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QUASI-LINEAR ELLIPTIC EQUATIONS IN THE INCOMPLETE NONLINEAR FORMULATION AND METHODS FOR THEIR PRECONDITIONING

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Introduction

We propose the incomplete nonlinear (IN) formulation[1] for one class of the divergent-type quasi-linear elliptic problems which arise in magnetostatic and nonlinear diffusion problems. The idea is that the domain of nonlinearity is partitioned into a finite number of subdomains Ω_i with equation coefficients equal to fixed constants to be defined which depend nonlinearly on the average value of the solution gradient in the substructure Ω_i. Thus we make the approximation of nonlinearity before the space discretization of the equation.

The uniqueness of solvability of the formulated nonlinear boundary value problem is studied as well as the estimates of errors resulting due to some simplifications concerned with IN - formulation. The problem in IN - formulation is transformed to the nonlinear boundary equation of the integral type defined only on internal boundaries of the subdomains Ω_i. The solution of this equation as well as of the corresponding Galerkin's schemes may be performed by means of the single iteration method with preconditioners, preconditioned conjugate gradient (PCG) method or by the Newton type methods.

The family of linear preconditioners is developed which provide the means for constructing the cost-effective methods for the solution of equations in IN - formulation. The construction is based on the domain decomposition method when the substructure boundaries contain the internal cross-points (decomposition of the "box" type). The rate of convergence of the iteration processes does not depend on the variation range of the coefficient of the modified initial nonlinear equation and weakly (logarithmically) depends on the step of the domain triangulation for some finite element Galerkin schemes.

The preconditioner operators are easily invertible both for parallel and for traditional computing architectures. In this paper we consider only the preconditioners for the two-dimensional case Some results for the three-dimensional case have been developed in [2,20,7]. Some approaches for the construction of block preconditioners for the difference elliptic equations have been developed in [3-14]

The case when the block-representation of unknowns corresponds to the domain decomposition with substructures of the "strip" type have been developed completely enough in [14-22]. Iteration methods for the difference elliptic equations with highly varying coefficients were developed in [4,14,23-25]. Preconditioners for the two-dimensional finite-element elliptic systems with the decomposition of the "box" type have been developed in [26-29], some results for the three-dimensional case were obtained in [27,2]. The multi-grid domain decomposition methods were developed in [30,39]. A method of domain decomposition with cross-points is used in in [31] for the solution of finite-difference elliptic boundary value problems in rectangle and in parallelepiped. Some numerical experiments were presented in [31]. It is also interesting to note that IN - formulation for the equations in the unbounded domain (see Chapter 3) may be constructed in the same way as in [1,22]. The problems of coupling BEM and FEM have been considered in [21,32,33].

§ 1. Equations in the IN-formulation

Equations in the IN-formulation (Problem IN) are some modification of the quasy-linear elliptic equations of the divergen-type (Problem FN)[1], which have broad applications in mathematical modelling.

Let the bounded domain $\Omega \in R^N$, $N = 2,3$ with a Lipschitz boundary Γ be a unification of two domains $\Omega = \Omega_o \cup \Omega_\mu$, having also Lipschitz boundaries Γ_o and $\Gamma_1{}^o$. We designate by $\Gamma_\mu = \Gamma_1{}^o \setminus (\Gamma \cup \Gamma_1{}^o)$.

PROBLEM FN. Find the finction $u(x)$, $x \in \Omega$, which satisfies the equation

$$Au = - \sum_{i=1}^{N} \frac{\partial}{\partial x_i} a_i(w) = 0$$

$$u_\Gamma = 0, \quad [u]_{\Gamma_\mu} = 0 \;, \quad [\partial u / \partial u]_{\Gamma_\mu} = \psi(x), \tag{1.1}$$

where $w = \text{grad } u \equiv (y_1, \ldots y_N)^T$, $[\;\cdot\;]_{\Gamma_\mu}$ is a jump of the function at Γ_μ, $\partial u / \partial n$ is a conormal derivative and for $i=1,\ldots,N$ we have

$$a_i(w) = \begin{cases} y_i, & x \in \Omega_o \\ \mu(|y|) y_i, & x \in \Omega_\mu ; \quad |y|^2 = \sum_{i=1}^{N} y_i{}^2 . \end{cases} \tag{1.2}$$

The given function $\mu(t)$, $t \in [0,\infty)$, satisfies to some of the conditions

$$\mu(t)\cdot t - \mu(r)\cdot r \ge m(t-r), \qquad t \ge r,\ m > 0, \quad (a)$$

$$|\mu(t)\cdot t - \mu(r)\cdot r| \le M|t-r|, \qquad (b) \quad (1.3)$$

$$\left|\frac{\partial}{\partial t}\mu(t)\cdot t\right| \le M . \qquad (c)$$

Let us consider the IN-formulation for the equations (1.1), (1.2). The domain Ω_μ is partitioned into a finite number p of subdomains Ω_i, $\Omega_\mu = \bigcup_{i=1}^{p} \Omega_i$, $\Omega_i \cap \Omega_j = \emptyset$, $i \ne j$, having the Lipschitz boundaries Γ_i.

