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V.P.Gerdt, W.Lassner*

VERIFYING ISOMORPHISMS OF FINITE DIMENSIONAL LIE ALGEBRAS BY GRÖBNER BASIS TECHNIQUE

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*Department of Computer Science, University of Leipzig, Augustusplatz 10-11, 0-7010 Leipzig, Germany

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1. Introduction

Lie algebras and groups play an important role in many areas of modern applied mathematics and theoretical physics. Among them there are soliton mathematics, quantum mechanics, nuclear and elementary particle physics, nonlinear optics, accelerator physics and others. One of the most exiting application of Lie algebra methods is symmetry analysis of differential equations, especially of nonlinear ones [1]-[10]. The construction of explicit form of infinitesimal symmetry generators and knowledge of their Lie algebra structure allows to get valuable information on the given ordinary or partial differential equations and often to simplify or even to integrate them by the method of symmetry reduction [5].

At each stage of construction of infinitesimal symmetry generators and investigation of their algebraic structure one has often to carry out tedious algebraic manipulations with symbolic mathematical objects. Intensive development of computer algebra algorithms and software over last years allows to provide a number of modern computer algebra systems such as REDUCE, AXIOM, MACSYMA with efficient software packages [6, 7, 8] for finding the explicit form of the classical or point Lie symmetry generators and their commutation relations. In addition to these packages a special-purpose computer algebra system DELiA [9] has been developed just for symmetry analysis of differential equations on an IBM PC. Computer algebra packages are also available (see [10] and refs. therein). for the construction of generalized or Lie-Bäcklund symmetries [2].

The infinitesimal symmetry generators being linear differential operators form a Lie algebra. The procedure of finding an explicit form of generators is very complicated one, and may lead to different mathematical expressions for generators of a given differential equation or a system of such equations. Therefore, it is very important to have an constructive approach to verifying whether two sets of generators determine essentially the same, i.e. isomorphic Lie algebras, or different ones. Furthermore, in paper [12] algorithm was developed for computation of the structure constants of symmetry Lie algebra without integrating determining systems of partial differential equations for generators. It underlines our idea to start with given structure constants. If a symmetry Lie algebra of differential equations can be identified as a member of available tables or data bases, one is able to use known properties for further investigations, reduction of the order of ODEs, similarity reduction of PDEs or even their explicit integration. In the case of isomorphism it may be useful to find an explicit form of basis transformation between both algebras.

Efficient computer-aided approach to that isomorphism verification for finite-dimensional Lie algebras and to computation of underlying basis transformation as well as knowledge of their complete automorphism group is important [11] in connection with recent achievements on construction of efficient algorithms and software packages for solving the fundamental problem of identification and classification of Lie algebras and its subalgebras (see, for example, review [5] and also [13]).

In the present paper we present a straightforward approach (*i*) to decide whether two finite-dimensional Lie algebras are isomorphic or not, (*ii*) to identify given Lie algebra as a member of known complete tables or data bases of Lie algebras and (*iii*) to construct a transformation matrix for two isomoprhic Lie algebras. These problems are transformed into commutative algebra as question on the existence and the construction of the solution of a related system of quadratic equations. This allows to use modern constructive techniques in the theory of polynomial ideals based on construction of Gröbner bases [14] and implemented in the form of appropriate computer algebra packages.

2. Basic System of Algebraic Equations

Let $\mathbf{F}$ be the field of real ($\mathbf{R}$) or complex ($\mathbf{C}$) numbers and L and $\tilde{L}$ be Lie algebras over $\mathbf{F}$ of dimension N defined by their structure constants c_{ij}^k and $\tilde{c}_{ij}^k$

$$L: \quad [X_i, X_j] = c_{ij}^k X_k, \qquad \tilde{L}: \quad [\tilde{Y}_i, \tilde{Y}_j] = \tilde{c}_{ij}^k \tilde{Y}_k, \tag{1}$$

in bases $\{X_i\}$ and $\{\tilde{Y}_i\}$ $(i = 1, 2, \ldots, N)$ of the Lie algebras L and $\tilde{L}$, respectively. Here and below we use a convention on summation over the repeated indices.

