated on the same curve from the family of straight lines or quadratic parabolas.

The variable coordinate $Y(\tau) \equiv Y_3$ can be expressed from (5) by means of vectors $\vec{D} = [d_1, d_2, d_3]^T$ and $\vec{Z} \equiv [Y_1, Y_2, Y_0]^T$:

$$Y(\tau) = (\vec{D}, \vec{Z}) = \sum_{i=1}^{3} d_i(\lambda, L, \tau)Z_i. \qquad (6)$$

If $x_0 = 0$, then taking into account (4) and $\tau = x$, the equation (6) can be written as

$$Y(x) = \frac{-Y_1}{\lambda(L-\lambda)}x(x - L) + \frac{Y_2}{L(L-\lambda)}x(x - \lambda) + \frac{Y_0}{\lambda L}(x - \lambda)(x - L).$$

This equation is *the parameteric* equation of the quadratic parabola or the straight line. The actual form of the curve depends upon the relative position of three points $(X_0, Y_0), (\lambda, Y_1), (L, Y_2)$ on the plane. All these coordinates are *direct measurement parameters* of the corresponding line.

Eqs.(5) and (6) define *the Discrete Projective Transformations* (DPT) or "4-points" transformations of the points, situated either on the stright line or on the quadratic parabola. If we fix on the curve (6) three points $(X_j, Y_j), j = 0, 1, 2$, then the fourth (variable) point (X, Y) of the curve will be projected by means of the weighted functions $p_i(\lambda, L, \tau)$ into point (X, Y_0), i.e. all points of the initial curve will be transformed on to points of the simpler geometrical line - the horizontal line $Y = Y_0$.

Using eq.(6), coefficients a and b of the curve $Y = aX^2 + bX + c$ can be written as follows:

$$a(\lambda, L) = \frac{1}{\lambda L(L-\lambda)}[\lambda \Delta Y_2 - L\Delta Y_1] \qquad (7)$$

and

$$b(\lambda, L) = \frac{1}{\lambda L(L-\lambda)}[L^2 \Delta Y_1 - \lambda^2 \Delta Y_2].$$

Fig.1 shows fragmentary graphics of functions p_i and $d_i, i = 1, 2, 3$ for fixed $\lambda_o = 1$ and for variable values L and τ.

Remark 2. Eqs.(5) and (6) can be used, in general, for arbitrary continuous curve $f(x)$. If we choose x_0 and fix on $f(x)$ two points

