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ON EXISTENCE OF A BOUND STATE IN AN L-SHAPED WAVEGUIDE

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1. Introduction

It is a distinguishing feature of classical problems that they can be a source of inspiration for many decades. In this note, we are going to treat such a problem, namely the existence of a new solution to the Helmholz equation in a particular spatial region and to discuss briefly its physical implications.

We dedicate this study to Professor Vaclav Votruba who is becoming octogenarian, though it is hard to believe that, as a token of our gratitude ; the senior among the authors is his student while the other two may be regarded as the second generation of his disciples. Prof.Votruba accomplished much during his career that started at the old times when nobody challenged the simple truth that a university professor should at the first place educate a new generation of scholars by reading lectures (maintaining a natural feedback) and pushing (by his own hands) the science forward in his field. Since we are firmly convinced that these virtues are essential for further development of Czech theoretical physics, we are grateful to Prof.Votruba also for demonstrating them permanently.

The problem we are going to discuss concerns classical electrodynamics lectured by Prof.Votruba for many years [1,2] as well as quantum mechanics which he lectured with the same passion and skill but never found time to turn his lectures into a textbook. On the classical side, it is related to the theory of waveguides, the quantum aspect of the problem concerns recent experiments with tiny structures of a pure semiconductor material which might be called *quantum waveguides*.

In a recent series of papers [3-5] we have proven that the Laplace operator with Dirichlet boundary conditions on a curved planar strip of a width d can have an eigenfunction (bound state) with the eigenvalue below the threshold of the continuous spectrum, provided d is small enough, the strip is asymptotically straight and its boundary is infinitely smooth. While unexpected and important, this result has some drawbacks. In particular, its possible physical implications depend crucially on the distance between the bound-state energy and the continuous-spectrum threshold about which we have no information. Except of that, the smoothness requirement is in part technical and one would like to know whether it is possible to get rid of it.

With this aim we treat here a solvable model concerning the Laplace operator on an L-shaped strip ; we shall show that the bound state exists in this case and corresponds to the eigenvalue

$$\lambda_0 = 0.93\ \lambda_1\ , \tag{1.1}$$

where λ_1 is the continuous-spectrum threshold, i.e., the first-transversal-mode energy in the strip.

2. Existence of the ground-state solution

For definiteness we shall speak about a quantum particle living on an L-shaped strip Ω (cf.Fig.1) of a width d ; it is not difficult to translate the following argument into classical terms. The state Hilbert space is therefore $\mathcal{H} = L^2(\Omega)$ with the usual Lebesgue measure on Ω . The particle may be confined in Ω, e.g., by infinitely high potential walls to which the Dirichlet boundary conditions on the boundary $\partial\Omega$ correspond. Mathematically speaking, it means to choose

$$H_\Omega = -\ \Delta_D^\Omega \tag{2.1}$$

as a Hamiltonian of the problem, where the Dirichlet Laplacian on the *rhs* is the self-adjoint operator associated with the quadratic form

$$h_\Omega\ :\ h_\Omega(\psi) = \int_\Omega |\vec{\nabla}\psi|^2\ dx^2 \tag{2.2}$$

with the domain $Q(-\Delta_D^\Omega) = C_0^\infty(\Omega)$; since Ω has the segment

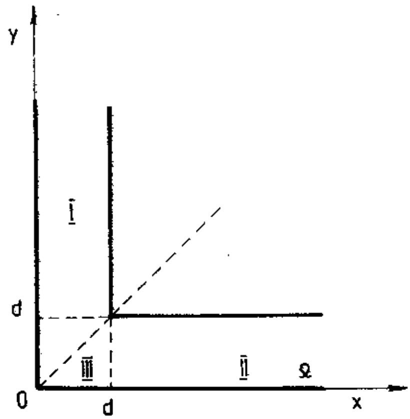

Fig.1. An L-shaped strip

property, the set of all $\psi \in \mathcal{H}$ that are infinitely smooth on the interior of Ω and vanish on its boundary forms a core of $-\Delta_D^{\Omega}$ - cf.[6], Sec.XIII.15. In (2.1) and further on, we set $\hbar^2/2m = 1$ for simplicity.