Assume the existence of an isomorphism

$$\phi : \tilde{L} \longrightarrow L.$$

We denote the image of the basis elements $\tilde{Y}_i$ by $Y_i := \phi(\tilde{Y}_i)$. The set $\{Y_i\}$ forms a basis of L with the structure constants $\{\tilde{c}_{ij}^k\}$:

$$[Y_i, Y_j] = \tilde{c}_{ij}^k Y_k. \tag{2}$$

So there exists a basis transformation represented by a $N \times N$ - matrix $A = (a_i^j)$ with $a_i^j \in \mathbf{F}$.

$$Y_i = a_i^j X_j, \quad det(A) \neq 0. \tag{3}$$

If we replace the $Y's$ in (2) according to (3), we get for the l.h.s. of (2)

$$[Y_i, Y_j] = a_i^k a_j^l [X_k, X_l] = a_i^k a_j^l c_{kl}^n X_n \tag{4}$$

and for the r.h.s. of (2)

$$[Y_i, Y_j] = \tilde{c}_{ij}^m Y_m = \tilde{c}_{ij}^m a_m^n X_n. \tag{5}$$

Comparison of (4) and (5) shows that the elements of the transformation matrix must satisfy the relations

$$a_i^k a_j^l c_{kl}^n - \tilde{c}_{ij}^m a_m^n = 0. \tag{6}$$

For given structure constants $\{c_{ij}^k\}$ of L and $\{\tilde{c}_{ij}^k\}$ of $\tilde{L}$ (6) is a system of at most $N \binom{N}{2} = N^2(N-1)/2$ quadratic equations in N^2 unknowns a_i^j. It follows from the anti-symmetry of the structure constants $c_{ij}^k = -c_{ji}^k$.

To use the Gröbner basis technique [14] we add one more variable d and one more equation $d - det(a_i^j) = 0$ to system (6). So we have shown that the answer to the isomorphism problem is given by the following theorem.

Theorem 1. Two N-dimensional algebras L and $\tilde{L}$ over $\mathbf{F}$ given by the structure constants $\{c_{ij}^k\}$ and $\{\tilde{c}_{ij}^k\}$ are isomorphic if and only if the system of equations

$$\begin{aligned} a_i^k a_j^l c_{kl}^n - \tilde{c}_{ij}^m a_m^n &= 0, \\ d - det(a_i^j) &= 0 \end{aligned} \tag{7}$$

in unknowns $\{a_i^j, d\}$, $(i, j = 1, \ldots, N)$ has a solution in $\mathbf{F}$ with $d \neq 0$.

But let us stress the following consequence of Theorem 1. If the Lie algebras L and $\tilde{L}$ are not isomorphic, then we must get $d = 0$ from (7).
Because of linearity of (7) in d a Gröbner basis G [14] of the ideal generated by the polynomial in the l.h.s. of (7) in the complex case $\mathbf{F} = \mathbf{C}$ shows this fact very explicitly, i.e. $d \in G$. To be more precise, this fact follows from the linearity of (7) in d and the possibility to choose an ordering with d having the highest precedence.
It is clear that (7) has always the trivial solution $a_i^j = 0$, $d = 0$. Moreover, the following theorem takes place.

Theorem 2. If the system (7) has a non-trivial solution, then it has infinitely many solutions.

Proof. Let matrix $A = (a_i^j)$ be a non-trivial solution of (7). For given two Lie algebras the system of equations (7) is completely determined by their structure constants. Hence, the solution space is invariant under transformations which will not change the structure constants $\{c_{ij}^k\}$ and $\{\tilde{c}_{ij}^k\}$. So, looking at the derivation of our system of equations (6) one immediately sees that one can act first with an automorphism of $\tilde{L}$ represented in the basis Y_i by a matrix $\tilde{T}$ and the structure constants $\{\tilde{c}_{ij}^k\}$ remain unchanged. In the same way one can after the transformation A in (3) apply an automorphism of L represented in the basis X_i by a matrix T and the structure constants $\{c_{ij}^k\}$ remain unchanged.
Therefore, for any solution A

$$B = \tilde{T}^{-1} A T, \tag{8}$$

is also a solution where T and $\tilde{T}$ are automorphism matrices of L and $\tilde{L}$ respectively.
Now, let T be an inner automorphism of the Lie algebra L. It has the following general form

$$T_x = exp(\alpha\, ad_x), \quad ad_x\, y = [x, y], \quad x, y \in L, \tag{9}$$

where α is a parameter with values in $\mathbf{F}$. If L is not an abelian Lie algebra, then there exists at least a one-parametric set of automorphisms and hence infinitely many solutions of (7). The case of abelian L can be neglected here as trivial one. □