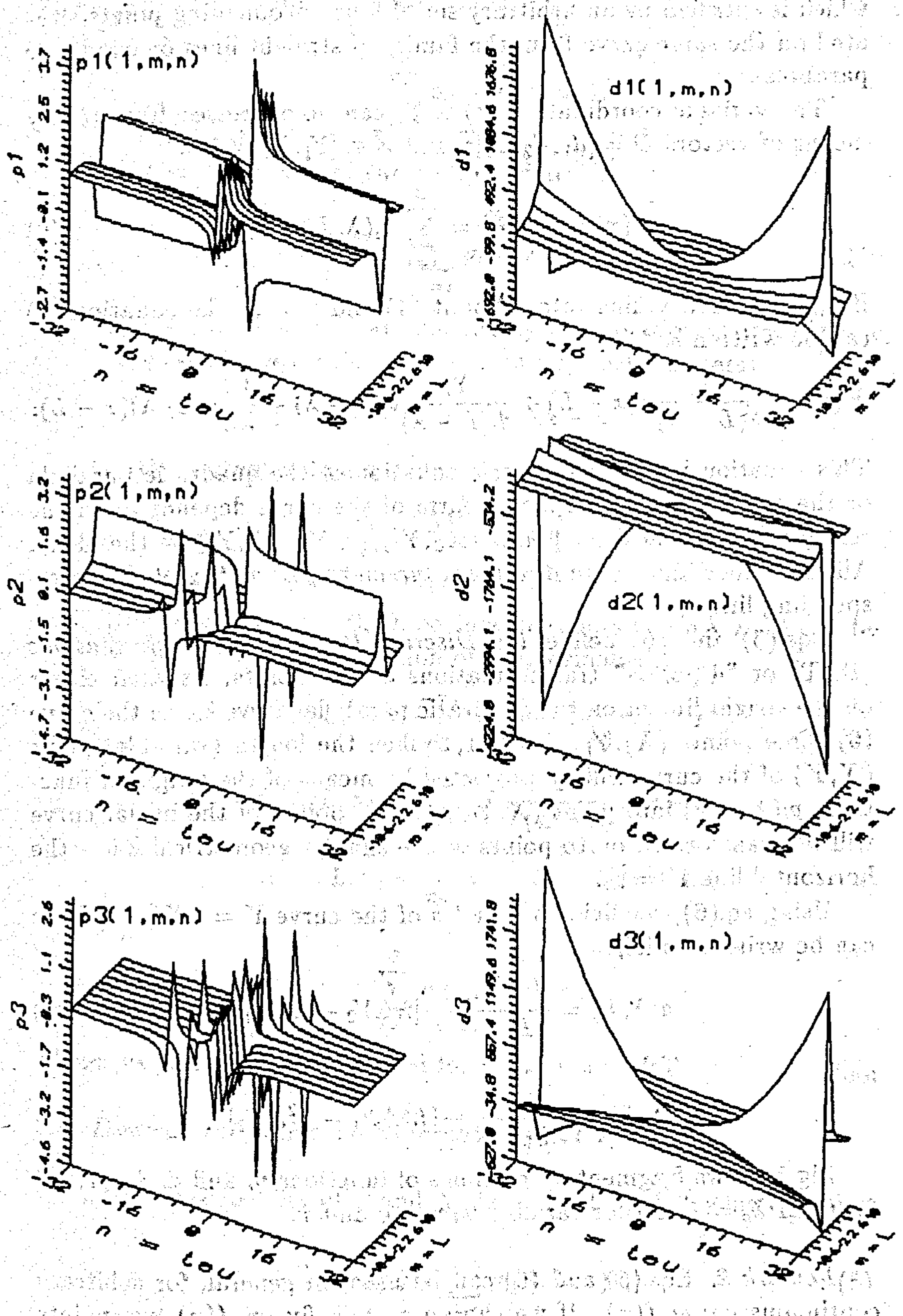


Fig.1 Fragments of graphics of *CR*-functions for $\lambda_o = 1$.

$f_1 \equiv f_\lambda = f(x_0 + \lambda)$ and $f_2 \equiv f_L = f(x_0 + L)$, then an arbitrary, not equally with the previous one, point $f_3 \equiv f_\tau = f(x_0 + \tau) \equiv f(x)$, is mapped by DPT into the corresponding point on the curve of the simpler geometrical form $h(\tau)$ as follows:

$$\mathcal{D}[f(x)] = h(\tau) = (\vec{P}, \vec{F}) \tag{8}$$

and vice versa

$$\mathcal{D}^{-1}[h(\tau)] = f(x) = (\vec{D}, \vec{H}),$$

where $\vec{P}$ and $\vec{D}$ are 3D vectors of weight functions and

$$\vec{F} \equiv [f_\lambda, f_L, f(x)]^T, \ \vec{H} \equiv [f_\lambda, f_L, h(\tau)]^T.$$

Let us note two main features of this transformation. First, the family of power functions $f(x) = x^n, n = 0, 1, 2, \cdots$ is of a special interest from the point of view of our transformations. It can be proved [1] that for $\vec{F} = [\lambda^n, L^n, \tau^n]^T, (x_0 = 0)$

$$h(\tau) = \mathcal{D}[x^n] = (\vec{P}, \vec{F}) = \lambda L \tau \sum_{i=1}^{n-1} \lambda^{i-1} \sum_{k=1}^{n-i-1} L^{k-1} \tau^{n-i-k-1},$$

i.e. h is the homogeneous function of λ, L and τ. It follows that the operation $\mathcal{D}[\cdot]$ decreases the power of multinomials by 2:

$$\mathcal{D}[\sum_{i=0}^{n} a_i x^{n-i}] = \sum_{i=0}^{n-2} b_i(\lambda, L) \tau^{n-2-i}.$$

In particular,

$$\mathcal{D}[c] = c, \mathcal{D}[bx + c] = bx_0 + c \text{ and } \mathcal{D}[ax^2 + bx + c] = ax_0^2 + bx_0 + c.$$

The second feature of DPT follows from (8) in transforming of the linear form $(x_0 = 0)$:

$$\mathcal{D}[\alpha\phi(x) + \beta\psi(x)] = \alpha h_\phi(x) + \beta h_\psi(x), \tag{9}$$

where $\phi(x), \psi(x)$ - arbitrary continuous functions and α, $\beta = Const.$ Eq.(6) allows to estimate the influence of measurement errors in coordinates (X, Y) on the result of transformations, that is important for using DPT in practice. Assuming a measurement of coordinates of actual points $(\tilde{X}, \tilde{Y})$ yields a random errors (ϵ_x, ϵ_y), which have the Gaussian distribution $N(0, \sigma)$, i.e.

$$\tilde{X} = X + \epsilon_x, \tilde{Y} = Y + \epsilon_y.$$

One can show, that the contribution in the total error of transformations owing to weight functions is small in the points remote from poles, therefore we can neglect ϵ_x, if $|\tau - \lambda| \gg 0$ and $|\tau - L| \gg 0$.
A total error in prediction of the coordinate $\hat{Y}(\tau)$ by (6) can be estimated as

$$|\epsilon_y(\tau)| < Q \cdot \epsilon_{max}, \quad (10)$$

where

$$Q(\lambda, L, \tau) = \sum_{i=1}^{3} |d_i(\lambda, L, \tau)|,$$

and

$$\epsilon_{max} = max\{|\epsilon_{y_1}|, |\epsilon_{y_2}|, |\epsilon_{y_0}|\}.$$

The coefficient Q can be used for choosing the parameters λ and L, which provide the permissible errors in the procedure of the starting prediction.

3. Adaptive Projective Filters

The new algorithm of discrete digital filters based on DPT and use of it for track segments finding is considered in this and next sections.