PROBLEM IN. Find the function $u(x)$, $x \in \Omega$, satisfying the equation (1.1) with the function $a_i(x)$, $i=1,\dots,N$ of the type

$$a_i(w) = \begin{cases} y_i, & x \in \Omega_0, \\ \mu\left(\left(\frac{1}{\text{mes}\,\Omega_j} \int_{\Omega_j} |w|^2 dx\right)^{1/2}\right) y_i, & x \in \Omega_j,\ j = 1,\dots,p. \end{cases} \qquad (1.4)$$

supplemented by the conditions at the boundaries Γ_{kj}, common for the domains Γ_k and Γ_j.

$$[u]_{\Gamma_{kj}} = [\partial u/\partial n]_{\Gamma_{kj}} = 0, \quad k,j = 1,\dots,p;\ \Gamma_{kj} \cap \Gamma_\mu \ne \emptyset . \quad (1.5)$$

Then without the loss of generality we suppose that $\Gamma \cap \Gamma_1^0 = 0$, i.e., $\Gamma_1^0 = \Gamma_\mu$. We denote by $(\cdot,\cdot)$ the scalar product in L_2 space in the corresponding domain of definition. Let us consider the general formulation of the problem IN. We introduce the following spaces and operators

$$H = L_2(\Omega), \quad V = \overset{\circ}{W}{}_2^1(\Omega), \quad Y = L_2^{(N)}(\Omega) = \underbrace{L_2 \times \dots \times L_2}_{N},$$

$$T: u \to (\text{grad}\, u), \quad T \in (V \to Y), \qquad (1.6)$$

$$A_{IN}: y \to (a_1(y), \dots a_N(y)) - \bar{g} ,$$

where the linear functional $\bar{g} \in Y^*$ according to the Han-Banach theorem is a continuation of the linear continuous functional g

$$g(Th) = \int_{\Gamma_\mu} \psi(s)\, \gamma_0(h(s))\, ds , \qquad \forall h \in V$$

into the whole space $L_2^{(N)}(\Omega)$. Here we denote by $\gamma_0 : V \to W^{1/2}_{2,g_0}(\Gamma_\mu)$ the trace operator /35/ on Γ_μ for a function from the space V, and suppose that the function

$\psi \in W^{-1/2}_{1,2}(\Gamma_\mu)$, where

$$W^{1/2}_{2,g_0}(\Gamma_\mu) = \{ u \in W^{1/2}_2(\Gamma_\mu),\ (u,g_0) = 0 \},$$

$$W^{-1/2}_{2,1}(\Gamma_\mu) = \{ v \in W^{-1/2}_2(\Gamma_\mu),\ (v,1) = 0 \},$$

and g_0 is the density of the Robin potential on Γ_μ. We define the Sobolev spaces $W^{1/2}_2(\Gamma_\mu)$ and $W^{-1/2}_2(\Gamma_\mu)$ according to /35/. The direct testing of the expansion properties /40/ leads to the following lemma

LEMMA 1. For an arbitrary function $\psi \in W^{-1/2}_{2,1}(\Gamma_\mu)$ the operators and spaces (1.6) define(?) the energy expansion $\bar{A} = T^* A_{IN} T$ for the operator (1.1), (1.4), (1.5).

The generalized formulation of the Problem IN is : Find such $u \in v$ that

$$(A_{IN} Tv,\ T\eta) = 0 , \quad \forall \eta \in V . \qquad (1.7)$$

If we denote by $g_i = \text{mes}\Omega_i$, $\tau_i(y) = (g_i^{-1} \int_{\Omega_i} |y|^2 dx)^{1/2}$ and define the constants $\mu_0 = 1$ and $\mu_i(u) = \mu(\tau_i(\nabla u))$, $i = 1,\dots,p$, then the equation (1.7) becomes

$$\sum_{k=0}^{p} \mu_k(u) \int_\Omega (\nabla u, \Delta v) dx - \int_\Gamma \psi(s)\, \gamma_0(v) ds = 0 , \quad \forall v \in V. \qquad (1.8)$$

The following lemma have been proved in /1/.

LEMMA 2. If the condition (1.3a) or (1.3b) is satisfied, the operator $\bar{A}$. is strongly monotonous or Lipschitz continuous, correspondingly.. Under the conditions (1.3a) and (1.3c) the operator $\bar{A}$ has the Gatoex derivative which is symmetric and positively defined with the constant m. The operator A_{IN} is of potential type with the potential

$$F_0(y) = F_0(0) + \sum_{i=0}^{p} g_i \int_0^{\tau_i(y)} \mu(s)\cdot s\, ds ,$$

where for $i = 0$ we determine $\mu(s) = 1$.

The following theorem is a simple consequence of Lemmas 1,2.

THEOREM 1. Let the conditions (1.3a) and (1.3b) be satisfied. Then for every $\psi \in W^{-1/2}_{2,1}(\Gamma_\mu)$ there exists a unique generalized solution $u \in V$ for the Problem (1.7) (or (1.8)).

§ 2 Error estimates

Let us consider the estimates of the differences between the generalized solutions u_{FN} and u_{IN} for the Problems FN and IN, correspondingly. We denote $u_\Delta = u_{FN} - u_{IN}$; $\mu_{CI}(x) = \mu_i(|\nabla u_{IN}|)$; $\mu_{DI}(x) = \mu_i(u_{IN})(x)$; $x \in \Omega_i$, $i=0,\dots p$; $L_q^i = L_q(\Omega_i)$, $1 \le q \le \infty$.

The following lemma is a simple consequence from the properties of the operator $\bar{\mathbf{A}}$.

LEMMA 3. There holds the estimate

$$\| u_\Delta \|_V \le m^{-1} \sum_{i=1}^{p} (\int_{\Omega_i} |\mu_{CI} - \mu_{DI}|^2 \, |\nabla u_{IN}|^2 \, dx)^{1/2} \qquad (2.1)$$

If the assumptions $|\nabla u_{IN}|^2 \in L_q^i$, $1 \le q \le \infty$ and $q^{-1} + \rho^{-1} = 1$ are satisfied, then the estimate

$$\| u_\Delta \|_V \le c \sum_{i=1}^{p} \| \mu_{CI} - \mu_{DI} \|_{L_{2p}^i} \, \| |\nabla u_{IN}| \|_{L_{2q}^i} \qquad (2.2)$$

takes place.