With respect to the proposed computer algebra approach to the isomorphism problem it seems of interest to mention the following corollary.

Corollary. Let L be a N-dimensional Lie algebra over $\mathbf{F}$ given by the structure constants $\{c_{ij}^k\}$ in an basis $\{X_i\}$. The automorphisms of L are given by the system

$$a_i^k a_j^l c_{kl}^n - c_{ij}^m a_m^n = 0,$$
$$d - det(a_i^j) = 0$$

in unknowns $\{a_i^j, d\}$, $(i,j = 1,\ldots,N)$ as solution in $\mathbf{F}$ with $d \neq 0$.

3. Gröbner Basis Technique and ASYS Package

To investigate and solve eqs.(7) we need a method which could be applied to nonlinear algebraic equations with infinitely many solutions. For $N > 3$ the system under consideration is probably too large to be solved by hand. Effective computer algebra methods, algorithms and software packages are necessary attributes of practical computations.

For our purposes we select the Gröbner basis technique as an universal, constructive and computer-aided tool of commutative polynomial algebra [14] which allows to deal with polynomial ideals of either zero or positive dimension. All the modern general-purpose computer algebra systems have special built-in modules for a Gröbner basis computation. However, most of them were designed mainly for zero dimensional ideals and have no special facilities for investigation and solving algebraic equations with infinitely many solutions.

In the present paper we use the ASYS package [15] written in REDUCE [19], based on the Gröbner basis technique [14] and especially developed to investigate and solve polynomial algebraic equations with infinitely many solutions. The package was already successfully used in integrability analysis of polynomial-nonlinear evolution equations with arbitrary parameters [10]. ASYS has a number of special facilities to attack complicated algebraic systems generating positive dimensional ideals:

- **Reduction into subsystems by maximal sets of independent variables**
  In practice, in order to deal with positive dimensional polynomial ideals one needs information on its dimension and on independent sets of polynomial variables [16] which could be treated as free parameters. If any of the maximal independent sets is considered as a parametric set, then the lexicographical Gröbner basis with respect to the remaining variables has a triangular form. Therefore, the problem is reduced to univariate polynomials over rational coefficient fields $Q(a_1,\ldots,a_k)$ where $\{a_1,\ldots,a_k\}$ is one of the maximal sets of independent variables (parameters) according to [16]. ASYS allows to compute all possible maximal sets of independent variables for a given ordering and to execute the corresponding reduction to an equivalent set of triangular subsystems over a rational function coefficient field.

- **Homogeneity reduction**
  If under an appropriate scale transformation of unknowns each monomial of each given polynomial of the initial algebraic system has the same scale factor, the system could be transformed into an equivalent set of subsystems with smaller numbers of variables [15]. Because of double exponential complexity of Buchberger algorithm in the number of variables such a reduction allows a drastically speed up in the computing time of investigating and solving algebraic equations by the Gröbner basis technique. The ASYS package has built-in facilities for homogeneity analysis and the reduction of polynomial equations systems into subsystems.

The user of ASYS may combine homogeneity reduction with the reduction by maximal independent sets to reduce a given system into smaller subsystems with finitely many solutions.

It should be noted that the structure constants in (7) may include parameters, which can be treated as additional unknowns in order to determine their values for providing an isomorphism. This problem arises, first, in symmetry analysis when free parameters are present in differential equations. Second, in known tables [18] different classes of isomorphic Lie algebras are sometimes summarized and represented by parametrized families of classes. The parameters occur in the structure constants and are real in the case of real Lie algebras and sometimes restricted to certain intervals of $\mathbf{R}$. In Sect.5 a relevant example of a one-parametric family of four-dimensional Lie algebras is considered which illustrates our method in the presence of parameters.