3.1 DPT as a Model of Linear System

We assume, that the model of track segments is defined by the linear-quatratic functions (*LQ-model*). Eqs.(3) and (5) are based on the *LQ*-model of TS and have the structure of *the linear regression*. Such structures are typical in the theory of the system identification.
It is well known [8], that the stationary linear system with constant parameters can be described by the pulse response or the weight function $G(\tau)$:

$$y(t) = \sum_{\tau=1}^{\infty} G(\tau)u(t-\tau) + \nu(t), \quad (11)$$

where $u(t), y(t)$ are input and output signals, $\nu(t)$ is the additive noise, and $t = 0, 1, 2, ...$ Often weight functions are presented by rational functions and usually are defined by special computing procedures.

In our case eq.(8) or (5) is the equation of the similar linear system, which gives satisfaction to (9) (additivity and homogeneity) with *nonconstant*, nonlinear functions p_i. As it follows from (2), these functions depend not only on $s_1 = (t - \lambda), s_2 = (t - L), s_3 = 0$, but also on the distance $(L - \lambda)$, on the coordinate t and on two fixed parameters - (λ, L). For discrete values $n = 0, 1, 2...$ and fixed λ, L the eq.(8) can be written as

$$h(n) = \sum_{i=1}^{3} G(n, s_i) f(n - s_i),$$

where $f(n)$ and $h(n)$ are input and output signals, which are connected by the correlation of the convolution.

The pulse response $G(n, s_i)$ of this system equals to p_3, i.e.

$$G = p_3(\lambda, L, n) = \frac{\lambda L}{(n - \lambda)(n - L)}, n \neq \lambda, n \neq L,$$

which is the reaction of the system (8) or (5) on the input unit signal u_0 :

$$u_0 = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0. \end{cases}$$

As it follows from (2), functions p_1 and p_2 are created from p_3 by means of formal substitutions: $\lambda \rightleftharpoons n$ and $L \rightleftharpoons n$ respectively.

In the case of the track segment, input signals are coordinates of measurement on TS points and output signals will be coordinates of points obtained by DPT, which for linear and parabolic shapes of TS are equal to the constant $(h = f_0)$.

For example, if $\tilde{Y}(n) - \tilde{Y}(n_0) = \tilde{\Delta}Y(n) = \Delta Y(n) + e(n)$ have measurement errors $e(n)$ with *zero means* and *normal* distribution, then following eq.(3), we get the structure of the system similar to the structure of the model of error equation [8]:

$$\varepsilon(n) = \sum_{i=1}^{3} G(n, s_i)\tilde{\Delta}Y(n - s_i) = 0 + \sum_{i=1}^{3} p_i(\lambda, L, n)e(n - s_i), \quad (12)$$

where $s_i = n - \lambda, n - L, 0.$

In eq.(12) the signal of the white noise is transformed by dynamical through the denominator of the system with the amplitude response $G = p_3(\lambda, L, n)$. Figs.2a, 2b show the graphics of $| G |$ on the discrete grid (m, n) for fixed $\lambda = \lambda_o$ in the logarithmic scale and corresponding contours of equal levels. It is necessary to note, that owing to properties of DPT, the choice of values n permits an arbitrary step, if only $n \neq \lambda$ and $n \neq L$.

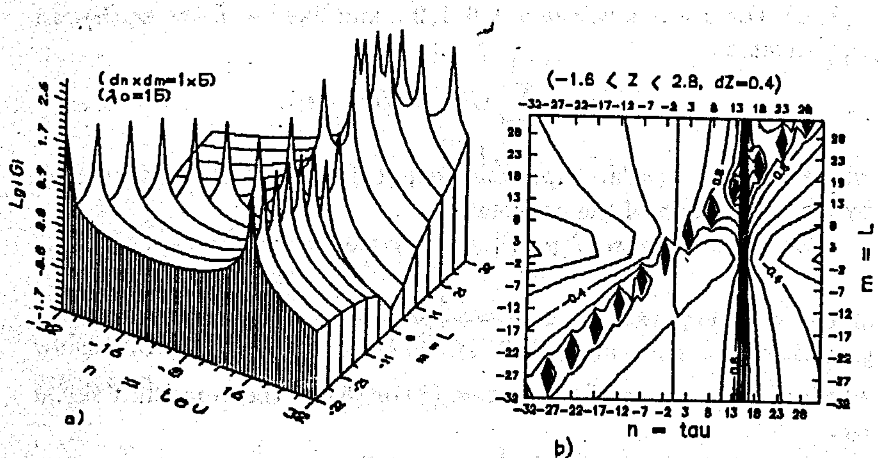


Fig.2 The family of pulse response functions:

(a) $| G |$ in the logarithmic scale, $\lambda_o = 1$;

(b) contours of equal levels for the same family.

The stability of the system (12) with respect to the measurement errors follows from the restriction of the input values and weight functions $p_i(\lambda, L, n)$ everywhere, exept two pole points situated on the real axes. As it is noted in [8], the *predictor* in such model reduces to the *linear regression.*

3.2 Algorithm of Adaptive Projective Filters

Above results are very good tools for solving the track finding problem. In most cases, actual data of a track image are sampled to form an image space consisting of $K_{max} \times M_{max}$ pixels or of discrete coordinates

$(X, \tilde{Y})$, which have a structure similar to the TV signals:

$$\mathcal{U} : \left\{ (X_k, \tilde{Y}_{km}) \mid k = 1, 2, ..., K_{max}; m = 0, 1, 2, ..., N_k \right\},$$

where K_{max} - a number of raws and N_k - numbers of points on k–th raw.