If we use the Sobolev's inequality [41], we can specify the estimate of the first factor in terms of (2.2)[1]. We denote $v = |\nabla u_{IN}|$.

LEMMA 4. Suppose that $c \cdot (\mathrm{diam}\, \Omega_i)^N \le \mathrm{mes}\, \Omega_i$, $i = 1,\dots,p$, and $| \nabla v | \in L_2^i$, $v^2 \in L_\infty^i$, $i = 1,\dots,p$. Then the estimate

$$\| u_\Delta \|_V \le c \, d_1 d_2 (\mathrm{mes}\, \Omega_1^o)^{1/2} \qquad (2.3)$$

is true, where

$$d_1 = \max_{1 \le i \le p} (\mathrm{diam}\, \Omega_i),$$

$$d_2 = \max_{1 \le i \le p} (\| |\nabla v| \|_{L_4^i} \cdot \| v \|_{L_4^i} \cdot \| v \|_{L_\infty^i} \cdot \| v \|_{L_2^i}^{-1})$$

If $|\nabla u_{IN}| \in \mathbb{C}^1(\bar{\Omega}_i)$, then the following estimate results from (2.2)

$$\| u_\Delta \|_V \le c \cdot d_1 \cdot (\sum_{i=1}^{p} \| v \|_{L_{2q}^i}).$$

Note, that the estimate of the type $\| u_\Delta \|_V \le c \cdot d_1$ apparently can't be improved for the chosen method of approximation of nonlinearity and formally justifies the use of the IN-model only for sufficiently small d_1. However in practical problems it is often interesting to know the approximate solution for u_{FN} only in some subdomain $\Omega_A \subset \Omega$ which does not intersect the domain of nonlinearity Ω_μ (for example in magnetostatic problems Ω_A is the aperture domain). In the last case we can see in numerical experiments that the solution u_{IN} is the sufficient approximation for the function u_{FN} even for small number of subdomains p and for a phenomenoligical choice of of the constants μ_i, $i = 1,\dots,p$.

Besides the direct use of the IN-formulation, we can utilize the operator of the problem as a preconditioner for accelerating the convergence of the iterative processes for solution of the nonlinear Problem FN.

§3 Problem in the unbounded domain

An existence of the unique solution for the nonhomogeneous (on Γ) boundary value problems of Dirichlet and Neuman types for the IN-formulation can be proved in a similar way as in Theorem 1. In particular, for the Neuman problem an analogous statement to that in Lemma 2 takes place, as well as the following theorem[1]:

THEOREM 2. For every $\psi \in W_{2,1}^{-1/2}(\Gamma_\mu)$ and $g \in W_{2,1}^{-1/2}(\Gamma)$ there is a unique generalized solution $u \in W_2^1(\Omega)$, $(u,g_0) = 0$ for the equation

$$\sum_{k=0}^{p} \mu_k(u) \cdot \int_{\Omega_k} (\nabla u, \nabla \eta) dx - \int_{\Gamma_\mu} \psi(s) \gamma_0(\eta) ds = \int_{\Gamma} g(s) \gamma_0(\eta) ds , \qquad (3.1)$$

for $\forall \eta \in W_2^1(\Omega)$, $(\eta, g_0)_{L_2(\Gamma)} = 0$. Here g_0 is the density of the Robin's potential on Γ.

We define the nonlinear Poincare-Steklov operator S_{IN} for the problem in IN-formulation according to Theorem 2, so that $S_{IN} \in (Z^* \to Z)$, where $Z = W_{2,0}^{1/2}(\Gamma)$, $Z^* = W_{2,1}^{-1/2}(\Gamma)$. This operator maps any function $g \in Z^*$ to the trace $\gamma_0(u) \in Z$ of the function u on the boundary Γ, where u is the solution of the equation (3.1), so that the following inequality holds:

$$(S_{IN} g, \eta) = (\gamma_0(u), \eta), \qquad \forall \eta \in Z^*. \qquad (3.2)$$

Using properties of the operator $\bar{\mathbf{A}}$, obtained in Lemma 2 and the technique, used in paper [22], where the Problem FN has been investigated , we can prove the following

THEOREM 3. Under the conditions (1.3a) and (1.3b) the operator S_{IN} is strongly monotonous, continuous and has the inverse S_{IN}^{-1}, which is Lipschitz continuous and strongly monotonous, $S_{IN}^{-1} \in (Z \to Z^*)$. Under the condition (1.3c) the operator S_{IN}^{-1} has the Gatoex derivative with positively defined and symmetrical $R = [S_{IN}^{-1}]'$. The operator S_{IN}^{-1} has the potential type with the potential

$$F(u) = F(0) + \sum_{i=0}^{p} g_i \int_0^{\tau_i(|\mathrm{grad}\, v|)} \mu(t) \cdot t \cdot dt, \qquad \gamma_0(v) = u .$$

The Theorem 3 as it was in [22] enables to construct equations for the domain decomposition method when the solution u of the boundary value problem IN is being found in the whole space $\mathbb{R}^N$

with the boundary condition $u(\infty)=0$. Let the integral operators K and L define the density of the simple and double-layer potentials on the auxiliary surface Γ covering the domain Ω_0^1. Then the following equation defined on the surface Γ is true

$$S^1_{IN}u_\Gamma + G^1 u_\Gamma = 0, \quad u_\Gamma = \gamma_0 u, \qquad (3.3)$$

where $G \in L(Z \to Z^*)$, $G = (E + K)^{-1}L$. Iterative methods for solution of the problem (3.3) are similar to those developed in [22] for the combined formulation of the Problem FN. Independently of the chosen method (simple iterations, gradient or Newton type methods) it is necessary at every step of iterations to solve the boundary integral equation with the operator E + K and to compute $S^{-1}_{IN}\cdot u$ for $u \in Z$. The last pr ocedure is reduced to the solution of nonhomogeneous Dirichlet problem (1.1), (1.4), (1.5) in the domain Ω, which can be easily transformed to homogeneous one, i.e., with the condition $u_\Gamma = 0$. Further we consider methods for solution of homogeneous Problem IN.