Let us start from some general observations. The only distinguished length in our problem is the strip width d, and therefore the results must be scaled with respect to this length. In particular, if there is a bound state of H_Ω below the continuous-spectrum threshold, its energy must equal $\lambda_0 = \varkappa_0\lambda_1$, where $\varkappa_0$ is a number from $[0,1)$ and $\lambda_1 = \pi^2/d^2$ is the energy of the lowest transversal mode ; it is not difficult to check, e.g., by Dirichlet-Neumann bracketing [6], that the continuous spectrum of H_Ω starts just at this value.

The symmetry of Ω implies that every bound state of H_Ω is either symmetric or antisymmetric with respect to the axis $x=y$. Using the bracketing technique again (on the diagonal of the subdomain III and on some distant cuts on the "arms") we see that the ground state, provided it exists, is symmetric with respect to the symmetry axis of Ω and non-degenerated.

In order to find equations for bound states, we decompose first the wave function on both arms (subdomains I,II indicated on Fig.1) of Ω with respect to the transversal modes

$$\psi_I(x,y) = \sum_{j=1}^{\infty} r_j(y)\phi_j(x) \quad , \quad \psi_{II}(x,y) = \sum_{j=1}^{\infty} t_j(x)\phi_j(y) \quad , \qquad (2.3)$$

where

$$\phi_j(t) = \left(\frac{2}{d} \right)^{1/2} \sin(\omega_j t/d) \qquad (2.4)$$

and the frequencies $\omega_j = j\pi$ refer to the transversal-mode energies $\lambda_j = \omega_j^2/d^2$, $j \in \mathbb{N}$ .Substituting this ansatz into the equation

$$H_\Omega \psi = \lambda\psi \qquad (2.5)$$

with $\lambda = \varkappa\lambda_1$ for some $\lambda \in (0,1)$, we get

$$\psi_I(x,y) = \sum_{j=1}^{\infty} (-1)^{j+1} \, r_j \, e^{q_j(1-y/d)} \phi_j(x) \ , \qquad (2.6a)$$

$$\psi_{II}(x,y) = \sum_{j=1}^{\infty} (-1)^{j+1} \, t_j \, e^{q_j(1-x/d)} \phi_j(y) \ , \qquad (2.6b)$$

where $q_j = \pi(j^2-\varkappa)^{1/2}$, $j \in \mathbb{N}$, while in the subdomain III the wavefunction can be written as

$$\psi_{III}(x,y) = \sum_{j=1}^{\infty} (-1)^{j+1} \left[r_j\alpha_j(y)\phi_j(x) + t_j\alpha_j(x)\phi_j(y) \right] \qquad (2.6c)$$

with $\alpha_j(x) = \mathrm{sh}(q_j x/d)/\mathrm{sh}\, q_j$ .Of course, to justify the term-by-term differentiation of the sums one has to know more about the coefficients r_j, t_j .

The wavefunction (2.6) is symmetric or antisymmetric iff $r_j = t_j$ or $r_j = -t_j$, respectively, holds for all j . With the above-mentioned argument in mind, we restrict our attention to the symmetric case. The norm of ψ is then expressed in terms of the coefficients r_j as

$$\|\psi\|^2 = d \sum_{j=1}^{\infty} \left(\frac{1 + \operatorname{cth} q_j}{q_j} - \frac{1}{\operatorname{sh}^2 q_j} \right) |r_j|^2 +$$

$$+ \left(\frac{2}{\pi} \right)^2 \sum_{j,k=1}^{\infty} \frac{kj}{(k^2+j^2-\varkappa)^2} r_k \bar{r}_j \tag{2.7}$$

or $\|\psi\|^2 = 2\|\psi_I\|^2 + \|\psi_{III}\|^2$, where

$$\|\psi\|^2 = \sum_{j=1}^{\infty} \frac{d}{2q_j} |r_j|^2 .$$

If $\|\psi\| < \infty$ the same is true for $\|\psi_I\|$ so the sequence $r \equiv \{r_j\}$ belongs to the Hilbert space $\ell^2(j^{-1})$ with the weighted norm $\|r\|^2 := \Sigma_j \; j^{-1}|r_j|^2$. Suppose, on the contrary, that $r \in \ell^2(j^{-1})$ so $\|\psi_I\| < \infty$. The other part of the wavefunction can be written as $\psi_{III} = \eta + R\eta$, where

$$\eta(x,y) = \sum_{j=1}^{\infty} (-1)^{j+1} r_j \alpha_j(y) \phi_j(x)$$

and R is the reflection operator, $(R\psi)(x,y) = \psi(y,x)$. The Schwarz inequality then gives $\|\psi_{III}\|^2 = 2\|\eta\|^2 + 2 \operatorname{Re}(\eta,R\eta) \le 4\|\eta\|^2$, and at the same time,

$$\|\eta\|^2 = \sum_{j=1}^{\infty} \frac{d}{2} \left(\frac{\operatorname{cth} q_j}{q_j} - \frac{1}{\operatorname{sh}^2 q_j} \right) |r_j|^2 < \infty ,$$

so we get $\|\psi\| < \infty$. We have proven therefore that a symmetric wavefunction ψ belongs to $\mathscr{H}$ iff the corresponding sequence of coefficients $r \in \ell^2(j^{-1})$.