According to (6), the track segment (linear or curve) is defined completely by its three points (*parameters*): *the basis and two pivot points.* These points give us the crossing point $(\tilde{Y}_0)$ of TS with raw $X = X_0$ and the curvature of TS, which is defined by (7). As the basis point, one can choose an arbitrary point on the basis raw, while the $\tilde{Y}_\lambda$ and $\tilde{Y}_L$ are positioned on the poles $X = X_\lambda$ and $X = X_L$ accordingly (Fig.3).

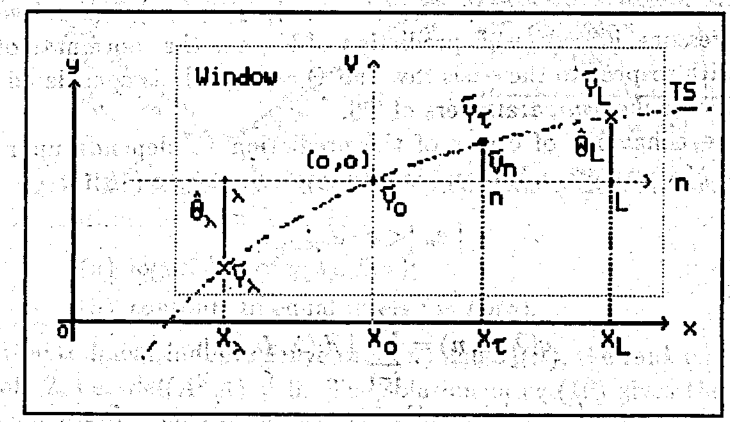


Fig.3 Sketch of main parameters of the track segment.

By shifting of the coordinate system to the basis point $(X_0, \tilde{Y}_0)$:

$$\tau_k = X_k - X_0,$$

$$\tilde{V}_{km} = \tilde{Y}_{km} - \tilde{Y}_0, \qquad (13)$$

we reduce the number parameters by 1, simultaneously changing errors $(e_v = \epsilon_y - \epsilon_0)$:

$$\lambda = X_\lambda - X_0,$$
$$L = X_L - X_0,$$
$$V_0 = 0,$$
$$\tilde{\theta}_\lambda = \tilde{Y}_\lambda - \tilde{Y}_0,$$
$$\tilde{\theta}_L = \tilde{Y}_L - \tilde{Y}_0. \qquad (14)$$

If $\Delta\tau$ is a step of discretization of values τ, then for $\tau_n = n\Delta\tau$ the weight functions p_i and d_i are defined in discrete points on raws $\pm 1, \pm 2, ..., \pm n, ...$ $(n \neq \lambda, n \neq L)$ in relation to the basis $n = 0$.

Using (12), (13) and (14), the $LQ-$ model of a track segment or *the predictor*, is written in the form

$$\hat{V}_n = (\vec{D}_n, \vec{\Theta}) + e(n), \qquad (15)$$

where $\vec{D}_n \equiv [d_1(n), d_2(n)]^T = [d_1(\lambda, L, n), d_2(\lambda, L, n)]^T$ is the vector of regressors, $\hat{V}_n = \hat{V}(n)$ is prediction of V_n, n is the coordinate of the raw with respect to the basis raw and $\vec{\Theta} = [\theta_\lambda, \theta_L]^T$ is considered as a vector of *unknown* parameters of TS.

The behaviour of errors of the prediction $\hat{V}_n$ depends upon the choice of parameters λ, L and is estimated similar to (10):

$$| e_v | < q \cdot e_{max},$$

where

$$q(\lambda, L, n) = \sum_{i=1}^{2} | d_i(\lambda, L, n) |,$$

and $\epsilon_{max} = max\{| e_{v_\lambda} |, | e_{v_L} |\}$. Figs.4a and 4b show the behaviour of $Lg(q)$ for fixed parameter λ_o. The model (15) is well known as a linear regression model, with given functions $d_i(n)$ (regressors) and unknown parameters θ_λ, θ_L. Discrete observations $\{\tilde{V}_n\}$ are obtained on the known subset of independent variables n with no measurement errors. If the data are unbiased and variances are equal to σ^2, then LSF-estimates of $\vec{\Theta}$ in (15) have important statistical properties: they are *unbiased* estimates and the least squares estimate has *the smallest* possible error (the Gauss-Marcov theorem) [10].

The normal equations of LSF for (15) are written in the matrix form

$$(A^T A)\vec{\Theta} = A^T \tilde{\vec{V}}, \qquad (16)$$

where A is $n \times 2$ matrix of $d_i(n), (i = 1, 2)$, $\vec{\Theta}$ is the vector of parameters and

$$\tilde{\vec{V}} = [\tilde{V}_1, \tilde{V}_2, ..., \tilde{V}_n]^T$$

is the vector of measurements.

