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ON THE INITIAL CONDITION FOR INSTANTON SOLUTIONS

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1. INTRODUCTION

This paper is addressed to the initial condition in the sense of Takasaki for both local and global instanton bundles. Throughout the paper the gauge group is assumed to be $U(r)$, $r \geq 2$; and it is well known that it can be always reduced to $SU(r)$ in the global case. In what follows r denotes the rank of the holomorphic or instanton bundle under consideration and $c_2 = c$ denotes its topological charge equal to the second Chern class. It appears to be convenient to treat the framed instanton bundles [1].

We shall prove that in the inverse scattering formalism we can distinguish a special "canonical" solution in every gauge equivalence class, and particularly we can introduce the notion of the canonical initial condition. Since the initial value problem has the unique solution [2] we can eliminate gauge freedom in this way. Moreover, the reality condition is retained in this treatment. Using twistors we shall also describe a geometric construction relating the canonical initial condition to a distinguished and again called canonical transition function. All considerations associated with the construction remain valid even in the more general case of framed holomorphic vector bundles over $\mathbb{P}^n$, $n \geq 2$, and the mentioned transition function appears to be a rational matrix function on $\mathbb{P}^n$ with some special properties.

Using this construction we are able to obtain explicit expressions for the canonical initial condition of the ADHM instantons and consequently to give the full transcription of the ADHM construction into the inverse scattering formalism. To the author's knowledge, despite of the fact that the ADHM construction became now classical such a transcription was nowhere derived and published until yet. Hopefully it will enable to check and further develop some previous concepts such as the Bäcklund transformation [3,4].

In other words we have found the explicit form for the embedding of the framed instanton moduli spaces $M(r,c_2)$ into the space $\mathcal{W}$ of canonical initial conditions. We advance this approach and show that there exists an injective holomorphic mapping of the moduli space into a finite-dimensional complex vector space with the image being a (locally) analytic set. Following Crane [5], in Sec.7 we shall consider the loop group action on the instanton transition functions. Particularly we shall discuss the infinitesimal action on the 1-instantons.

2. PRELIMINARIES

We choose in $\mathbb{C}^4$ a basis (the standard one) $\{e_1, \dots, e_4\}$ and hence the coordinates $(z_1, \dots, z_4)$, and also the real structure τ: $(z_1, z_2, z_3, z_4) \longmapsto (-\bar{z}_2, \bar{z}_1, -\bar{z}_4, \bar{z}_3)$. This real structure is transferred to the projective space $\mathbb{P}^3 = \mathbb{P}(\mathbb{C}^4)$ as well as to the Grassmann manifold $\mathbb{G}_2 = \mathbb{G}_2(\mathbb{C}^4)$ consisting of lines in $\mathbb{P}^3$, and also to the flag manifold $\mathbb{F}_{1,2} = \mathbb{F}_{1,2}(\mathbb{C}^4) \subset \mathbb{P}^3 \times \mathbb{G}_2$. Every point in $\mathbb{P}^3$ lies on the unique real line. The manifold of real lines in $\mathbb{P}^3$ is the sphere S^4 and the projection $\pi: \mathbb{P}^3 \longrightarrow S^4$ is the Penrose twistor transformation. We have the real analytic embeddings $S^4 \hookrightarrow \mathbb{G}_2$ and $\mathbb{P}^3 \hookrightarrow \mathbb{F}_{1,2}$. Denote by $\mathbb{P}^2$ the 2-dimensional projective space embedded into $\mathbb{G}_2$ and consisting of those lines in $\mathbb{P}^3$ which contain the point $P_o = \operatorname{span} e_4$. Clearly, $S^4 \cap \mathbb{P}^2 = \{x_o\}$. The manifold $pr_2^{-1}(\mathbb{P}^2) \subset \mathbb{P}^3 \times \mathbb{P}^2$ is the blow-up of $\mathbb{P}^3$ at the point P_o and it will be denoted by $\tilde{\mathbb{P}}^3$.

We distinguish the following objects in $\mathbb{P}^3$: the points $P_o = \operatorname{span} e_4$, $P_\infty = \operatorname{span} e_3$, the real lines $L_o = \mathbb{P}((e_3, e_4)) = \overline{P_o P_\infty}$, $L_\infty = \mathbb{P}((e_1, e_2))$, the planes $H_\infty = \mathbb{P}((e_1, e_2, e_3))$, $H_o = \mathbb{P}((e_1, e_3, e_4))$. The real lines L_o, L_∞ considered as points in S^4 will be denoted by x_o, x_∞ (or 0, ∞), respectively.