§4. Nonlinear equations at the inner boundaries

We divide the domain $\Omega = \bigcup_{i=1}^{M} \Omega_i$ into subdomains Ω_i with Lipschitz boundaries using "chess" (or black-white) division which in two-dimensional case (N=2) is topologically equivalent to division of a rectangular Ω to checks by (m_x-1) vertical and (n_y-1) horisontal lines. Thus in two-dimensional numeration of domains Ω_{ij}, $i \le m_x$, $j \le n_y$ we obtain representations $\Omega = \Omega_B \cup \Omega_W$, $\Omega_B = \bigcup_{i+j=2l} \Omega_{ij}$, $\Omega_W = \bigcup_{i+j=2l+1} \Omega_{ij}$. In the three-dimensional case (N=3) the subdivision is similar. Further we use one-dimensional indexation for subdomains Ω_i. Denote by I_B the set of indexes i for which $\Omega_i \in \Omega_B$ and the same for $i \in I_W$, if $\Omega_i \in \Omega_W$. Designate by $\Gamma_i = \partial\Omega_i$, $\Gamma = \partial\Omega$ and $\Gamma_I = \bigcup_{i\in I_B} \Gamma_i \setminus \Gamma = \bigcup_{i \in I_W}\Gamma_i\setminus\Gamma$ the set of inner boundaries. For any function $u \in \overset{\circ}{W}{}^1_2(\Omega)$ we define the traces[35] $u_i = \gamma_i u$ of this function on $\tilde\Gamma_i = \Gamma_i \cap \Gamma_I$, $i \le M$. The space of functions u_i is denoted by $V_i^{1/2} = \tilde W_2^{1/2}(\Gamma_i)$ (with the norm from $W_2^{1/2}(\Gamma_i)$). In the same way the space of the normal derivative traces $\gamma_i^1 u = \frac{\partial u}{\partial n}\Big|_{\Gamma_i}$, $u \in V$, $i = 1,\dots M$ are denoted by $V_i^{-1/2} = \tilde W_2^{-1/2}(\Gamma_i)$ (with the norm from $W_2^{-1/2}(\Gamma_i)$). Here the sign ~ implies some subspace of the corresponding function space (with the domain

of definition Γ_i) defined according to the position of the boundaries Γ_i and Γ. Next we introduce the following spaces
$X_B = \sum_{i\in I_B}\oplus V_i^{1/2}$, $X_W = \sum_{i\in I_W} \oplus V_i^{1/2}$, $X_B^* = \sum_{i\in I_B}\oplus V_i^{-1/2}$, $X_W^* = \sum_{i\in I_W}\oplus V_i^{1/2}$. Further $S_{\Delta,i}$ are the operators of the Poincare-Steklov type for the Laplacian in Ω_i, where $D(S_{\Delta,i}) = \{v \in X_B^* : (v,1)_{\Gamma_i} = 0\}$ and the inverse operator $S^{-1}_{\Delta,i} \in \mathcal{L}(V_i^{1/2} \to V_i^{-1/2})$ with $\mathrm{Ker}\, S^{-1}_{\Delta,i} = \{u \in V_i^{1/2} : u=\mathrm{const},\ x\in\Omega_i\}$. The following operators are defined

$$S^{-1}_{B,\Delta} = \sum_{i\in I_B}\oplus S^{-1}_{\Delta,i}, \quad S^{-1}_{B,\Delta}\in\mathcal{L}(X_B\to X_B^*) \qquad (4.1)$$

$$S^{-1}_{W,\Delta} = \sum_{i\in I_W}\oplus S^{-1}_{\Delta,i}, \quad S^{-1}_{W,\Delta}\in \mathcal{L}(X_W\to X_W^*),$$

as well as the permutation operator $\mathbb{U}: X_B \to X_W$, $\mathbb{U}\mathbb{U}^* = E$, which transforms every function from from X_B into the function from X_W by interchanging the corresponding blocks of elements u_i[1,2]. According to Green's formula, we have

$$\int_{\Omega_i} |\nabla u|^2 dx = (S^{-1}_{\Delta,i}\cdot u, u),$$

and consequently

$$\tau_i(\nabla u) = [1/g_i (S^{-1}_{\Delta,i}\cdot u_i, u_i)]^{1/2}, \quad u_i = \gamma_i u. \qquad (4.2)$$

We define constants $\mu_i(u) = \mu(\tau_i(\nabla u))$, $i = 1,\dots,M$ for $u\in V$ according to (4.2). The diagonal operators are $\mathbb{M}_B = \sum_{i\in I_B}\oplus \mu_i E_i$ and $\mathbb{M}_W = \sum_{i\in I_W}\oplus\mu_i E_i$, where E_i are the identity operators in $V_i^{1/2}$. Let us consider the nonlinear operator $\mathbb{A}_I = \mathbb{A}_1 + \mathbb{A}_2$, where

$$\mathbb{A}_1 = \mathbb{M}_B\cdot S^{-1}_{B,\Delta}, \quad \mathbb{A}_2 = \mathbb{U}^*\mathbb{M}_W S^{-1}_{W,\Delta}\mathbb{U} \qquad (4.3)$$