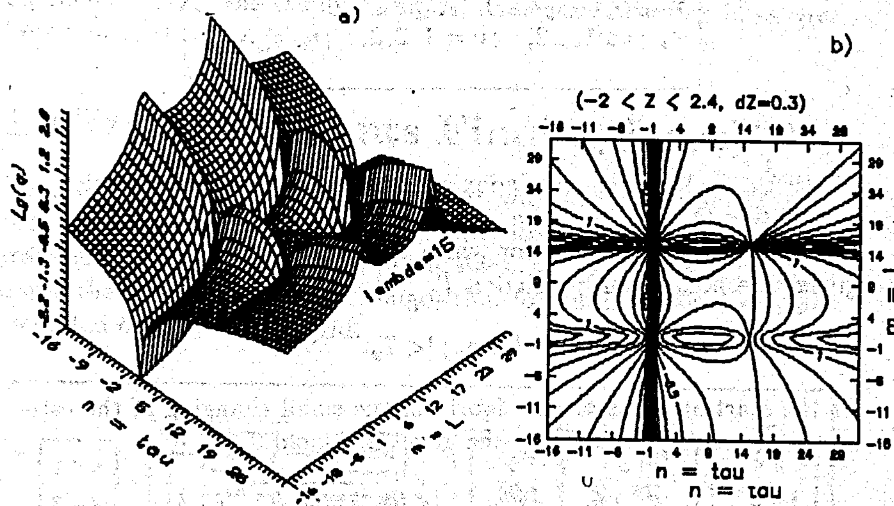


Fig.4 The estimation of the error prediction q as a function of τ and the parameter L for $\lambda_o = 15$:

(a) logarithm of $q(\lambda_o, L, \tau)$;

(b) contours of equal levels for $Lg(q)$.

Due to linear independence of $d_1(n)$ and $d_2(n)$, the rank of $(A^T A)$ equals 2, i.e. $det(A^T A) \neq 0$. The solution of eq.(16) gives the LSF-estimate of $\vec{\Theta}$ *(the corrector)* in the form

$$\hat{\vec{\Theta}} = (A^T A)^{-1} A^T \tilde{\vec{V}}. \qquad (17)$$

The difference or the residual r between predicted and measurement coordinates is defined as

$$r(n) = \hat{V}_n - \tilde{V}_n. \qquad (18)$$

Measurement coordinates $\tilde{Y}_0, \tilde{Y}_\lambda$ and $\tilde{Y}_L$ are used for finding the starting values $\tilde{\theta}_\lambda$ and $\tilde{\theta}_L$ by (14). Now, using equations (15),(17) and (18), we can construct the *recurrent algorithm* for finding the fourth coordinate of TS on the $n_\nu - th$ raw ($\nu - 1$ is the number points, which have been included in TS before).

$$
\begin{array}{l}
\hat{V}_{n_\nu} = (\vec{D}_{n_\nu}, \hat{\tilde{\Theta}}_{n_{\nu-1}}), \\
r(n_\nu) = \hat{V}_{n_\nu} - \tilde{V}_{n_\nu}, \\
\hat{\tilde{\Theta}}_{n_\nu} = (A^T_{n_\nu} A_{n_\nu})^{-1} A^T_{n_\nu} \vec{\tilde{V}}_{n_\nu}, \\
n_\nu = \pm 1, \pm 2, ...; \nu = 1, 2, 3, ...; n_\nu \neq \lambda, L,
\end{array}
\tag{19}
$$

where $\hat{\tilde{\Theta}}_{n_0} = [\tilde{\theta}_\lambda, \tilde{\theta}_L]^T$, the index n_ν points out that in estimation of parameters $\vec{\Theta}$ were used the values of regressors on n_ν–th raw and the actual coordinate $\tilde{V}_{n_\nu}$, for which the residual $r(n_\nu)$ is less than a given threshold of the prediction T_p

$$| r(n_\nu) | < T_p.$$

At the starting steps of the algorithm the small changing of the estimates $\hat{\theta}_\lambda$ and $\hat{\theta}_L$ is tested by the given threshold T_θ

$$| \hat{\theta}_{in_\nu} - \hat{\theta}_{i,n_{\nu-1}} | < T_\theta, i = \lambda, L.$$

Remark 3. When three parameters are used then $\vec{\Theta} = [Y_\lambda, Y_L, Y_0]^T$, $\vec{D} = [d_1(n), d_2(n), d_3(n)]^T$ and A will be $n \times 3$ matrix of $d_i(n)$, which are also linear independent $(i = 1, 2, 3)$.

Elements of the matrix A_k and of vector $\vec{D}_k$ are tabulated as L.U.T. inside the selected window. After that the calculation of the new prediction by (19) needs only three short operations. To define the new $\hat{\tilde{\Theta}}_{n_\nu}$ the accumulated on the previous steps sums are used. This procedure takes about 18 short and one long arithmetical operations. It should be noted, that after achievement of the sufficient accuracy of estimates, the correction procedure is turned off. The total number of operations per step of the algorithm (19) is estimated by 27. This number can be essentialy reduced by parallel or systolic procedures.

Algorithm (19) has a structure of the *adaptive* algorithm [8], in which the prediction on the next step uses the 'information state', accumulated before. It is clear, that the 'information state' is contained in $\hat{\vec{\Theta}}$ and in measurements, which have been selected before. Filters, which use the adaptive algorithm, are colled *adaptive filters*. The *'information feedback'* is utilized in the creation of that filters.