The restriction $\pi: H_\infty \setminus L_\infty \longrightarrow S^4 \setminus \{x_\infty\}$ induces a complex structure on $S^4 \setminus \{\infty\}$ which we shall regard as the standard one. We choose complex coordinates y, z on $S^4 \setminus \{\infty\} \cong \mathbb{C}^2$ via the identification $\operatorname{span}(ye_1 + ze_2 + e_3) \longmapsto (y, z)$, and we introduce coordinates $\xi = z_1/z_4$, $\eta = z_2/z_4$, $\zeta = z_3/z_4$ on $\mathbb{P}^3 \setminus H_\infty$. Denoting by $\lambda = z_3/z_4$ the coordinate on $L_o \setminus \{P_\infty\}$ we have the transformations $y = (\xi\bar{\zeta} + \bar{\eta})/(1 + \zeta\bar{\zeta})$, $z = (\eta\bar{\zeta} - \bar{\xi})/(1 + \zeta\bar{\zeta})$, $\lambda = \zeta$, and $\xi = \lambda y - \bar{z}$, $\eta = \lambda z + \bar{y}$. So we have the possibility to express functions on $\mathbb{C}^3$ in the coordinates y, z, λ or ξ, η, ζ. Provided the latter ones are used the corresponding function will be underlined.

The fundamental theorem due to Atiyah, Hitchin and Singer [6] relates to every (local) self-dual gauge field a (local) instanton bundle F with $\sigma: \tau^* \bar{F} \longrightarrow F$ being the holomorphic isomorphism inducing a Hermitian structure. We shall restrict our considerations to the framed instanton bundles with a distinguished orthonormal frame over the line L_o. To any such a bundle there corresponds a gauge equivalence class of germs of local transition functions. Every local transition function G is defined on an open set $\mathcal{U} \times \mathcal{V} \subset \mathbb{C}^3$ with $\mathcal{U}$ and $\mathcal{V}$ being neighbourhoods of the origin in $\mathbb{C}^2$ and the unit circle in $\mathbb{C}$, respectively, and it has properties: (i) $G(0,\lambda) = 1$, (ii) $\underline{G}(\xi,\eta,\zeta)$ is holomorphic, (iii) $G(\mathbf{x},-1/\bar{\lambda})^\dagger = G(x,\lambda)$. The space of germs of local transition functions fulfilling (i-iii) will be denoted by $\mathcal{G}_u$. This correspondence can be established even if all reality conditions are omitted. The larger space of germs of local transition functions satisfying only the conditions (i),(ii) will be denoted by $\mathcal{G}$.

The objects we are dealing with are real analytic in some neighbourhood of the origin in $\mathbb{C}^2$ and hence they can be locally extended from $\mathbb{C}^2$ to $\mathbb{C}^4$. In what follows the symbol $\mathbf{x}$ stands for four complex variables $y,z,\bar{y},\bar{z}$ in this order. Let $\mathcal{S}$ denote a subspace of $gl(r,\mathbb{C}[[\mathbf{x}]]) \times gl(r,\mathbb{C}[[x,\lambda]]) \times gl(r,\mathbb{C}[[x,\lambda^{-1}]])$ consisting of those matrices $(J,W,\hat{W})$ of formal power series which satisfy

$$W(\mathbf{x},0) = \hat{W}(\mathbf{x},\infty) = 1 \quad , \tag{2.1}$$

$$J(0) = W(0,\lambda) = \hat{W}(0,\lambda) = 1 \quad , \tag{2.2}$$

and solve

$$\partial_z W - \lambda J \partial_{\bar{y}}(J^{-1}W) = \partial_y W + \lambda J \partial_{\bar{z}}(J^{-1}W) = 0 \quad , \tag{2.3}$$

$$\partial_{\bar{y}}\hat{W} - \lambda^{-1}J^{-1}\partial_z(J\hat{W}) = \partial_{\bar{z}}\hat{W} + \lambda^{-1}J^{-1}\partial_y(J\hat{W}) = 0 \quad . \tag{2.4}$$