4. Computational Strategy

A general case, heuristically optimal computational strategy for verifying an isomorphism for given two Lie algebras consists of the following successive steps:

1. Generation of the system of nonlinear algebraic equations (7).

2. Analysis of homogeneity properties of the system and, in the case of their existence, reduction of the system into subsystems with reduced number of variables treating the remaining (homogeneous) variables as free parameters.

3. Construction of the Gröbner basis for the ideals generated by the subsystems of step 2. At this step the lexicographical ordering with d in (7), being of "highest" variable such that the other variables are arranged in heuristically optimal ordering [17], is preferable. If one obtains d as an element of the Gröbner basis this means that the corresponding subsystem does not contribute to an isomorphism. Such subsystems are to be omitted. If there are no other subsystems in the output this means that the Lie algebras are not isomorphic as complex ones. Otherwise, if at least one subsystem does not contain "d" in the list of the Gröbner basis elements, then the complex Lie algebras are isomorphic. In the

case of real Lie algebras and absence of d in the Gröbner basis of a subsystem one has to go to the next step.

4. Because of zero-dimensionality of the ideals for the subsystems obtained at step 3 one has to investigate successively each subsystem starting from their univariate polynomial in the "lowest" variable with respect to the chosen ordering. Generally, these polynomials have rational function coefficients with respect to the parametric set.

We observed that in numerous concrete cases of isomorphism problem the following interesting fact. If the Lie algebras are not isomorphic, then in each subsystem the univariate polynomial in the "lowest" variable is a quadratic one with a negative discriminant for $d \neq 0$. Consequently, if that heuristic criterion is violated in at least one subsystem then the Lie algebras are isomorphic as real ones.

This interesting observation that the nonexistence of real solutions of a subsystem is already expressed by that property of the last equation, could be understood in relation to the paper [20].

5. Demonstration of the Method: Four-dimensional Lie Algebras

In this section we consider two examples of four-dimensional Lie algebras. Initial algebraic systems as well as results of the computations in accordance with the strategy of Sect.4 are given just in the form of the REDUCE input for the ASYS package and of its output.

Example 1

Identification of the Lie Algebra generated by classical (Lie) symmetry analysis [1, 2, 3, 4] of the well-known Korteweg-de Vries equation

$$u_t + u_{xxx} + u_x u = 0. \tag{10}$$

The symmetry Lie algebra of (10) is generated by differential operators

$$\begin{aligned} v_1 &= \partial_x \\ v_2 &= \partial_t \\ v_3 &= t\partial_x + \partial_u \\ v_4 &= x\partial_x + 3t\partial_t - 2u\partial_u \end{aligned}$$

with a commutator table

	v_1	v_2	v_3	v_4
v_1	0	0	0	v_1
v_2		0	v_1	$3v_2$
v_3			0	$-2v_3$

We try to identify this Lie algebra as one of the table [18] of all real four-dimensional Lie algebras up to isomorphism. The explicit form of the algebraic equations (7) for the isomorphism problem of this symmetry Lie algebra and one of the class Lie algebra $A^b_{4,9}$ taken from that table in the classification of paper [18]

$$[e_2, e_3] = e_1, \ [e_1, e_4] = (1+b)e_1, \ [e_2, e_4] = e_2, \ [e_3, e_4] = be_3, \ (-1 < b \le 1)$$