It follows from the definition of the operators $\mathbb{A}_1$ and $\mathbb{A}_2$ that if $u \in V$ is a generalized solution of the Problem IN, satisfying (1.8), then its trace $u_\gamma = \gamma_I u$, defined on Γ_I satisfies the equation

$$\mathbb{A}_I\cdot u_\gamma = \psi, \quad u_\gamma \in X_B \qquad (4.4)$$

in the sense of equality in X_B^*. The function $\psi \in W^{-1/2}_{2,1}(\Gamma_\mu)$ was defined earlier supposing that $\Gamma_\mu \in \Gamma_I$. Note that (4.4) is the equation of the domain partition for the Problem IN for the decomposition $\Omega = \Omega_B\cup\Omega_W$. The nonlinearity structure of the operator $\mathbb{A}_I$ is given by the diagonal factors M_B and M_W. Norms in the space X_B are defined by

$$\|u\|^2_{X_B} = \sum_{i\in I_B}\|u_i\|^2_{V_i^{1/2}} + \sum_{i\in I_W}\|(\mathbb{U}\cdot u)_i\|^2_{V_i^{1/2}}.$$

§ 5. Solution of the nonlinear boundary equation

Let us consider methods of solution for the equation (4.4). Assume that the constants $\mu_i > 0$, $i = 1,\dots,M$, defining the operator A_I in (4.3) don't depend on the function u, i.e., $\bar{u}_o = (\mu_1,\dots,\mu_M)^T$ is a fixed vector. The corresponding linear operator is denoted by $A(\bar\mu_o)$. Then the following Lemma is true /1/.

LEMMA 5. The operator $A(\bar\mu_o) \in \mathcal{L}(X_B \to X_B^*)$ is symmetric and positively defined. If we use the equivalent norms

$$\|u\|_X^2 = (M_B(\bar\mu_o)\, S_{B,\Delta}^{-1} u, u) + (M_W(\bar\mu_o)\, S_{W,\Delta}^{-1} Tu, Tu),$$
$$\|v\|_{X^*}^2 = (A(\bar\mu_o)^{-1} v, v), \qquad (5.1)$$

then the operator $A(\bar\mu_o)$ is a dual mapping of the spaces X_B and X_B^*, i.e.,

$$(A(\bar\mu_o)\cdot u, u) = \|u\|_{X_B}^2 = \|A(\bar\mu_o)u\|_{X_B^*}^2 .$$

Consider the nonlinear operator A_I. From /1/ it follows

LEMMA 6. Under the conditions (1.3a) and (1.3b) the operator $A_I \in (X_B \to X_B^*)$ is strongly monotonous with the constant m, Lipschitz continuous with the constant 3M, has the potential and the Gateaux derivative which is the symmetric positively defined operator.

CONSEQUENCE 1. For any function $\psi \in X_B^*$ there is a unique solution $u_\gamma \in X_B$ of the equation (4.4) which coincides with the trace $\gamma_I u$ of the generalized solution of the equation (1.8) on Γ_I.

Using Lemmas 5, 6 one can formulate the convergence conditions for the Newton type and gradient methods for solution of the equation (1.4) as well as for simple iteration methods with preconditioners. Let us consider the latter group of methods. From the theorem 4.5 /40/ it easily follows

THEOREM 4. Let for $\mu_o \in R^M$-an arbitrary vector with positive components the following inequalities hold

$$(A_I u - A_I v,\ u-v) \ge m\,(A(\bar\mu_o)(u-v), u-v),$$
$$\|A_I u - A_I v\|_{X_B^*}^2 \le M_o^2 (A(\bar\mu_o)(u-v), u-v). \qquad (5.2)$$

Then for $\tau \in (0, 2M_o^{-1})$ the iterational process

$$A(\bar\mu_o)\,\frac{u_{n+1} - u_n}{\tau} = -A_I u_n + \psi, \qquad n = 0,1,\dots \qquad (5.3)$$

converges to the solution u_γ with the rate

$$\|u_n - u_\gamma\|_{X_B} \le \frac{\tau\cdot q^n}{1-q}\,\|A_I u_o - \psi\|_{X_B^*} ,$$

where $q = \max(1-\tau m_o, 1-\tau M_o)$, and $u_o \in X_B$ is an arbitrary vector. At $\tau = 2(m_o+M_o)^{-1}$ we have $q(\tau) = (M_o - m_o)\cdot(M_o + m_o)^{-1}$.

Let's construct the finite-dimensional approximation of the equation (4.4) by the Galerkin's method and obtain for it the analoques of the Theorem 4 and the Consequence 1.

Let $X_n \subset X$ be a linear subspace in X with the norm induced from X, where $h_1,\dots,h_n$ is a complete linear independent set of basic functions in X_n. Designate by $I_n \in (X_n \to X)$ the imbedding operator of X_n into X and the conjugate operator by $I_n^* \in (X^* \to X_n^*)$. Consider the system of equations for the Galerkin's approach $u_n \in X_n$

$$(A_I u_n, h_i) = (\psi, h_i), \qquad i = 1,\dots,n. \qquad (5.4)$$

Defining the operator $A_n = I_n^* A_I I_n \in (X_n \to X_n^*)$ and the element $\psi_n = I_n^*\psi \in X_n^*$, (5.4) can be written as an operator equation in X_n /40/

$$A_n \cdot u_n = \psi_n \qquad (5.5)$$

According to Lemma 1.4 /40/ and since $\|I_n u_n\| = \|u_n\|$, the properties of the operator A_n are similar to those of A_I. From Theorem 3.3 /40/ it follows

LEMMA 7. Under the conditions (1.3a) and (1.3b) the equation (5.5) has a unique solution for which the following estimate is true

$$\|u_n - u_\gamma\|_X \le \frac{3M}{m} \inf_{v\in X_n} \|v - u_\gamma\|_X \qquad (5.6)$$

With given operator $A_n(\bar\mu_o) = I_n^* A(\bar\mu_o) I_n$ and the equivalent norms in X_n and X_n^* defined according to (5.1), where $A(\bar\mu_o)$ is changed for $A_n(\bar\mu_o)$ the algorithm for solving the equation (5.5) is the same as for equation (4.4). In particular, the analoque to the Theorem 4 is true with the same constants m_o and M_o but with A_n, $A_n(\bar\mu_o)$ and X_n instead of A_I, $A(\bar\mu_o)$ and X_B, correspondingly.