The relations (2.6) show that ψ is continuous throughout the region Ω if only the coefficients r_j decrease with j fast enough. If ψ should solve the equation (2.5) its normal derivative must be continuous on the borderlines between the subdomains I,III and II,III too. This requirement yields formally the equation

$$r = Cr \ , \tag{2.8a}$$

where $C = (C_{jk})$ is a matrix operator,

$$C_{jk} = \frac{1}{\pi} \left\{ 1 - e^{-2\pi(j^2-\varkappa)^{1/2}} \right\} \frac{jk}{(j^2-\varkappa)^{1/2}(j^2+k^2-\varkappa)} \ . \tag{2.8b}$$

At this moment it is necessary to point out the difference between the quantum and classical cases ; though the equation is the same, we look for its solution in different classes of function. In the classical case, the requirement of energy finiteness means that the sequence r should fulfil $\Sigma_j \, j|r_j|^2 < \infty$ [7], i.e., it should belong to the space $\ell^2(j)$ which is a dense subspace in $\ell^2(j^{-1})$. Hence it is useful to solve the quantum problem first and then to look whether the solution obeys the classical restriction.

There is one more subtlety concerning the classical form of the eq.(2.8). In the waveguide literature such equations are solved numerically approximating C by a sequence of truncated matrices. Such a procedure is correct provided the operator C is compact on $\ell^2(j)$, and this claim actually appeared [8]. Unfortunately, it is false as we shall demonstrate in a while. To make the argument easier, we pass to an operator on ℓ^2 using the natural unitary map between the two spaces. Another simplification is possible due to the fact that compactness is not affected by multiplication by a bounded operator. Hence it is only necessary to prove the following assertion.

<u>Proposition</u> : The matrix operator $A = (A_{jk})$ with $A_{jk} = \sqrt{jk}/(j^2+k^2)$ on the Hilbert space ℓ^2 is non-compact.

<u>Proof</u> : Denote by P_n the projection on the subspace in ℓ^2 spanned by the first n basis vectors. We apply the following criterion [9] : A is compact iff $\lim_{n\to\infty} \|A - P_nA\| = 0$. Our aim is to show that there is $c > 0$ such that to each $n \in \mathbb{N}$ one can find $x \in \ell^2$ for which $\|(A - P_nA)x\|^2 \geq c\|x\|^2$. We set $x = \{x_j\}$, $x_j = j^{-(1+\varepsilon)/2}$, with a number $\varepsilon \in (0,1)$ to be determined later.

The following estimates hold :

$$\|x\|^2 = \sum_{j=1}^{\infty} j^{-(1+\varepsilon)} \le 1 + \int_1^{\infty} x^{-(1+\varepsilon)}dx = \frac{1+\varepsilon}{\varepsilon} \le \frac{2}{\varepsilon} ,$$

$$\sum_{j=n}^{\infty} j^{-(1+\varepsilon)} \ge \int_n^{\infty} x^{-(1+\varepsilon)}dx = \frac{n^{-\varepsilon}}{\varepsilon} ,$$

$$\left| \sum_{k=1}^{\infty} A_{jk}x_k \right| = \sqrt{j} \sum_{k=1}^{\infty} \frac{k^{-\varepsilon/2}}{j^2+k^2} \ge \sqrt{j} \int_1^{\infty} \frac{x^{-\varepsilon/2}}{j^2+x^2}\, dx =$$

$$= j^{-(1+\varepsilon)/2} \int_{1/j}^{\infty} \frac{t^{-\varepsilon/2}}{1+t^2}\, dt \ge c_1 j^{-(1+\varepsilon)/2}$$

for some positive c_1 . Then we have

$$\|x\|^{-2}\|(A - P_nA)x\|^2 \ge \frac{\varepsilon}{2}\, c_1^2 \sum_{j=n+1}^{\infty} j^{-(1+\varepsilon)} \ge c(n+1)^{-(1+\varepsilon)}$$

with $c := \frac{1}{2}\, c_1^2 > 0$. Since $(n+1)^{-1/n} \to 1$ as $n \to \infty$, it is sufficient to choose $\varepsilon = 1/n$. ∎

