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THE GRAVITATIONAL POTENTIAL OF PERTURBED ELLIPSOIDAL INHOMOGENEOUS CONFIGURATIONS WITH THE ACCOUNT OF THE "FIFTH" FORCE

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I. INTRODUCTION

At present the theoretical and experimental justification of the "fifth" force is a vital problem. The "fifth" force can be explained as the corrections, predicted from the quantum gravitation, to the law of reverse squares. The most developed model is the account for all recent experimental results.

It is of interest to investigate the role of the 'fifth' force in astrophysics [2,3]. Thus, the authors of GHN show the influence of the 'fifth' force both on stationary and moving objects [4,5,6]. However the study of spherical-symmetrical case only reduces the value of results achieved, as perturbed configurations present the greatest practical interest [7,8].

For configurations in question the analytical representations of internal and external potential are obtained. This is the main difficulty of many astrophysical problems, particularly the question of the role of the 'fifth' force in astrophysics.

In the second section of this paper the mathematical formulation of the problem and determinations of some designations are made. The analytical representation of the internal potential is obtained in the third section, also the technique developed in this section is used in the forth section devoted to the external potential. The main results are discussed in the conclusion.

II. THE MATHEMATICAL FORMULATION OF THE PROBLEM

In GHN model the gravitational potential is:

$$\Phi_5 = -G\int \frac{\rho(\bar{r}')}{|\bar{r}'-\bar{r}|}\left\{1 - a\exp\left(-\varepsilon_1|\bar{r}-\bar{r}'|\right) + b\exp\left(-\varepsilon_2|\bar{r}-\bar{r}'|\right)\right\} d\bar{r}'^{3}, \tag{1}$$

where Yukava terms conform the contribution of the spin-0 graviscalar and spin-1 graviphoton. The main aim is the obtaining of the representations for the potential (1) for the internal and external parts of perturbed ellipsoidal configuration D. The equation of the surface of the D can be given in the following way [8]:

$$X_1^2+X_2^2+X_3^2+\varkappa Z(X_1,X_2,X_3)=1\ ;\qquad 0\le\varkappa\ll 1\ , \tag{2}$$

where: $X_k=X^k/a_k$, $a_1\ge a_2\ge a_3$ are the semi-axes of the parent ellipsoid in coordinates X^k.

Let the function $Z(X1,X2,X3)$,governing the perturbation of the surface, and the distribution of density $\rho(X_1,X_2,X_3)$ be the continuous functions and approximate them, according to the well-known Stone-Weirstrasse theorem, by the polynomials in X_k of degrees P and L accordingly:

$$Z=\sum_{i,j,k}^{L} Z_{ijk}\,X_1^i X_2^j X_3^k\ ;\qquad \rho=\sum_{a,b,c}^{P}\rho_{abc}\,X_1^a X_2^b X_3^c\ . \tag{3}$$

It is obvious that instead of eqs.(1) one can consider the equivalent eqs.(4):

$$\Phi=-A\int\frac{\rho(\bar r')\exp\{-\varepsilon|\bar r-\bar r'|\}}{|\bar r-\bar r'|}\,d\bar r'^{3}\ , \tag{4}$$

because each term in eqs.(1) can be obtained from eqs.(4) with a proper parameters A and ε. That is why, it is natural to reduce the whole problem to the investigation of the eqs.(4), which will be termed later on as the potential in accorfance with its physical sense.

For the essential reduction of the notations the following operator is used [8]:

$$\begin{bmatrix} i_1 i_2\dots i_n\\ k_1 k_2\dots k_n\end{bmatrix}\overset{def}{=\!=}\sum_{k_1=0}^{i_1}\sum_{k_2=0}^{i_2}\dots\sum_{k_n=0}^{i_n} C_{i_1}^{k_1}\cdot C_{i_2}^{k_2}\dots C_{i_n}^{k_n}\ , \tag{5}$$

where: $C_i^k=\dfrac{i!}{(i-k)!\,k!}$, moreover $C_i^k\equiv 0$, if $k>i$ or $k<0$.