with all other commutators being zero, is generated by the separate REDUCE program ISOLIE and consists of 23 equations in 18 unknowns including b.

```
Z1 := 3*(A11*A24 - A14*A21);
Z2 := A11*A23 + A12*A24 - A13*A21 - A14*A22;
Z3 := - 2*(A13*A24 - A14*A23);
Z4 := 3*(A11*A34 - A14*A31);
Z5 := A11*A33 + A12*A34 - A13*A31 - A14*A32;
Z6 := - 2*(A13*A34 - A14*A33);
Z7 := - (B*A11 + A11 - 3*A11*A44 + 3*A14*A41);
Z8 := - (B*A12 + A12 - A11*A43 - A12*A44 + A13*A41 + A14*A42);
Z9 := - (B*A13 + A13 + 2*A13*A44 - 2*A14*A43);
Z10 := - A14*(B + 1);
Z11 := - (A11 - 3*A21*A34 + 3*A24*A31);
Z12 := - (A12 - A21*A33 - A22*A34 + A23*A31 + A24*A32);
Z13 := - (A13 + 2*A23*A34 - 2*A24*A33);
Z14 := - A14;
Z15 := - (A21 - 3*A21*A44 + 3*A24*A41);
Z16 := - (A22 - A21*A43 - A22*A44 + A23*A41 + A24*A42);
Z17 := - (A23 + 2*A23*A44 - 2*A24*A43);
Z18 := - A24;
Z19 := - (B*A31 - 3*A31*A44 + 3*A34*A41);
Z20 := - (B*A32 - A31*A43 - A32*A44 + A33*A41 + A34*A42);
Z21 := - (B*A33 + 2*A33*A44 - 2*A34*A43);
Z22 := - B*A34;
Z23 := DET MAT ((A11,A12,A13,A14),(A21,A22,A23,A24),
                (A31,A32,A33,A34),(A41,A42,A43,A44)) - D;
```

Then the ASYS package is applied with the switches **setord, setdim, setgb, scale** on, where

setord generates heuristically optimal ordering [17] for (7).

scale realizes homogeneity reduction. System (7) is splitted into subsystems such that any solution of the subsystem is a solution of the whole system.

setdim computes all the maximal independent sets of variables (parameters).

setgb makes the further reduction to the subsystems over rational function field and computes their Gröbner bases.

The combination of **scale** and **setgb** is the powerful instrument which leads to the following result of the considered example. The output consists of various subsystems. They are explicitly characterized first by those variables of the original system which has to be zero, second, by those variables of the original system which can be considered as free parameters, and third, by a small number of variables occurring in the Gröbner basis of the subsystem.

As mentioned in Sect.3, there are two types of parameters in the final subsystems [15]. Parameters coming from homogeneity cannot take the value zero. Parameters resulting from an independent set could take any value in **F**.

The first output line gives the heuristically optimal ordering

Order=(D A22 A32 A42 A12 A21 A23 A31 A33 A41 A43 A11 A13 B A44 A24 A34 A14)

If one therefore neglects all output subsystems with $d = 0$, one is focussed immediately to the following output of subsystem

---

Variables = (D A12 B A44)
Parameters = (A21 A33) % nonzero, from homogeneity
Zeros = (A11 A13 A34 A14 A41 A43 A24 A22 A32 A42 A23 A31)

GROEBNER BASIS

$$
\begin{aligned}
G(1) &= D + \frac{1}{3} * A21^2 * A33^2 \\
G(2) &= A12 - A21 * A33 \\
G(3) &= B + \frac{2}{3} \\
G(4) &= A44 - \frac{1}{3}
\end{aligned}
\tag{11}
$$

We emphasize the following facts.

- Subsystem (11) determines the parameter b to the value -2/3.
- It shows immediately that the transformation is a real one and so it identifies the KdV symmetry algebra as $A_{4,9}^{-2/3}$.
- One immediately obtains the real isomorphism matrix. The last equation is linear in the single variable $A44$. Already this observation in *one* subsystem with $d \neq 0$ ensures the existence of real isomorphism.
  Each variable occurs linearly as leading term in one of the equation. This reflects a general observation on the structure of our subsystems and is in agreement with results of paper [20].

For zero-dimensional ideal generated by (11) there is not necessity to make reduction by maximal independent sets.