We shall write $W(\mathbf{x},\lambda) = 1 + \sum_{j=1}^{\infty} W_j(x)\lambda^j$, $\hat{W}(x,\lambda) = 1 + \sum_{j=1}^{\infty} \hat{W}_j(\mathbf{x})\lambda^{-j}$, where $W_j(x)$, $\hat{W}_j(x) \in gl(r,\mathbb{C}[[\mathbf{x}]])$ and $W_j(0) = \hat{W}_j(0) = 0$. An involution denoted again by σ acts on $\mathcal{S}$. It interchanges $J(\mathbf{x})$ with $J(\mathbf{x})^\dagger$ and $\hat{W}(\mathbf{x},\lambda)$ with $(W(\mathbf{x},-1/\bar{\lambda})^\dagger)^{-1}$. The σ-invariant subspace consisting of $(J,W,\hat{W})$ fulfilling

$$J(\mathbf{x})^\dagger = J(\mathbf{x}) \ , \quad \hat{W}(\mathbf{x},\lambda)^{-1} = W(\mathbf{x},-1/\bar{\lambda})^\dagger \tag{2.5}$$

will be denoted by $\mathcal{S}_u$. The gauge transformations

$$\begin{aligned} J(\mathbf{x})^{-1} &\longmapsto \Gamma(\mathbf{x},0)^\dagger J(\mathbf{x})^{-1} \Gamma(\mathbf{x},0) \quad , \\ W(\mathbf{x},\lambda) &\longmapsto \Gamma(\mathbf{x},0)^{-1} W(\mathbf{x},\lambda) \Gamma(\mathbf{x},\lambda) \end{aligned} \tag{2.6}$$

make sense on $\mathcal{S}_u$ provided $\Gamma \in gl(r,\mathbb{C}[[\mathbf{x},\lambda]])$ satisfies $\Gamma(0,\lambda) = 1$ and

$$(\lambda\partial_{\bar{y}} - \partial_z)\Gamma = (\lambda\partial_{\bar{z}} + \partial_y)\Gamma = 0 \quad . \tag{2.7}$$

For $G \in \mathcal{G}$ we choose the Birkhoff decomposition in the form

$$G(x,\lambda) = (J(x)\hat{W}(x,\lambda))^{-1}W(x,\lambda) \quad , \tag{2.8}$$

where $W(x,\lambda)$ and $\hat{W}(x,\lambda)$ are holomorphic in λ on neighbourhoods of the discs $\{|\lambda|\le 1\}$ and $\{|\lambda^{-1}|\le 1\}$, respectively, provided x is close enough to 0, and they are normed by $W(x,0) = \hat{W}(x,\infty) = 1$. It is well known [7,8] that in this way we get embeddings $\mathcal{G}\subset\mathcal{S}$ and $\mathcal{G}_u\subset\mathcal{S}_u$.

3. THE CANONICAL INITIAL CONDITION

We can exclude J from (2.3) : $J\partial_{\bar{y}}J^{-1} = \partial_z W_1$, $J\partial_{\bar{z}}J^{-1} = -\partial_y W_1$. Takasaki's approach provides a method how to solve these equations in the realm of formal power series together with a given initial condition

$$W(y,z,0,0,\lambda) = W^{(0)}(y,z,\lambda) \quad . \tag{3.1}$$

In accordance with (2.1),(2.2) the initial condition $W^{(0)} \in gl(r,\mathbb{C}[[y,z,\lambda]])$ is required to fulfil $W^{(0)}(y,z,0) = W^{(0)}(0,0,\lambda)=1$. The initial value problem has the unique solution $W(x,\lambda)$ unambiguously determined by the condition

$$W(x,\lambda)W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)^{-1} \in gl(r,\mathbb{C}[[x,\lambda^{-1}]]) \quad . \tag{3.2}$$

We shall complete this result.

Proposition 3.1. It holds

$$W(x,\lambda)W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)^{-1} = J(x)\hat{W}(x,\lambda) \quad ,$$

where $\hat{W}$ is normed by $\hat{W}(x,\infty) = 1$ and $(J,W,\hat{W})$ solve (2.3),(2.4) and (2.2), i.e., $(J,W,\hat{W})\in\mathcal{S}$.

Proof. In fact we shall prove also the Takasaki's result in an alternative way. Put $H(x,\lambda) = W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)^{-1}\in gl(r,\mathbb{C}[[x,\lambda,\lambda^{-1}]])$. Then $H(0,\lambda) = 1$ and the m-th homogeneous term H_m, in the variables x does not contain powers of λ lower than λ^{-m} . Consequently $H(x,\lambda)$ is well defined. It also follows that for any $R \in gl(r,\mathbb{C}[[x]])$ there exists the unique solution $X \in gl(r,\mathbb{C}[[x,\lambda]])$ to the following problem: $X(x,\lambda)H(x,\lambda)\in gl(r,\mathbb{C}[[x,\lambda^{-1}]])$, $X(0,\lambda) = R(0)$ and $X(x,0) = R(x)$. Really, write X in the form $X(x,\lambda) = R(0) + \sum_{k=1}^{\infty} X_k(x,\lambda)$ with the terms X_k, being k-homogeneous in the variables x , to get the relations $X_k + \sum_{j=1}^{k} H_j X_{k-j} \in gl(r,\mathbb{C}[[x,\lambda^{-1}]])$, $k\ge 1$, which enable to compute recursively and together with the condition $X(x,0) = R(x)$ unambiguously all terms X_k . Hence the condition (3.2) together with $W(0,\lambda) = W(x,0) = 1$ has the unique solution W and it can be easily seen that this W also fulfils the initial condition (3.1). Now decompose