Digital filters (19) depend on two parameters (λ, L) and are based on the determinated regressors $d_i(\lambda, L, \tau), i = 1, 2$, which are the projective invariants, defined by cross-ratio of four collinear points (1). Therefore, (19) is the family of filters, named as *Adaptive Projective Filters* or APF.

4. Track Segments Finding by APF

An arbitrary image of multitrack events with backgrounds and gaps can be projected on some pixel grid, where discrete coordinates of separate points are arranged by raws and lines (see subsection 3.2). Fig.5 shows the diagram of the APF-algorithm for finding track segments presented on the discrete grid.

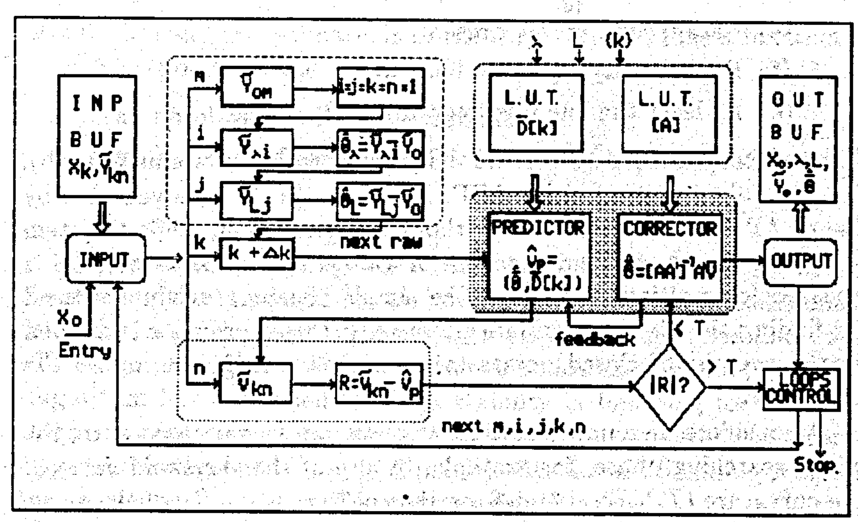


Fig.5 The diagram of use APF-algorithm for track segments finding.

The position and the shape of the individual TS on such image are defined only by three pivot points (triplet). These points $(\tilde{Y}_0, \tilde{Y}_\lambda, \tilde{Y}_L)$ we take on the three different raws: the basis and two poles, which remote from the basis on distances λ and L. In the real situation we have N_0, N_λ and N_L points (including backgrounds) on every raw respectively. Let us assume, that $N_0 \times N_\lambda \times N_L \neq 0$. Generally, the separate basis coordinate $\tilde{Y}_0$ is the common point for $N_{\lambda L} = N_\lambda \times N_L$ possible track segments.

If the triplet $T_{ij}, (i = 1, 2, ..., N_\lambda, j = 1, 2, ..., N_L)$ of such pivot coordinates are coordinates of true points on the actual TS, then all other points of TS are find by the APF-algorithm with the simultaneous correction of pivot coordinates. For look over all possible T_{ij} in case of the only basis point the number of operations is proportional to $N_{\lambda L}$ and strongly increases for large N_λ and N_L.

Generally, main structured peculiarities of the track segment are:

(a) TS have LQ-model
(b) Points are arranged irregular along TS
(c) Diametrical coordinates are scattered
(d) The minimal number of points on TS is limited by 4
(e) Background points present
(f) High density of tracks
(g) At least the one pivot coordinate is absent.

If true pivot coordinates of the TS are known, then items (a), (b), (c) and (e) are removed by APF. The item (d) is removed also by use of APF for one point but without the correction, while the item (f) creates great problems because of a huge number of wrong TS is apeared for multitrack events. The simple claster procedure is used for finding the pivot coordinate (item (g)): this coordinate is defined as the average of closed points taken on raws, neighbouring to the pivot raw.

A considerable number of 'false triplets' can be rejected yet on the initial searching phase, for example, by use of the threshold value of the curvature (T_c). Eq. (7) defines the coefficient $a(\lambda, L)$ of the curve passing over three points. Thise value is proportional to the curvature of the possible TS and can be used for rejecting of 'false triplets' by the effective criterion

$$| a_{ij}(\lambda, L) | \leq T_c,$$

where ij point out indices of pivot points. Figs.6a and 6b show the behavior of $Z = | a(M, J) |$ and of it contours of equal levels, obtained for fixed λ_o and $I_o (\lambda_o = -25, I_o = 15, T_c = 0.1, \Delta Z = 0.003)$, where $M = L - \lambda_o$, $I_o = \tilde{\Delta} Y_{io}$ and $J = \tilde{\Delta} Y_j$. For example, if we chose the

parameter $M = L - \lambda_o = 10$, then almost 70% of the total number of possible triplets can be rejected as wrong (Fig.6b).