III. INTERNAL POTENTIAL

3.1 Upon expanding the exponential in eqs.(4) into the series in the powers of $|\bar{r}-\bar{r}'|$ and collecting the terms with even and odd powers the internal potential can be divided into two parts:

$$\varphi_I = \varphi_I^1 + \varphi_I^2 \ , \tag{6}$$

$$\varphi_I^1 = A \sum_{n_0=0}^{\infty} \frac{\varepsilon^{2n_0+1}}{(2n_0+1)!} \int \rho(\bar{r}') \, |\bar{r}-\bar{r}'|^{2n_0} \, d\bar{r}'^{\,3} \tag{7}$$

$$\varphi_I^2 = - A \sum_{n_0=0}^{\infty} \frac{\varepsilon^{2n_0}}{(2n_0)!} \int \rho(\bar{r}') \, |\bar{r}-\bar{r}'|^{2n_0-1} \, d\bar{r}'^{\,3} \ . \tag{8}$$

It is necessary because of principal differences between even and odd powers [8]. The representation of φ_I^1 and φ_I^2 will be obtained in items 3.2 and 3.3 respectively.

3.2 THE REPRESENTATION OF THE φ_I^1

In the new system of coordinates:

$$x_k = a_k \alpha_k \tilde{R} \ ; \quad \alpha_1 = \sin\theta\cos\varphi \ , \ \alpha_2 = \sin\theta\sin\varphi \ , \ \alpha_3 = \cos\theta \ ; \tag{9}$$

upon integrating with the respect to the $\tilde{R}$ subject to eqs.(3) the eqs.(7) takes the form:

$$\varphi_I^1 = A a_0^3 \sum_{n_0=0}^{\infty} \frac{\varepsilon^{2n_0+1}}{(2n_0+1)!} \sum_{a,b,c}^{\ell} \begin{bmatrix} n_0 & m & m-\ell & \nu & n_0-m & p & \ell & w \\ m & \ell & \nu & t & p & q & w & \upsilon \end{bmatrix} \cdot \rho_{abc} \, a_1^{A_1} \cdot a_2^{A_2} \cdot a_3^{A_3} \times$$

$$\times (-2)^{\ell} (x^1)^{2(n_0-m-p)+\ell-w} (x^2)^{2(p-q)+m-\nu} (x^3)^{2q+\upsilon} \cdot \frac{1}{h+3} \int R^{h+3} \alpha_1^{A_1+a} \alpha_2^{A_2+b} \alpha_3^{A_3+c} \, d\Omega \ , \tag{10}$$

where: $A_1 = 2(m-\nu) - \ell - w$, $A_2 = 2(\nu - t) + w - \upsilon$, $A_3 = 2t + \upsilon$,

$d\Omega = \sin\theta \, d\theta \, d\varphi$, $a_0 = (a_1 a_2 a_3)^{1/3}$, $h = a + b + c + 2m - \ell$;

but R can be defined from the equation of the surface:

$$R^2 + \varkappa Z(R, \theta, \varphi) = 1 \ . \tag{11}$$

For this purpose it is necessary to use the method of the Burman-Lagrange series [9].

Thus if $Z(\xi)$ - an analytical function in the circle

$|\xi - 1| < \eta_0$, $\eta = \eta_0 \exp\{i\chi\}$ and:

$$\eta_0 |2-\eta_0| > \varkappa M_{\eta_0} , \quad M_{\eta_0} = \max_{|\eta| = \eta_0} |Z(\eta)| , \quad \eta_0 < 2 ; \qquad (12)$$

then the function R^{h+3} can be expanded into absolutely convergent series:

$$R^{h+3} = 1 + (h+3) \sum_{s=1}^{\infty} \frac{\varkappa^s (-1)^s}{s!} \frac{d^{s-1}}{dR^{s-1}} \left(\frac{R^{h+2} Z^s}{(R+1)^s} \right) \Bigg|_{R=1} . \qquad (13)$$