The cost-efficiency of the algorithm (5.3) as well as of the Newton type and gradient methods depends on the cost-effectiveness of the inversability of the linear operator $A_n(\bar\mu_o)$. Therefore we further consider the construction of the easily invertible preconditioners B for the operator $A_n(\bar\mu_o)$, i.e., such symmetric linear operators $B > 0$, for which

$$c_2(B\cdot u, u) \ge (A_n(\bar\mu_o)\cdot u, u) \ge c_1(B\cdot u, u), \qquad u \in X_n .$$

where the constants c_1, $c_2 > 0$ are weakly dependent or quite independent on the dimension of the space X_n. In what follows we consider the two-dimensional case , while the preconditioners for the three-dimensional case with the checker-board subdivision has been investigated in /2/.

§6. Construction of the linear preconditioner

We consider the "chess" subdivision of the domain Ω mentioned in §4. Without the loss of generality we suppose that all subdomains Ω_k are rectangles, though the results are true for the convex quadrangles also. Let us consider an exactly inner rectangle , i.e., $\Gamma_i \cap \Gamma = \emptyset$. We partition the component $u_i \in V_i^{1/2}$ of the element $u \in X_B$ into four components $u_i = \{u_i^k\}$, $k = 1,\dots,4$ each defined only on one side of the rectangle Ω_i. Correspondingly the operator $S_{\Delta,i}^{-1}$ may be written in a block form

$$S_{\Delta,i}^{-1} = \{ S_i^{km} \}, \qquad k,m = 1,\dots,4 \qquad (6.1)$$

For the domains of the boundary layer the block dimension of the operator $S_{\Delta,i}^{-1}$ equals to 2 or 3, while the block representation here can be obtained by deleting the block rows and columns corresponding to zero components u_i^k of the vector u_i.

We imply that the considered subdivision of the domain $\Omega = \bigcup_{i=1}^{M} \Omega_i$ form a grid which determines the first order finite elements of the "serendipity" - type . The dimension of the corresponding basic function space , which can be denoted by X_L equals q_I of the inner vertexs of the subdomains Ω_i. Denoting $A_2 = A(\bar\mu_0)$, define the spaces

$$X_{2,i} = \sum_{k=1}^{4} \oplus \overset{\infty}{H}{}^{1/2}(\Gamma_i^k) , \qquad X_2 = \sum_{i \in I_B} \oplus X_{2,i} , \qquad (6.2)$$

where the space $\overset{\infty}{H}{}^{1/2}(\Gamma)$ is defined, for example, in /35/. Let $X_2 \subset \bar X_2$ be some subspace of $\bar X_2$ and $X_0 = X_L \oplus X_2$. Then for any $u \in X_0$ the unique representation $u = u_1 + u_2$, $u_1 \in X_L$, $u_2 \in X_2$ is correct. For any $u, v \in X_0$ we have

$$(A_2 u, v) = \sum_{i=1}^{2} \sum_{j=1}^{2} (A_2 u_i, v_j) . \qquad (6.3)$$

In constructing the operator $\mathbb{B}$ we exclude from (6.3) the cross-point terms and replace the second summand by its block-diagonal part. Define the operator

$$\operatorname{diag} A_2 = M_B \cdot \sum_{i \in I_B} \oplus \left(\sum_{k=1}^{4} \oplus S_i^{kk} \right) + M_W \cdot \sum_{i \in I_W} \oplus \left(\sum_{k=1}^{4} \oplus S_i^{kk} \right) , \qquad (6.4)$$

according to block representation (6.1). Using the representation (6.4), define the preconditioner operator $\mathbb{B} u$ for $u \in X_0$ by

$$(\mathbb{B}u, v) = (\operatorname{diag} A_2 u_2, v_2) + (A_2 u_1, v_1), \quad \forall\, v \in X \qquad (6.5)$$

Let us state some auxiliary definitions. Let $X = X_1 \oplus X_2$ be a direct sum of some Hilbert type spaces X_1 and X_2. Consider the following numerical characters

$$\alpha(X, X_1) = \sup \alpha \ge 0 \quad , \quad \alpha(X, X_2) = \sup \beta \ge 0 \qquad (6.6)$$

for the constants $\alpha \ge 0$, $\beta \ge 0$, satisfying the estimate

$$\alpha \| x_1 \|^2 + \beta \| x_2 \|^2 \le \| x_1 + x_2 \|^2 , \ \forall\, x_1 \in X_1,\ x_2 \in X_2 .$$

These characteristics are analoque to the notion of a spread (opening) of a pair of a Hilbert spaces.