Hence one must be careful when solving numerically the eq.(2.8). It is, of course, possible to sweep the problem under the rug and to speak about the "relative convergence" phenomenon [10] trying to pick up a "right" approximating sequence, but it is clearly a not very honest way. Instead, we are going to demonstrate that (2.8) can be reformulated into a properly posed problem.

To begin with, we impose on solution of the system (2.8a) a slightly stronger requirement than $r \in \ell^2(j^{-1})$, namely we demand

$$r_j = j^{-s}a_j \quad , \quad a = \{a_j\} \in \ell^{\infty} \qquad (2.9)$$

with some positive s to be determined later. In fact, we have excluded up to now only those sequences $r \in \ell^2(j^{-1})$ with a slower than power-like decrease. Now we put the first equation of the system (2.8a) aside and solve the remaining ones ; since the system is linear, we may set

$$r_1 = a_1 = 1 \ . \qquad (2.10)$$

Substituting (2.9) and (2.10) into (2.8a), we get

$$a = b + Ka \ , \qquad (2.11a)$$

where $a = \{a_j\}_{j=2}^{\infty}$ is the sought sequence, while $b = \{b_j\}_{j=2}^{\infty}$ and $K = (K_{jk})_{j,k=2}^{\infty}$ are given by

$$b_j = \frac{1}{\pi}\left[1 - e^{-2\pi(j^2-\varkappa)^{1/2}}\right] \frac{j^{s+1}}{(j^2-\varkappa)^{1/2}\ (j^2+1-\varkappa)} \ , \qquad (2.11b)$$

$$K_{jk} = \frac{1}{\pi}\left[1 - e^{-2\pi(j^2-\varkappa)^{1/2}}\right] \frac{j^{s+1}k^{s-1}}{(j^2-\varkappa)^{1/2}\ (k^2+j^2-\varkappa)} \ . \qquad (2.11c)$$

Next one has to estimate the norm of the operator K in the Banach space ℓ^{∞} ; if $\|K\| < 1$, then there is $(I-K)^{-1} = I + K + K^2 + \dots$ and the equation (2.11a) has a unique solution, namely

$$a = (I-K)^{-1}b \ . \qquad (2.12)$$

In the following estimates, one has to assume $1 \le s < 2$. In that case, we have

$$|K(x_j)| = \left| \frac{1}{\pi}\left[1 - e^{-2\pi(j^2-\varkappa)^{1/2}}\right] \frac{j^{s+1}}{(j^2-\varkappa)^{1/2}} \sum_{k=2}^{\infty} \frac{k^{1-s}}{j^2+k^2-\varkappa}\, x_k \right| \le$$

$$\leq \|x\|_\infty \; \frac{1}{\pi} \left(1 - \frac{\varkappa}{j^2} \right)^{-(s+1)/2} \int_{(j^2-\varkappa)^{1/2}}^{\infty} \frac{dy}{(1+y^2)y^{s-1}} \; .$$

Since $\varkappa \in (0,1)$ and $j \geq 2$, it follows

$$\|K\| \leq \frac{1}{\pi} \left(\frac{4}{3} \right)^{(s+1)/2} \int_0^\infty \frac{dy}{(1+y^2)y^{s-1}} =: N(s) \; . \qquad (2.13)$$

We have $N(1) = 2/3$ so $\|K\| < 1$ for $s=1$. Moreover, the last estimate is continuous in s , and therefore there is $\delta > 0$ such that $N(1+\delta) < 1$. In this way, we have proven that the system (2.11) has a unique solution within the class of sequences fulfilling the condition (2.9) with $s=1$. This solution is of the form (2.12) and decreases as $O(j^{-(1+\delta)})$ for some $\delta > 0$.

It remains to solve the first equation of the system (2.8). Substituting from (2.9) and (2.10), we get

$$1 = \frac{(1 - e^{-2\pi\sqrt{1-\varkappa}})}{\pi\sqrt{1-\varkappa}} \left[\frac{1}{2-\varkappa} + \sum_{j=2}^{\infty} \frac{a_j}{j^2+1-\varkappa} \right] =: F(\varkappa) \qquad (2.14)$$

We shall show that F is a continuous increasing function on $[0,1]$ and $F(0) < 1 < F(1)$ so there is a unique point $\varkappa_0 \in (0,1)$ such that $F(\varkappa_0) = 1$.