Substitution now of eqs.(13) into eqs.(10) with the changing: $R=y+1$, $-2<y\le 0$, gives:

$$\Phi_I^1 = A a_0^3 \sum_{n_0=0}^{\infty} \frac{\varepsilon^{2n_0+1}}{(2n_0+1)!} \sum_{s=0}^{\infty} \varkappa^s \sum_{a,b,c}^{\ell} \sum_{i_1 j_1 k_1 \dots k_s}^{6L} \begin{bmatrix} n_0 & m & m-\ell & \nu & n_0-m & p & \ell & \omega \\ m & \ell & \nu & t & p & q & \omega & v \end{bmatrix} \cdot f_{abc} \times$$

$$\times a_1^{A_1} a_2^{A_2} a_3^{A_3} \left(\prod_{r=0}^{s} Z_{i_r j_r k_r} \right) \frac{(-2)^{\ell - s}}{s!} \cdot I_{B_1 B_2 B_3} \cdot F_s \cdot (X^1)^{2(n_0-m-p)+\ell-\omega} (X^2)^{2(p-q)+\omega-v} (X^3)^{2q+v} \qquad (14)$$

$$2B_1 = A_1 + a + i , \quad 2B_2 = A_2 + b + j , \quad 2B_3 = A_3 + c + k ;$$

where:

$$I_{B_1 B_2 B_3} = \int \alpha_1^{2B_1} \alpha_2^{2B_2} \alpha_3^{2B_3} d\Omega = \frac{2\pi (2B_1-1)!!(2B_2-1)!!(2B_3-1)!!}{(2(B_1+B_2+B_3)-1)!!} \qquad (15)$$

$$i = \sum_{r=0}^{s} i_r , \ j = \sum_{r=0}^{s} j_r , \ k = \sum_{r=0}^{s} k_r , \ i_0 = j_0 = k_0 = 0 , \ Z_{000} = 1 , \ F_0 = (h+3)^{-1} ;$$

$$F_s = \frac{d^{s-1}}{dy^{s-1}} \left[\frac{(1+y)^{h+i+j+k+2}}{(1+y/2)^s} \right] \Bigg|_{y=0} .$$

For the determination of the Fs and later on the following assertion will be necessary:

ASSERTION

$$\frac{d^a}{dy^a}\left[\frac{(1+y)^b}{(1+y/2)^{a+1}}\right]\Bigg|_{y=0} = \prod_{r=1}^{a}(b+1-2r) . \qquad (16)$$

PROOF

The right part in eqs.(16) can be expressed as an integral:

$$\frac{d^a}{dy^a}\left[\frac{(1+y)^b}{(1+y/2)^{a+1}}\right]\Bigg|_{y=0} = \frac{1}{2\pi i}\oint_{0\in\Gamma}\frac{(1+z)^b}{(1+z/2)^{a+1}}\,\frac{dz}{z^{a+1}} .$$

By integrating a times by part, one can easily achieve the assertion.

As a corollary $\quad Fs=\dfrac{(a+b+c+2m-l+i+j+k-1)!!}{(a+b+c+2m-l+i+j+k-2s+3)!!} \quad ; \; s > 0.$

Now after changing the order of summation the eqs.(14) take the form:

$$\Phi_I^1 = 2\pi a_0^3 A\sum_{n_0=0}^{\infty}\frac{\varepsilon^{2n_0+1}}{(2n_0+1)!}\sum_{s=0}^{\infty}x^s\sum_{a,b,c}^{p}\sum_{i_1j_1k_1\ldots k_s}\sum_{\alpha,\beta,\gamma}\sum_{\alpha+\beta+\gamma>2(n_0-m)}\cdot\rho_{abc}\;\times \qquad (17)$$

$$\times\left(\prod_{r=0}^{s} Z_{i_rj_rk_r}\right)\cdot C_{abc}^{\alpha\beta\gamma}(i_1j_1k_1;\ldots i_sj_sk_s)\cdot(X^1)^{\alpha}(X^2)^{\beta}(X^3)^{\gamma} \quad ,$$

where:

$$C_{abc}^{\alpha\beta\gamma} = \begin{bmatrix} n_0-m & p & 2n_0-m-\alpha-\beta-\gamma & \nu \\ p & q & \nu & t \end{bmatrix}\cdot C_{n_0}^{m}\cdot C_{m}^{A_1+A_2+A_3-m}\cdot C_{2m-A_1-A_2-A_3}^{A_1+A_2-2\nu}\cdot C_{A_1+A_2-2\nu}^{2A_3-2t}\;\times$$

$$\times\; a_1^{A_1}a_2^{A_2}a_3^{A_3}\cdot\frac{(-2)^{2m-A_1-A_2-A_3-s}}{s!}\;\frac{(A_1+i+a-1)!!\,(A_2+j+b-1)!!\,(A_3+k+c-1)!!}{(A_1+A_2+A_3+i+j+k+a+b+c+3-2s)!!} .$$