Example 2

Isomorphism analysis of two four-dimensional Lie algebras $A_{4,8}$ and $A_{4,10}$ taken from [18] with the following non-zero commutators

$$
\begin{aligned}
A_{4,8}: &\ [e_2, e_3] = e_1, \ [e_2, e_4] = e_2, \ [e_3, e_4] = -e_3, \\
A_{4,10}: &\ [e_2, e_3] = e_1, \ [e_2, e_4] = -e_3, \ [e_3, e_4] = e_2.
\end{aligned}
\tag{12}
$$

If one consider (12) as real Lie algebra, they belong to different isomorphic classes [18] and therefore are not isomorphic ones. Let us consider them as complex algebras and verify whether they are still isomorphic or not. In this case the ISOLIE module generates the system of 22 equations in 16 unknowns

```
Z1 := A12*A23 - A13*A22;
Z2 := A12*A24 - A14*A22;
Z3 := - (A13*A24 - A14*A23);
Z4 := A12*A33 - A13*A32;
Z5 := A12*A34 - A14*A32;
Z6 := - (A13*A34 - A14*A33);
Z7 := A12*A43 - A13*A42;
Z8 := A12*A44 - A14*A42;
Z9 := - (A13*A44 - A14*A43);
Z10 := - (A11 - A22*A33 + A23*A32);
Z11 := - (A12 - A22*A34 + A24*A32);
Z12 := - (A13 + A23*A34 - A24*A33);
Z13 := - A14;
Z14 := A31 + A22*A43 - A23*A42;
Z15 := A32 + A22*A44 - A24*A42;
Z16 := A33 - A23*A44 + A24*A43;
Z17 := A34;
Z18 := - (A21 - A32*A43 + A33*A42);
Z19 := - (A22 - A32*A44 + A34*A42);
Z20 := - (A23 + A33*A44 - A34*A43);
Z21 := - A24;
Z22 := det mat((A11,A12,A13,A14),(A21,A22,A23,A24),
        (A31,A32,A33,A34),(A41,A42,A43,A44)) - D;
```

Using the ASYS package just in the same way as in previous example, we obtain among output subsystems the following one

---

Variables = (D A11 A21 A31 A42 A43 A44 A12 A22 A23)
Parameters = (A32 A33) % non-zero
Zeros = (A34 A24 A14 A41 A13)

SUBBASIS FOR SET (A42 A43) % maximal independent set (arbitrary parameters)

GROEBNER BASIS

$$
\begin{aligned}
G(1) &= D - 4 * A23 * A33^2 * A32 \\
G(2) &= A11 + 2 * A23 * A32 \\
G(3) &= A21 + A33 * A42 - A32 * A43 \\
G(4) &= \frac{A31 * A33 - A23 * A33 * A42 - A23 * A32 * A43}{A33} \\
G(5) &= \frac{A44 * A33 + A23}{A33} \\
G(6) &= A12 \\
G(7) &= \frac{A22 * A33 + A23 * A32}{A33} \\
G(8) &= A23^2 + A33^2
\end{aligned}
$$

One can immediately see that

- Because d is not an element of the Gröbner basis, the two above Lie algebras are isomorphic as complex ones.
- The fact that they are not isomorphic as real Lie algebras leads to absence of real solutions of the last (univariate) polynomial in $A23$. However, in order to check that real Lie algebras are not isomorphic, one has to look at the last polynomial of *each* subsystem.
- Explicit form of complex transformation mätrix can be easy constructed.

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References

[1] Ovsiannikov L.V. Group Analysis of Differential Equations, Academic Press, New York, 1982.

[2] Ibragimov N.H. Transformation Groups Applied to Mathematical Physics, Reidel, Boston, 1985.

[3] Olver P.J. Applications of Lie Groups to Differential Equations, Springer-Verlag, New York, 1986.

[4] Bluman G.V., Kumei S. Symmetries and Differential Equations, Springer-Verlag, New York, 1989.

[5] Winternitz P. Group Theory and Exact Solutions of Partially Integrable Differential Systems. In: Partially Integrable Nonlinear Evolution Equations and Their Physical Applications (eds. Conte R. and Boccara N.), Kluwer Academic Publishers, Dordrecht, 1990, pp.515-567.

[6] Schwarz F. Symmetries of Differential Equations: from Sophus Lie to Computer Algebra, *SIAM Rev.* 30, 3, 450-481, 1988.

[7] Schwarz F. Programming with Abstract Data Types: The Symmetry Packages SODE and SPDE in Scratchpad, *Lecture Notes in Computer Science* 296, Springer-Verlag, Berlin, New-York, 1988, pp. 167-176.