§7. General estimate of the condition number

Denote similar to (6.2) $X_2 = \sum_{i \in I_B} \oplus X_{2,i}$. For the function $v \in X_{2,i}$ given on Γ_i^k , $k = 1,\dots,4$ define the functionals

$$\bar f_k(v) = \left(\int_{a_0}^{a_1} v^2(x) \left(\frac{1}{|x-a_0|} + \frac{1}{|x-a_1|} \right) dx \right)^{1/2}, \ x \in \Gamma_i^k,$$

where a_0 and a_1 are the ends of the segment Γ_i^k, and then for the function $u_i = \sum_{k=1}^{4} \oplus u_i^k \in X_{2,i}$ we construct the functional

$$f_i^2(u_i) = \sum_{k=1}^{4} \bar f_k^{\,2}(u_i^k) \qquad (7.1)$$

The condition number $K(\mathbb{B}^{-1} A_2)$ is estimated in terms of some general characteristics of the space X_2, not related with the operators $\mathbb{B}$ and A_2. We use the following hypothesises:

H1. There is such a constant $g(X_2) > 0$, that for any function $u \in X_0$ satisfying the condition $u(\xi_0) = 0$ for some $\xi_0 \in \Gamma_i$ the inequality holds

$$\| u \|^2_{L^\infty(\Gamma_i)} \le g(X_2) \cdot \int_{\Omega_i} |\nabla \bar u|^2 dx , \qquad (7.2)$$

H2. For any function $u \in X_2$ the following inequality holds

$$\bar f_k^{\,2}(u_i^k) \le \varepsilon(X_2) \cdot \max_{x \in \Gamma_i^k} | u_i^k |^2 ,$$

where Γ_i^k passes through all inner edges and $\varepsilon(X_2) > 0$.

Here and in what follows $\bar u$ designates a harmonic in Ω_i function having a trace u on Γ_i.

According to [2] for the constants defined in (6.6) the following estimates are true

LEMMA 8. Under the hypothesis H1 the following estimate is true

$$\min\left(\alpha(\mathbf{X}_0,\mathbf{X}_1),\alpha(\mathbf{X}_0,\mathbf{X}_2)\right) \geq c\cdot(1+g(\mathbf{X}_2))^{-1},$$

where the constant $c>0$ is defined only by the shape of domains Ω_i.

LEMMA 9. Assuming the hypothesises H1 and H2 are true, for any functions $u_L \in \mathbf{X}_1$ and $u_i \in \mathbf{X}_{2,1}$ the estimate holds

$$f_i^2(u_i) \leq c\cdot g(\mathbf{X}_2)\cdot \varepsilon(\mathbf{X}_2) \int_{\Omega_i} |\nabla(\bar{u}_i+\bar{u}_L)|^2 dx. \qquad (7.5)$$

Using Lemmas 8,9, the inequalities for the function traces from H^1 on the Lipschitz surfaces and the results of sticking together the functions from H^r, $0 < r < 1$, defined at the surface of the quadrangle [34] in [2] the following theorem is proved

THEOREM 5. If the hypothesises H1 and H2 hold, then for the preconditioner $\mathbb{B}$ defined according to (6.5) the following estimates

$$c_1(\mathbb{A}_2 u,u) \leq (\mathbb{B}\, u,u) \leq c_2(1 + g(\mathbf{X}_2))\cdot(1 + \varepsilon(\mathbf{X}_2))(\mathbb{A}_2 u,u) \qquad (7.6)$$

take place for all $u \in \mathbf{X}_0$ where the constants $c_2, c_1 > 0$ depend only on the shape of the domain Ω_i.

Note that the problem of inversing the operator $\mathbb{B}$ resulting to solution of the equation

$$(\mathbb{B}\cdot u^*, v) = (f,v), \qquad \forall\, v \in \mathbf{X}_0$$

is equivalent to solving of two rather simple problems . First find the function u_2 from the equation

$$(\mathrm{diag}\, \mathbb{A}_2 u_2, v) = (f,v) \quad , \qquad \forall\, v \in \mathbf{X}_2 \qquad (7.7)$$

and the function u_1 from the equation

$$(\mathbb{A}_2 u_1, v) = (f,v) \;, \qquad \forall\, v \in \mathbf{X} \qquad (7.8)$$

for which the equality $u_1 + u_2 = u^*$ holds. The problem (7.7) results to independent solution of combined problems for the Laplace operator in domains Ω_i with the Neumann condition on one side of the domain and with the homogeneous Dirichlet condition for three other sides. For the corresponding subspaces $\mathbf{X}_2$ such problems are solved either by the FFT method or by the approach proposed in [38]. The problem (7.8) corresponds to finite-element sets of linear algebraic equation with the dimension M for the "serendipity"-type elements of the first order formed by partition of the domain

$\Omega = \bigcup_{i=1}^{M} \Omega_i$. To solve the equation (7.8) the direct methods or the preconditioned conjugate gradients method (PCG) can be used.

§8. Results for the specific subspaces

Let us consider the estimates of the constants $g(\mathbf{X}_2)$ and $\varepsilon(\mathbf{X}_2)$ for two finite dimensional subspaces $\mathbf{X}_2$.