3.3 REPRESENTATION OF THE Φ_I^2

In the new system of coordinates connected with the point of observation:

Now upon the changing $R=-T+U(y+1)$; $-2<y\leq 0$ from eqs.(21) with taking into account eqs.(20) transpires the form:

$$\Phi_I^2=-Aa_0^3\sum_{n_0=0}^{\infty}\frac{\varepsilon^{2n_0}}{(2n_0)!}\sum_{s=0}^{\infty}\frac{(-2)^{-s}\varkappa^s}{s!}\sum_{a,b,c}^{\ell}\sum_{l_1j_1k_1\ldots k_s}^{sL}\begin{bmatrix}abc\, l_1j_1k_1\ldots k_s & N+1\\ dfg\, n_1m_1e_1\ldots e_s & \mu\end{bmatrix}\cdot\rho_{abc}\ \times$$

$$\times\left(\prod_{r=0}^{s}Z_{l_rj_rk_r}\right)\cdot F_s\cdot X_1^{i-n}X_2^{j-m}X_3^{k-e}\int(-1)^{\mu}T^{\mu}U^{N-\mu-2(s-1)}d_1^{n}d_2^{m}d_3^{e}d^{2n_0-1}d\Omega,\tag{22}$$

where: $i=a+\sum_{r=0}^{s}l_r,\ j=b+\sum_{r=0}^{s}j_r,\ k=c+\sum_{r=0}^{s}k_r,\ n=d+\sum_{r=0}^{s}n_r,$

$$m=f+\sum_{r=0}^{s}m_r,\ e=g+\sum_{r=0}^{s}e_r,\ i_0=j_0=k_0\equiv 0,\ Z_{000}=1,\ N=n+m+e+2n_0,$$

$$F_0=(h+2)^{-1}\qquad,\qquad F_s=\frac{d^{s-1}}{dy^{s-1}}\left[\frac{(1+y)^{N+1-\mu}}{(1+y/2)^{s}}\right]\Bigg|_{y=0}.$$

It is obvious from eqs.(16) that:

$$F_s=\frac{(N-\mu)!!}{(N-\mu-2(s-1))!!}\quad;\ s>0.\tag{23}$$

It is essential to note, that if $(N-\mu)<2(s-1)$ and $(N-\mu)$ are odd, then $F_s=0$. Therefore there are only non-negative powers of the U in eqs.(22).

Thus, from eqs.(22) and eqs.(23) the representation of the Φ_I^2 can be obtained:

$$\Phi_I^2=-a_0^3A\sum_{n_0=0}^{\infty}\frac{\varepsilon^{2n_0}}{(2n_0)!}\sum_{s=0}^{\infty}\varkappa^s\sum_{a,b,c}^{\ell}\sum_{\alpha,\beta,\gamma}\sum_{i+j+k\geq\alpha+\beta+\gamma+2(s-1)}\cdot\rho_{abc}\quad\times$$

$$\times\left(\prod_{r=0}^{s}Z_{i_rj_rk_r}\right)\cdot C_{abc}^{\alpha\beta\gamma}(l_1j_1k_1;\ldots l_sj_sk_s)\cdot X_1^{\alpha}X_2^{\beta}X_3^{\gamma},\tag{24}$$

$$X_k' = X_k + \alpha_k \tilde{R}\ , \quad \alpha_1 = \sin\Theta\cos\varphi,\ \alpha_2 = \sin\Theta\sin\varphi,\ \alpha_3 = \cos\Theta\ ; \qquad (18)$$
$$\tilde{R} = \left[\sum_{i=1}^{3}(X_i' - X_i)^2\right]^{1/2},$$

and upon the integrating the eqs.(8) with respect to the $\tilde{R}$ one can obtain the following expression:

$$\Phi_I^2 = -Aa_0^3 \sum_{n_0=0}^{\infty} \frac{\varepsilon^{2n_0}}{(2n_0)!} \sum_{a,b,c}^{p} \begin{bmatrix} a\,b\,c \\ d\,f\,g \end{bmatrix} X_1^{a-d} X_2^{b-f} X_3^{c-g} \frac{1}{h+2} \int R^{h+2} \alpha_1^{d} \alpha_2^{f} \alpha_3^{g} \alpha^{2n_0-1}\, d\Omega\ , \qquad (19)$$

where: $h = d + f + g + 2n_0\ , \quad \alpha = (\alpha_1^2 a_1^2 + \alpha_2^2 a_2^2 + \alpha_3^2 a_3^2)^{1/2}$.

R can be defined from the equation of the surface, which according to the eqs.(18) takes the form:

$$R = R_0 + \varkappa\, \Psi(R)\ ,$$

where: $R_0 = u - T,\ u = (T^2 + Q)^{1/2},\ T = \sum_{k=1}^{3} \alpha_k X_k,\ Q = 1 - \sum_{k=1}^{3} X_k^2$,

$$\Psi(R) = -Z(R)/(R + T + u)\ .$$

If $\Psi(\xi)$ is an analytical function in the circle

$|\xi - R_0| < \eta_0 \quad , \ \eta = \eta_0 \exp(i\chi)$ and :

$$|\eta|\cdot|2u - \eta| > \varkappa M_{\eta_0}\ ; \quad M_{\eta_0} = \max_{|\eta| = \eta_0} |Z(\eta)|\ , \quad \eta_0 < 2u\ , \qquad (20)$$

then the function R^{h+2} can be expanded in this circle into absolutely-convergent series:

$$R^{h+2} = (u - T)^{h+2} + (h+2) \sum_{s=1}^{\infty} \frac{\varkappa^s (-1)^s}{s!} \frac{d^{s-1}}{dR^{s-1}} \left(\frac{R^{h+2} Z^s}{(R + u + T)^s} \right) \Bigg|_{R = u - T} . \qquad (21)$$

where: $$C_{abc}^{\beta\gamma} = \begin{bmatrix} a\,b\,c\; i_1 j_1 k_1 \ldots k_s & N+1 & (N-\mu-2s+2)/2 & N-2(\nu+s-1) & \lambda \\ d\,f\,g\; n_1 m_1 l_1 \ldots l_s & \mu & \nu & \lambda & \tau \end{bmatrix} (-1)^{\omega+\mu} \times$$

$$\times C_{\nu}^{\omega} \cdot C_{\omega}^{\nu} C_{\sigma}^{t} \cdot \frac{(N-\mu)!!}{(N-\mu-2(s-1))!!} \; \frac{(-2)^{-s}}{s!} \cdot \mathcal{N}_{Q_1 Q_2 Q_3}^{n_0} ,$$

$$2\omega = d+f+\gamma-i-j-k+2(s+\nu-1), \; 2\sigma = \beta+\gamma-j-k-\lambda+m+l, \; 2t = \gamma+c-k-\tau,$$

$$2Q_1 = 2n+m+l-2\nu-\lambda-2(s-1), \; 2Q_2 = m+\lambda-\tau, \; 2Q_3 = l+\tau ;$$

$$\mathcal{N}_{Q_1 Q_2 Q_3}^{n_0} = \int \alpha_1^{2Q_1} \alpha_2^{2Q_2} \alpha_3^{2Q_3} \alpha^{2n_0-1} \, d\Omega .$$

It is necessary to point out that eqs.(24) is valid if U=0, but in this case another expanding of R^{h+2} is needed.

3.4 The eqs.(17) and eqs.(24) taking into account the relations (6-8) are, as a matter of fact, the complete solution of the analytical representation of the potential of configuration D in the form of series in the parameter of the perturbation æ, the coefficients of which are the polynomials in coordinates of power $2n_0+P+s(L-2)+3$. If $n_0=0$, the eqs.(24) describe the Newtonian potential.