First of all, we notice that $\varkappa \longmapsto b(\varkappa)$ is continuous in the ℓ^∞-norm and $\varkappa \longmapsto K(\varkappa)$ is continuous in the corresponding operator norm. The functions $b_j(.)$ have bounded derivatives and $b_j(\varkappa) \to 0$ as $j \to \infty$ uniformly with respect to $\varkappa$ so there is d_1 independent of $\varkappa$ such that $\|b(\varkappa_1)-b(\varkappa_2)\|_\infty \leq d_1|\varkappa_1-\varkappa_2|$ for any $\varkappa_1,\varkappa_2 \in [0,1]$; in a similar way one can check $\|K(\varkappa_1)-K(\varkappa_2)\| < d_2|\varkappa_1-\varkappa_2|$ for some d_2 . It follows readily from (2.12) that $\varkappa \longmapsto a(\varkappa)$ is continuous in the ℓ^∞-topology. Moreover, since all the $b_j(.)$ and $K_{jk}(.)$ are positive increasing functions, $a_j(.)$ is also positive and increasing for every $j = 2,3,\dots$, and the same is true for the function F . Using the estimates

$$\|a\|_\infty \le \frac{1}{1 - \|K(\varkappa)\|_\infty} \|b(\varkappa)\|_\infty \le 3\|b(\varkappa)\|_\infty ,$$

$$\|b(0)\|_\infty \le \frac{1}{\pi} \sup_{j\ge 2} \frac{j}{j^2+1} = \frac{2}{5\pi}$$

we get

$$F(0) = \frac{1}{2\pi}(1-e^{-2\pi}) \left[1 + \sum_{j=2}^{\infty} \frac{2}{j^2+1} a_j(0) \right] \le \frac{1}{2\pi} \left(1 + \frac{12}{5\pi} \sum_{j=2}^{\infty} j^{-2} \right)$$

$$\le \frac{1}{2\pi} \left[1 + \frac{12}{5\pi} \left(\frac{\pi^2}{6} - 1 \right) \right] = 0.237\ldots < 1 ,$$

while

$$F(1) = 2 \left(1 + \sum_{j=2}^{\infty} j^{-2} a_j \right) > 2 > 1 .$$

Concluding the argument, we have shown that there is a unique symmetric solution ψ to the eq.(2.5) in the class (2.9) with $s=1$, and moreover, that the coefficients decrease as $r_j = O(j^{-(1+\delta)})$ for some $\delta > 0$, i.e., ψ obeys the classical restriction mentioned above.

3. The numerical solution

The eq.(2.11) can be solved by iteration. We set $a^{(0)} = b$ and $a^{(j+1)} = b + Ka^{(j)}$, the number of iterations will be denoted as m. The numerical solution requires, of course, to truncate the system. Let $P \equiv P_n$ be the projection introduced in the previous section, the one can define $\tilde{a}^{(0)} = Pb$ and $\tilde{a}^{(j+1)} = Pb + PKP\tilde{a}^{(j)}$ for $j=0,1,\ldots,m$. Since $\|PKP\| \le \|K\| < 1$, the sequence $\{\tilde{a}^{(m)}\}_{m=0}^{\infty}$ converges to a unique solution to the equation $\tilde{a} = Pb + PKP\tilde{a}$.

One has to estimate the error due to the truncation, in particular, the quantities $w_j := \|Pa - \tilde{a}^{(j)}\|_\infty$. We have

$$Pa - \tilde{a}^{(j+1)} = PK\ (a - \tilde{a}^{(j)}) = PK\ (Pa - \tilde{a}^{(j)}) + PK(I-P)a \quad (3.1)$$