$W(x,\lambda)H(x,\lambda) = J(x)\hat{W}(x,\lambda)$ with the given normalization. Clearly, $(\partial_z-\lambda\partial_{\bar y})H = 0$ and hence $(\partial_z-\lambda\partial_{\bar y})(J^{-1}(x)W(x,\lambda))\,H(x,\lambda) = (\partial_z-\lambda\partial_{\bar y})\hat{W}(x,\lambda) \in gl(r,\mathbb{C}[[x,\lambda^{-1}]])$. At the same time it holds $(\partial_z-\lambda\partial_{\bar y})(J^{-1}(x)W(x,\lambda))_{x=0} = \partial_z(J^{-1}(x)W(x,0))_{x=0} = \partial_z J^{-1}(0)$ and $(\partial_z-\lambda\partial_{\bar y})(J^{-1}(x)W(x,\lambda))_{\lambda=0} = \partial_z J^{-1}(x)$. But $(\partial_z J^{-1}(x))\,W(x,\lambda)$ fulfils the same relations and so according to the above observation we have the equality $(\partial_z-\lambda\partial_{\bar y})(J^{-1}(x)W(x,\lambda)) = (\partial_z J^{-1}(x))\,W(x,\lambda)$. Analogously we get $(\partial_y+\lambda\partial_{\bar z})(J^{-1}(x)W(x,\lambda)) = (\partial_y J^{-1}(x))\,W(x,\lambda)$, i.e., the equations (2.3) are satisfied. Further, $(\partial_z-\lambda\partial_{\bar y})\hat{W}(x,\lambda) = (\partial_z J^{-1}(x))\,W(x,\lambda)H(x,\lambda) = (\partial_z J^{-1}(x))\,J(x)\hat{W}(x,\lambda)$, and analogously $(\partial_y+\lambda\partial_{\bar z})\hat{W}(x,\lambda) = (\partial_y J^{-1}(x))\,J(x)\hat{W}(x,\lambda)$. Hence the equations (2.4) are satisfied as well. The rest of the proof is evident.

As a rule the gauge equivalence in the inverse scattering approach is usually quoted but not systematically developed. In the reminder of this section we shall try to show that such a more detailed discussion leads to some rather useful consequences. The condition (2.7) on Γ means that $\Gamma(x,\lambda) = \underline{\Gamma}(\lambda y-\bar z,\lambda z+\bar y,\lambda)$, where $\underline{\Gamma}(\xi,\eta,\zeta) \in gl(r,\mathbb{C}[[\xi,\eta,\zeta]])$. Hence $\Gamma^{(0)}(y,z,\lambda) := \Gamma(y,z,0,0,\lambda) = \underline{\Gamma}(\lambda y,\lambda z,\lambda) = 1 + \sum_{j=1}^{\infty} \Gamma_j^{(0)}(y,z)\,\lambda^j$, where $\Gamma_j^{(0)}(y,z)$ is a polynomial with the zero absolute term and of degree at most equal to j. Clearly, knowing $\Gamma^{(0)}$ we are able to reconstruct $\underline{\Gamma}$ and hence Γ as well. The gauge transformation of the initial condition takes the form $W^{(0)} \longmapsto W^{(0)}\Gamma^{(0)}$. It follows that for each $(J,W,\hat{W}) \in \mathcal{S}_u$ there exists the unique gauge transformation Γ_c such that $W^{(0)}$ having been transformed satisfies $W_j^{(0)}(y,z) \in gl(r,(y,z)^{j+1})$, where $W_j^{(0)}(y,z) := W_j(y,z,0,0)$ and $(y,z)^{j+1}$ is the (1+j)-th power of the ideal $(y,z) \subset \mathbb{C}[[y,z]]$ generated by y,z . This condition eliminates gauge freedom. The unique gauge transformation Γ_c will be called <u>canonical</u> and we shall derive an explicit formula for it.