IV. EXTERNAL POTENTIAL

4.1. Upon expanding the eqs.(4) into the three-dimensional Taylor series, converging at least beyond the surrounding sphere, and upon integrating with respect to the $\tilde{R}$ in the system of coordinates (9) one can obtain the following representation of the external potential:

$$\Phi_e = -a_0^{1} A \sum_{a,b,c}^{P} \sum_{M=0}^{\infty} \sum_{d+f+g=M} \frac{a_1^{d} a_2^{f} a_3^{g} (-1)^{d+f+g}}{d!\,f!\,g!} \cdot \rho_{abc} \times$$
$$\times \left(\frac{\partial^M}{\partial (x^1)^d \, \partial (x^2)^f \, \partial (x^3)^g} \, \frac{\exp(-\varepsilon r)}{r} \right) \frac{1}{h+3} \int R^{h+3} \alpha_1^{a+d} \alpha_2^{b+f} \alpha_3^{c+g} \, d\Omega , \qquad (25)$$

where: $h = a+b+c+d+f+g$; $d\Omega = \sin\theta \, d\theta \, d\varphi$,

but R=R(θ,φ) can be defined, as in the previous case, from the equation of the surface (11). While fulfilling the conditions (12) the function R^{h+3} can be expanded into Burman-Lagrange series (13). So, in such a way from eqs.(25) taking into account the eqs.(13),(15),(16) it is obvious that the external potential can be represented in the following way:

$$\varphi_e = -2\pi a_1^3 A \sum_{s=0}^{\infty} \frac{x^s(-1)^s}{s!\,2^s} \sum_{i_1 j_1 k_1 \dots k_s}^{sL} \left(\prod_{r=0}^{s} Z_{i_r j_r k_r} \right) \times$$
$$\times \frac{(a+d+i-1)!!(b+f+j-1)!!(c+g+k-1)!!}{(a+b+c+d+f+g+i+j+k+3-2s)!!} \cdot M_{dfg}^{abc} \quad , \tag{26}$$

where: $i = \sum_{r=0}^{s} i_r$, $j = \sum_{r=0}^{s} j_r$, $k = \sum_{r=0}^{s} k_r$, $i_0 = j_0 = k_0 = 0$, $Z_{000} = 1$,

$$M_{dfg}^{abc} = \sum_{a,b,c}^{L} \sum_{M=0}^{\infty} \sum_{d+f+g=M} \frac{(-1)^{d+f+g}}{d!\,f!\,g!} \rho_{abc} \left(\frac{\partial^M}{\partial(x^1)^d \partial(x^2)^f \partial(x^3)^g} \frac{\exp(-\varepsilon r)}{r} \right) a_1^d a_2^f a_3^g .$$

Now the problem reduces to defining the differential form:

$$I_{dfg} = \frac{\partial^d}{\partial(x^1)^d} \frac{\partial^f}{\partial(x^2)^f} \frac{\partial^g}{\partial(x^3)^g} \left\{ \frac{\exp(-\varepsilon r)}{r} \right\} . \tag{27}$$

4.2 Let expand $\exp(-\varepsilon r)$ into the series:

$$\exp(-\varepsilon r) = \sum_{n_0=0}^{\infty} \frac{(-1)^{n_0}}{n_0!} \varepsilon^{n_0} . \tag{28}$$

Then upon using the Rodrigo formula [10] one can obtained:

$$I_{dfg} = \sum_{k=0}^{[d/2]} \sum_{\ell=0}^{[f/2]} \sum_{m=0}^{[g/2]} K_{dfg}^{km\ell} \frac{(x_1)^{d-2m}(x_2)^{f-2\ell}(x_3)^{g-2k}}{(r)^{2\delta+1}} , \tag{29}$$

where: $n_0 = 0$, $z = d+f+g-k-m-\ell$;

$$K^{km\ell}_{dfg} = (-1)^{z}\, C^{2k}_{d}\cdot C^{2\ell}_{f}\cdot C^{2m}_{g}\cdot(2m-1)!!\,(2\ell-1)!!\,(2k-1)!!\,(2z-1)!! \ .$$