Next we use the fact that $r_j = O(j^{-(1+\delta)})$, i.e., that

$$0 < a_j < cj^{-\delta} \qquad (3.2)$$

for some $\delta > 0$ and all j . For $2 \le j \le n$ we get

$$0 < (PK(I-P)a)_j < \frac{j^2}{\pi(j^2-1)^{1/2}} \sum_{k=n+1}^{\infty} \frac{1}{j^2+k^2-1} \frac{c}{j^\delta} \le$$

$$\le c_0 j^{-\delta} \int_{n/j}^{\infty} \frac{dx}{x^{2+\delta}} = c_1 j n^{-(1+\delta)} = c_1 n^{-\delta}$$

for appropriately chosen c_0, c_1 , and therefore $\|PK(I-P)a\|_\infty \le c_1 n^{-\delta}$ for some positive c_1 . The relation (3.1) then yields $w_{j+1} \le \|K\| w_j + c_1 n^{-\delta}$ and by induction we obtain

$$w_m \le \|K\|^m w_0 + \frac{c_1}{1-\|K\|} n^{-\delta} . \qquad (3.3)$$

Since $\|K\| \le 2/3 < 1$, the sought solution a is approximated by $\tilde{a}^{(m)}$ in the sense that

$$\lim_{m,n\to\infty} \|P_n a - \tilde{a}^{(m)}\|_\infty = 0 . \qquad (3.4)$$

More explicitly, choosing m,n large enough, the difference $a_j - \tilde{a}^{(m)}$ can be made smaller than a given ε for $j = 1,\dots,n$ due to (3.3), while for $j = n+1, n+2, \dots$ it will be small due to (3.2) and the fact that $\tilde{a}^{(m)} = 0$.

Furthermore, approximating the function F by

$$\tilde{F}(\varkappa) := \frac{1 - e^{-2\pi\sqrt{1-\varkappa}}}{\pi\sqrt{1-\varkappa}} \left[\frac{1}{2-\varkappa} + \sum_{j=2}^{\infty} \frac{\tilde{a}^{(m)}}{j^2+1-\varkappa} \right] \qquad (3.5)$$

we get the estimate

$$|\tilde{F}(\varkappa) - F(\varkappa)| \le 2\left(\sum_{j=n+1}^{\infty} j^{-2} \right)\|a\|_{\infty} + 2\left(\sum_{j=2}^{\infty} j^{-2} \right) w_m \le$$

$$\le c_2\|a\|_{\infty} n^{-1} + c_3 w_m .$$

The inequality (3.3) yields

$$|\tilde{F}(\varkappa) - F(\varkappa)| \le c_4 n^{-1} + c_5\|K\|^m + c_6 n^{-\delta} ,$$

where c_4, c_5, c_6 are some positive constants.

For numerical solution, we have gone with the number of iterations up to $n = 20$ and with the order of truncation up to $m = 100$; it appears that both $\{a_m\}$ and the function F are very well stabilized at these values. In order to illustrate this, we present in the following table the values of $F(0.93)$ calculated for various n and m .

m \ n	10	20	40	70	100
2	.992	.995	.995	.996	.996
3	.996	1.000	1.001	1.001	1.001
5	.998	1.002	1.003	1.004	1.004
10	.998	1.002	1.004	1.004	1.004
20	.998	1.002	1.004	1.004	1.004

The function F is plotted on Fig.2 . The eq.(2.14) is then solved by

$$\varkappa_0 = 0.9291... \qquad (3.6)$$

which yields the result expressed by the relation (1.1). Using the calculated coefficients a_j together with (2.6) and (2.9), one can find also the eigenfunction corresponding to this eigenvalue. In particular, one can calculate the probability density in such a state ; it is plotted on Fig.3

4. Concluding remarks

The states discussed here can be manifested physically in various ways. The quantum mechanical applications are particularly interesting. One of them concerns the behaviour of electrons in ultrathin layers of a pure semiconductor material over a sharply edged substrate. One can see easily that the electrons (which move as free particles of some effective mass in the crystallic lattice) with energy below the first transversal mode remain localized near the edge ; it suggests a possible existence of *edge-confined currents* [4]. Such an effect has not been observed

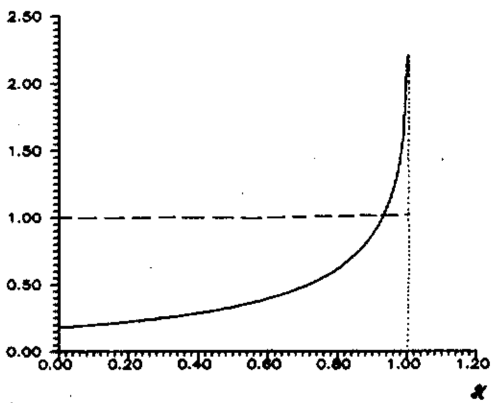

Fig.2. The function F

up to now, but there is a recent experimental evidence for similar currents in relief-surface semiconductor layers [11].