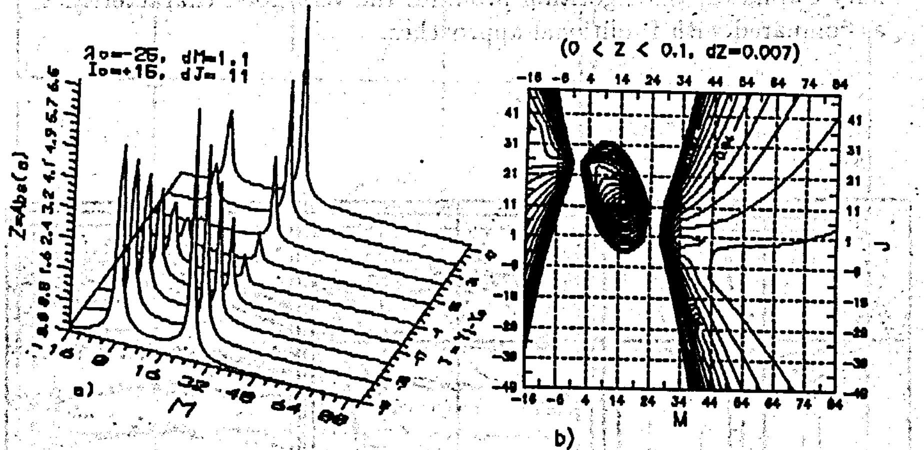


Fig.6 Three-points estimations of the curvature of TS as a function of $J = \Delta Y_j$ (points on the L raw) and of the parameter $M = L - \lambda_o$:

(a) absolute values of estimations;
(b) contours of equal levels for $0 <| a(M, J) |< 0.1, \Delta Z = 0.007$.

The performance of the APF has been realize as the TP5-program for finding the simulated on the image space (100 × 100 pixels) linear and/or quadratic track segments with errors, backgrounds and gaps. The examples of runs of the program are showen on Figs.7, 8, 9. The algorithm has confirmed the high stability in following for the track segment with respect to the wide variations of measurement errors, gaps and backgrounds ($| \epsilon_y | \leq 5$ pixels). Results of TS finding program are presented by means of histogramming of values

$$N_c = N_o + \hat{\theta}_\lambda p_1(\lambda, L, n) + \hat{\theta}_L p_2(\lambda, L, n) + \tilde{V}_n p_3(\lambda, L, n),$$

where $N_o = \frac{N}{2}$, N is the number of pixels on the raw in the window. In accordance with eq.(3), the track segment with $N_p \geq N_{min}$ points gives the pick of the histogram in the neighbourhood of $N_c = N_o$.

The speed of APF-algorithm depends on the number of TS-points, which are used in the dynamical process of the adaptation and on the number of background points. The efficiency of the pattern recognition can be tuned by the choice of λ and L and other parameters of the algorithm $(T_c, T_p, T_\theta,$ etc.). In spite of the obtained preliminary estimates, the algorithm promises the very good characteristics as compared with traditional approaches.

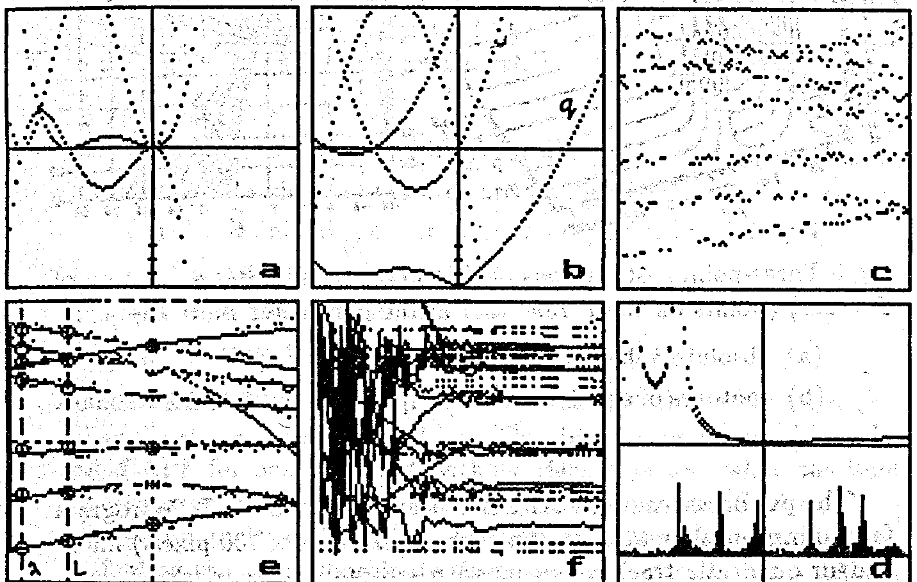


Fig.7 Examples of the output of the TS finding program for simulated images ($\lambda = -46, L = -34, |\epsilon_y| \leq 4$) on the grid 100×100 pixels:

- (a) L.U.T. for matrix A;
- (b) L.U.T. for vectors $\vec{D}$ and the function $q(\lambda_o, L_o, k)$;
- (c) the simulated input image;
- (d) histogramming picks of the detected track segments;
- (e) pivot raws, pivot points and intermediate output of the APF-algoritm;
- (f) the reaction of the system on actual signals and estimations for $\hat{\Theta}_\lambda, \hat{\Theta}_L$ (horisontal tracks).