Consider the subspace $\mathbf{X}_{2,h} \subset \mathbf{X}_2$ consisting of piece-vice linear on Γ_i, $i = 1,\ldots,M$, functions with zeroes at the vertex of the rectangles Ω_i. Partition the edges Γ_i^k, $k = 1,\ldots,4$ to $n_{i,k}+1$ segments Δ_j, $j=0,\ldots,n_{i,k}$, so that for $h>0$ there are constants c_0, c_1, independent on j, k, h, for which $c_0 h \leq |\Delta_j| \leq c_1 h$ for all j. Define subspaces

$$\mathbf{X}_{i,k} = \{\, u \in \mathbb{C}(\bar{\Gamma}_i^k),\;\; u|_{\Delta_j} \in P_1(x);\; u(x)=0,\; x \in \partial(\Gamma_i^k)\,\}, \quad (8.1)$$

where $P_1(x)$ is a set of linear polynomials. Then the space $\mathbf{X}_{2,h}$ is defined by

$$\mathbf{X}_{2,h} = \sum_i \oplus\, \mathbf{X}_{2,i}; \qquad \mathbf{X}_{2,i} = \sum_{k=1}^{4} \oplus\, \mathbf{X}_{i,k} \qquad (8.2)$$

Define also the space $\mathbf{X}_{2,s}$ similar to (8.1) so that at every edge Γ_i^k the functions from $\mathbf{X}_{2,s}$ take the form $(\sin \dfrac{\pi p x}{l_i^k})$, $p = 1,\ldots,n_{i,k}$, $l_i^k = |\Gamma_i^k|$, $x \in \Gamma_i^k$. It is convinient to use the space $\mathbf{X}_{2,s}$ for the rectangular domains while the space $\mathbf{X}_{2,h}$ is well-suitable for the finite-element approximation of the functions which are harmonic within the quadrangle Ω_i of the general type. In both cases we designate $n_i = \max_k n_{i,k}$, $n_M = \max_i n_i$,

where $n_{i,k} = \dim(\mathbf{X}_{i,k})$.

Let us consider the following auxiliary confirmation.

LEMMA 10. Assume that for the function $u(x) \in H^1(\Omega_i)$, continuous at the convex quadrangle Ω_i the following estimate holds

$$h\, \|\nabla u\|_{L^\infty(\Omega_i)} \leq c\cdot \max_{x\in\Omega_i} |u(x)| \qquad (8.2)$$

for some $h>0$ and there is $\zeta_0 \in \Gamma_i$, for which $u(\zeta_0) = 0$.

Then

$$\max_{x\in\Omega_i} |u(x)|^2 \leq c\,(1 + \ln(d/h)) \int_{\Omega_i} |\nabla u|^2 dx, \qquad (8.3)$$

where the constant c doesn't depend on h and $d = \mathrm{diam}\,(\Omega_i)$.

This lemma generalizes the well-known estimates for the functions from the finite-element subspaces [26,37] and can be proved using the technique from [26].

LEMMA 11. For any of the subspaces $X_{2,h}$ and $X_{2,e}$ the constants $\varepsilon(X_2)$ and $g(X_2)$ defined in hypothesises H1 and H2 are estimated by

$$\max(\varepsilon(X_2), g(X_2)) \le c(1 + \ln n_M) \qquad (8.4)$$

where the constant c doesn't depend on n_M ∎ This confirmation is proved on the basis of Lemma 10 similar to Lemma 7 from [2].

From the Theorem 5 and Lemma 11 it follows

THEOREM 6. Assume that X_2 is one of the spaces $X_{2,h}$ or $X_{2,e}$. Then for any $u \in X_2 \oplus X_L$ the estimate is true

$$c_1(\mathbb{A}_2 u, u) \le (\mathbb{B} \cdot u, u) \le c_2(1 + (\ln n_M)^2)(\mathbb{A}_2 u, u) \qquad (8.5)$$

where constants c_1, c_2 doesn't depend on n_M.

Note that the estimate (8.5) for the case $X_2 = X_{2,h}$ coincides with the similar estimates of preconditioners for the finite element elliptic systems of equations [25,26,28,29].

REMARK 1. For the domains Ω_i being not rectangles in calculations it is convinient to replace the components of the operator $\mathbb{B}$ in the subspace $X_{2,i} \subset X_{2,h}$ by spectrally equivalent operators $\mathbb{C} = \left(-\frac{d^2}{dt^2}\right)^{1/2}$, which define the norm equivalent to $\|\cdot\|_{H^{1/2}_{00}(\Gamma_i^k)}$ [35]. Such operators are used in [26].

REMARK 2. It is easy to show that estimates (7.6) and (8.5) are true also if we use instead of harmonic functions in Ω_i the "h-harmonic" functions, i.e., those satisfying the finite element algebraic systems for the corresponding triangulation of the domains Ω_i obtained by the Galerkin scheme.

REMARK 3. It can be easily seen that for the partition of the "strips" type , i.e., at $n_y=1$ the quantity $K(\mathbb{B}^{-1}\mathbb{A}_2)$ doesn't depend on the dimension n_M for the considered subspaces.

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Квазилинейные эллиптические уравнения в неполно-нелинейной постановке и методы их переобуславливания

Рассматриваются краевые задачи для уравнений в неполно-нелинейной (IN) постановке, являющихся модификацией квазилинейных эллиптических уравнений дивергентного типа. На основе клеточной декомпозиции области для уравнений в IN-постановке построено семейство линейных переобуславливающих операторов, которые легко обратимы /как на параллельных, так и на последовательных ЭВМ/ и близки по спектру к соответствующему линеаризованному оператору с переменными /сильно меняющимися/ коэффициентами. Рассмотренные переобуславливатели дают возможность эффективно решать уравнения магнитостатики как в IN-постановке[1], так и в постановке уравнений Максвелла для скалярного потенциала.

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Quasi-Linear Elliptic Equations in the Incomplete Nonlinear Formulation and Methods for Their Preconditioning

We propose a boundary value problems for equations in the incomplete-nonlinear (IN) formulation, which are some modification for quasi-linear equations of the divergent-type. The family of linear preconditioners is developed which are easily invertible (both for parallel and for traditional computers) and spectrally close to the corresponding linearized operator with highly variable coefficients. The construction is based on the domain decomposition method with checkerboard subdivision. This preconditioners provide means for cost-effective solution of magnetostatic equations for IN-formulation, as well as in formulation of the Maxwell equation for the scale potential representation.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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