If n_0 - even:

$$I_{dfg} = \sum_{n_0=0}^{\infty}\frac{(-1)^{n_0}}{n_0!}\varepsilon^{n_0}\sum_{\omega=0}^{(n_0-1)/2}\sum_{\tau=0}^{\omega}\cdot C^{\omega}_{(n_0-1)/2}\cdot C^{\tau}_{\omega}\ \times$$

$$\times\frac{(n_0-1-2\omega)!\,(2(\omega-\tau))!\,(2\tau)!}{d!\,f!\,g!}(X_1)^{n_0-1-2\omega-d}(X_2)^{2(\omega-\tau)-f}(X_3)^{2\tau-g} \ . \tag{30}$$

If n_0 - odd:

$$I_{dfg} = \sum_{n_0=0}^{\infty}\frac{(-1)^{n_0}}{n_0!}\varepsilon^{n_0}\sum_{i=0}^{d}\sum_{j=0}^{f}\sum_{k=0}^{g} C^{i}_{d}\cdot C^{j}_{f}\cdot C^{k}_{g}\ \times$$

$$\times\left(\frac{\partial^{d-i}}{\partial(X_1)^{d-i}}\frac{\partial^{f-j}}{\partial(X_2)^{f-j}}\frac{\partial^{g-k}}{\partial(X_3)^{g-k}}\ \frac{1}{r}\right)\cdot\left(\frac{\partial^{i}}{\partial(X_1)^{i}}\frac{\partial^{j}}{\partial(X_2)^{j}}\frac{\partial^{k}}{\partial(X_3)^{k}}\ r^{n_0-1}\right), \tag{31}$$

where: (n_0-1) is even, and expressions between the brackets are defined according to eqs.(29) and (30).

If $n_0=1$, then:

$$M^{abc}_{dfg} = a_0^{3}\sum_{a,b,c}^{p} \rho_{abc}\, X_1^{a+1}\, X_2^{b+1}\, X_3^{c+1} \ . \tag{32}$$

4.3 Eqs.(26) with eqs.(29-32) determine the external potential of configuration D in the form of series in the parameter of perturbation æ. It is important to note that the representation of the form (27) has been obtained using the expanding of the exponential, however it is evident that it contains the multiplier $\exp(-\varepsilon r)$ and the coefficients of the expanding the eqs.(27) have the form of polynomials in X_k .

V. CONCLUSIONS

In the present paper the analytical representations of the potential of the perturbed inhomogeneous configuration D in the form of series in the parameter of perturbation $\varkappa$ are obtained.

As to the 'fifth' force, it is necessary to point out that the neutron stars are the most interesting objects for investigation, because of their rapid rotations ($v/c \sim 0{,}1$ at the equatorial surface) and size comparable with supposed range of the new force. The consideration, in this case, of the ellipsoidal configurations is of necessity in principle.

The authors believe that the results of this paper will find a lot of applications in astrophysics, geophysics and other fields of science due to its universal character and the opportunity of using the methods of numerical calculations and analytical transformations [11].

REFERENCES

1. Nieto M.M., Hughes R.J., Goldman T. Phys.Lett.B, 212, 1, p.18-22.(1988)
2. Glass E.N., Szamosi G. Phys.Rev.D, 36, 4, p.1054-1057.(1989)
3. Glass E.N., Szamosi G. Phys.Rev.D, 35, p.1205-1209.(1987)
4.,5.,6. Nieto M.M., Goldman T., Hughes R.J. Phys.Rev.D, 36, 12, pp. 3684-3688, 3689-3693, 3694-3698.(1987)
7. Chandrasekhar S. The Ellipsoidal Figures of Equilibrium. New Haven - London, Yale Univ. Press, 1969, IX.
8. Tsirulev A.N., Tsvetkov V.P. Astron.Journal (USSR), 59, pp.475-482, 666-676.(1982)
9. Whitteker E.T., Watson G.N. A Course of Modern Analysis. v1. Cambridge: At the Univ. press.(1927)
10. Antonov V., Timoshkova E., Kholshevnikov K. Introduction to the Theory of the Newtonian Potential. Moscow: Nauka, 1988. (In Russian).
11. International Conference on Systems and Techniques of Analitical Computing and Their Applications in Theoretical Physics, Dubna, September 1979. JINR, D11-80-13, Dubna, 1980.

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