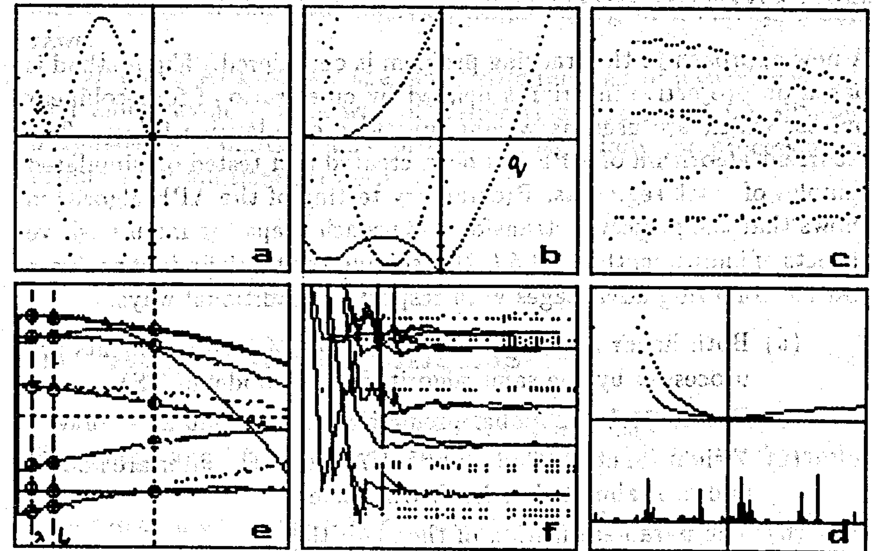


Fig.8 The case, when two TS with no basis points are not detected ($\lambda = -45, L = -39, |\epsilon_y| \leq 2$).

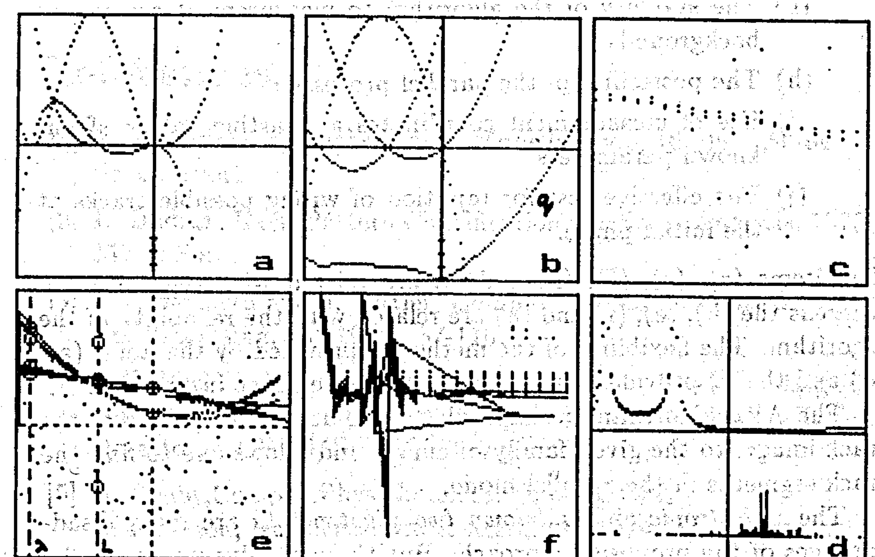


Fig.9 The example of detecting of two closed track segments ($\lambda = -45, L = -20, |\epsilon_y| \leq 1$).

5. Conclusion

A new approach to the tracking problem is considered. The method is based on projective invariants defined by cross ratio of four collinear points, which are used as weight functions of adaptive filters. The recurrent algorithm of APF has been created and tested on simulated samples of track segments. Preliminary testing of the APF-algorithm shows that the projective transform approach keeps the main positive aspects of known methods [4,5,6,7] in solving the track finding problem and has following advantages with respect to traditional ways:

(a) Both linear and/or quadratic shapes of track segments are processed by the same algorithm (LQ-model of TS).

(b) Use of local and global predictions

(c) Weight functions and matrix elements of the filter are known and are tabulated in Look Up Tables

(d) The parameterization of the algorithm

(e) The high accuracy of the prediction of new point on TS

(f) Use of the information feedback in the track following procedure

(g) The stability of the algorithm to measurement errors and backgrounds

(h) The possibility of the parallel processing

(i) Use of measurement coordinates as starting values of unknown parameters

(j) The effective test for rejection of wrong possible tracks at the initial phase.

The items (c), (e), (f), (h) and (j) are releted with the speed of, whereas the (b), (e), (f) and (g) are related with the reliability of the algorithm. The flexibility of the method is provided by the items (a), (d) and the (i) provides the ease and the convenience in practice.

The APF algorithm is adaptable to the actual conditions of the track image, to the given family of curves and allows one to find the track segments in the parallel mode.

The *unisotropic* and *unknown two pivot* points are main disadvantages of the proposed approach. But these disadvantages can be removed by simple procedures of the rotation of the coordinate system

and of choice of the pivot point on neighbouring with basis and pole raws.

DPT and APF satisfy the many requirements of pattern recognition and provide a wide way of elaborating algorithms for different purposses in a digital image processing, especially for processing of complicated multitrack events.

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