[8] Hereman W. Champaign B., Hereman W., Winternitz P. The Computer Calculation of Lie Point Symmetries of Large Systems of Differential Equations, *Comp. Phys. Comm.* 2&3, 319-340, 1991.

[9] Bocharov A.V., Bronstein M.L. Efficiently Implementing Two Methods of the Geometrical Theory of Differential Equations, *Acta Appl. Math.* 16, 143-166, 1989. Bocharov A.V. Will DELiA Grow into an Expert System? *Lecture Notes in Computer Science* 429, Springer-Verlag, Berlin, New-York, 1988, pp. 167-176.

[10] Gerdt V.P. Computer Algebra Tools for Higher Symmetry Analysis of Nonlinear Evolution Equations. Preprint JINR E5-402-91, Dubna, 1991. To be published in Proceedings of the IFIP Working Conference on Programming Environments for High-Level Scientific Problem Solving (23-27 September 1991, Karlsruhe, Germany).

[11] Lassner W. Symmetrien von Differentialgleichungen und computergestützte Identifikation und Klassifikation von Lie-Algebren. Talk given at the V. Internationales Kolloquium über Aktuelle Problem der Rechentechnik (19-22 March 1991, Dresden).

[12] Reid G. Finding Symmetries of Differential Equations without Integrating Determining Equations, To appear in the *European Journal of Applied Mathematics.*

[13] Kornyak V.V. Classification of Subalgebras of Finite Dimensional Lie Algebra by Computer. In the Proceedings of International Conference on Computer Algebra and its Applications in Theoretical Physics (17-20 September, 1985), JINR D11-85-791, Dubna, 1985, pp. 339-344 (in Russian). Glazunov N.M., Karpinsky F.G., Kornyak V.V. Solving Some Problems of Algebra, Analysis and Mathematical Physics Using Computer Algebra Systems. Cybernetics, No.2, 1991, pp.23-29 (in Russian)

[14] Buchberger B. Gröbner Bases: an Algorithmic Method in Polynomial Ideal Theory. In: (Bose N.K., ed.) Recent Trends in Multidimensional System Theory, Reidel, 1985.

[15] Gerdt V.P., Zharkov A.Yu., Khutornoy N.V. ASYS: A Computer Algebra Package for Analysis of Systems of Nonlinear Algebraic Equations. JINR R5-92-100, Dubna, 1992.

[16] Kredel H., Weispfenning V. Computing Dimension and Independent Sets for Polynomial Ideals, *J. Symb. Comp.* 6, 231-247, 1988.

[17] Böge W., Gebauer R., Kredel H. Some Examples for Solving Systems of Algebraic Equations by Calculating Gröbner Bases, *J. Symb. Comp.* 2, 83-98, 1986.

[18] Patera J., Sharp R.T., Winternitz P. Invariants of Real Low Dimension Lie Algebras, *J. Math. Phys* 17, 6, 986-994, 1976.

[19] Hearn A.C. REDUCE User's Manual. Version 3.4. The Rand Corporation, Santa Monica, 1991.

[20] Kobayashi H., Moritsugu S., Hogan R.W. On Radical Zero-Dimensional Ideals, *J. Symb. Comp.* 8, 545-552, 1989.

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Гердт В.П., Ласснер В. E5-92-145

Проверка изоморфизма конечномерных алгебр Ли с помощью техники базисов Гребнера

В настоящей работе представлен компьютерно-алгебраический подход к решению задачи проверки изоморфизма конечномерных алгебр Ли и построения явного вида матрицы преобразования для изоморфных алгебр. Подход основан на исследовании систем квадратично-нелинейных алгебраических уравнений для матричных элементов матрицы преобразования с помощью метода базисов Гребнера.

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Gerdt V.P., Lassner W. E5-92-145

Verifying Isomorphisms of Finite Dimensional Lie Algebras by Gröbner Basis Technique

In this paper we present a computer-aided approach to verify the isomorphism between finite-dimensional Lie algebras and to construct an explicit form of an transformation matrix in the case of isomorphism. Our approach is based on the direct investigation of quadratic algebraic equations for matrix elements of a transformation matrix by the Gröbner basis method.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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