Another indirect indication for existence of curvature-related bound states comes from the experiments with the so-called quantum wires [12]. One has to recall that in a bent finite-length wire (strip, tube) the bound state turns into a resonance ;one expects the transmission coefficient to vary strongly around the infinite-length bound-state energy. Since the transmission coefficient is related to conductance through the Landauer formula [13], one expects that bending of a wire can affect its conductance. This is precisely one of the results of a recent experiment by Timp et al.[14] with many-probe junctions ; they have shown that the resistance between a pair of probes depends on the number of right-angle turns the electron path contains.

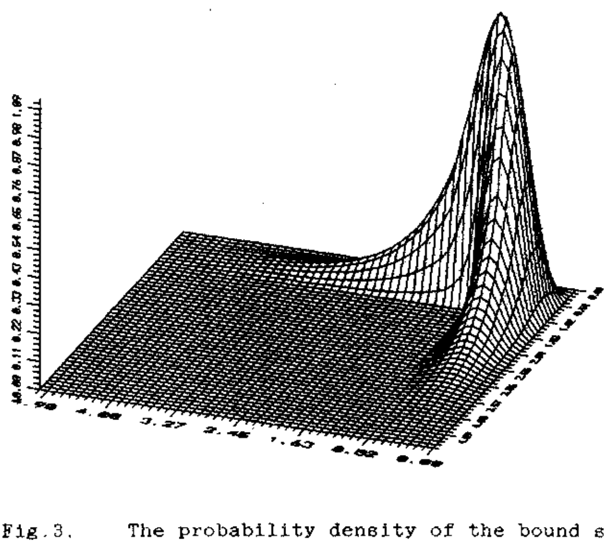

Fig.3. The probability density of the bound state

From the mathematical point of view there are many interesting problems related to the subject of this paper - cf.[15]. An immediate generalization worth of attention concerns the case of a strip broken at an arbitrary angle $\alpha \in (0,\pi)$; its solution could help to answer the question whether the critical width below which the curvature-related bound states appear is finite or not.

References

1 V.Votruba, C.Muzikář : Theory of the Electromagnetic Field, 2nd edition, Academia, Prague 1958 (in Czech).

2 V.Votruba : Foundations of Special Relativity, Academia, Prague 1969 (in Czech).

3 P.Exner, P.Šeba : Bound states in curved quantum waveguides, preprint BiBoS 298/87, Bielefeld 1987 ; submitted to J.Math. Phys.

4 P.Exner, P.Šeba, P.Stovicek : The edges can bind electrons, preprint SFB 237, Bochum 1987 ; submitted to Phys.Rev.Lett.

5 P.Exner, P.Šeba : Trapping modes in curved electromagnetic waveguides, preprint JINR E5-88-379, Dubna 1988 ; submitted to Phys.Lett.A.

6 M.Reed, B.Simon : Methods of Modern Mathematical Physics, IV.Analysis of Operators, Academic Press, New York 1978.

7 A.A.Kirilenko, V.R.Litvinov, L.A.Rudj, Radiotechnika i elektronika 24(1979),1043-1052.

8 A.A.Kirilenko, L.A.Rudj, V.I.Tkatchenko, Radiotechnika i elektronika 30 (1985),918-924.

9 L.V.Kantorovich, G.P.Akilov : Functional Analysis in Normed Spaces, Fizmatgiz, Moscow 1959 ; Sec.XI.2 (in Russian).

10 R.Mehran, IEEE Trans.:Microwave Theory and Technique 26 (1978),400-405.

11 T.Hundhausen, T.Ichiguchi, Y.Shiraki, Appl.Phys.Lett.53 (1988),110-112.

12 H.Sakaki : Advances in microfabrication and microstructure

physics, in Proc.Int.Symp."Foundations of Quantum Physics in the Light of New Technology" (S.Kamefuchi et al., eds.), Physical Society of Japan, Tokyo 1983 ; pp.94-110.

13 R.Landauer, Phys.Lett.A85 (1981),91-93.

14 G.Timp et al., Phys.Rev.Lett.60 (1988),2081-2084.

15 P.Exner, P.Šeba : Electrons in semiconductor microstructures: a challenge to operator theorists, in "Schroedinger Operators, Standard and Non-Standard" (P.Exner and P.Šeba, eds.), World Scientific, Singapore 1989.

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