<u>Lemma 3.2</u>. Assume that $(J,W,\hat{W}) \in \mathcal{S}$ and $J(y,z,0,0) = \hat{W}(y,z,0,0,\lambda) = 1$. Then

(i) $\hat{W}_k(x) \in gl(r,(\bar y,\bar z)^{k+1})$, $k \geq 1$, where $(\bar y,\bar z)$ is now an ideal in $\mathbb{C}[[x]]$, and

(ii) $W(x,\lambda)\,W^{(0)}(y-\lambda^{-1}\bar z,z+\lambda^{-1}\bar y,\lambda)^{-1} = J(x)\hat{W}(x,\lambda)$. (3.3)

<u>Proof</u>. (i) We have to show that $\partial_{\bar y}^{\,i}\partial_{\bar z}^{\,j}\,\hat{W}_k(y,z,0,0) = 0$ for $0 \leq i+j \leq k$. According to the assumption the assertion is valid for $i=j=0$. The equations (2.4) are equivalent to

$$\partial_{\bar y}\hat{W}_k = \partial_z\hat{W}_{k-1} + (J^{-1}\partial_z J)\hat{W}_{k-1} \quad , \quad \partial_{\bar z}\hat{W}_k = -\partial_y\hat{W}_{k-1} - (J^{-1}\partial_y J)\hat{W}_{k-1} \quad , \quad k \geq 1.$$

Put $\bar{y}=\bar{z}=0$ in these relations to verify the assertion for $k = 1$. Further we proceed by induction in k. It suffices to differentiate these equations by $\partial_{\bar{y}}^{i-1}\partial_{\bar{z}}^{j}$ or by $\partial_{\bar{y}}^{i}\partial_{\bar{z}}^{j-1}$, respectively.
(ii) According to Proposition 3.1 the left-hand side in (3.3) equals to $J'(x)\hat{W}'(x,\lambda)$ with $(J',W,\hat{W}') \in \mathcal{S}$. It is sufficient to show that the initial condition on $J,\hat{W}$ determines them unambiguously. But this is an immediate consequence of the following two easily verifiable assertions: Let (J,W) and $(J',\hat{W})$ solve (2.3) and satisfy the corresponding boundary conditions in (2.1),(2.2). Then $J' = JX$ with $X \in gl(r,\mathbb{C}[[y,z]])$, $X(0) = 1$. Let $(J,\hat{W})$, $(J,\hat{W}')$ solve (2.4) and satisfy the corresponding boundary conditions in (2.1),(2.2). Then $\hat{W}'(x,\lambda) = Y(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)\hat{W}(x,\lambda)$ with $Y(\xi,\eta,\zeta) \in gl(r,\mathbb{C}[[\xi,\eta,\zeta^{-1}]])$ and $Y(0,0,\zeta) = 1$.

<u>Theorem 3.3</u>. Let $(J,W,\hat{W}) \in \mathcal{S}_u$ and $J,\hat{W}$ fulfil

$$J(y,z,0,0) = \hat{W}(y,z,0,0,\lambda) = 1 \quad . \tag{3.4}$$

Then $W^{(0)}$ satisfies

$$W_j^{(0)}(y,z) \in gl(r,(y,z)^{j+1}) \quad , \; j \geq 1 \; , \tag{3.5}$$

and

$$W^{(0)}(\bar{\lambda}\bar{z},-\bar{\lambda}\bar{y},-1/\bar{\lambda})^{+} = W^{(0)}(y,z,\lambda) \quad . \tag{3.6}$$

On the contrary let $W^{(0)}$ satisfy (3.5),(3.6). Then there exists the unique solution $(J,W,\hat{W}) \in \mathcal{S}_u$ with $W^{(0)}$ being the initial condition for W and, moreover, this solution fulfils (3.4).

<u>Remark</u>. $W^{(0)}(\bar{\lambda}\bar{z},-\bar{\lambda}\bar{y},-1/\bar{\lambda})^{+}$ makes sense owing to (3.5). (3.6) is the anounced reality condition.

<u>Proof</u>. ($\Longrightarrow$) Validity of (3.5) follows from Lemma 3.2 ad(i), and from the equality $W^{(0)}(y,z,\lambda)^{-1} = \hat{W}(0,0,\bar{y},\bar{z},-1/\bar{\lambda})^{+}$. By the assumption the equality (3.3) holds. Set $y=z=0$ in it and make use of (3.4),(2.5) to get $W^{(0)}(-\lambda^{-1}\bar{z},\lambda^{-1}\bar{y},\lambda)^{-1} = \hat{W}(0,0,\bar{y},\bar{z},\lambda)$. Now it is sufficient to replace λ by $-1/\bar{\lambda}$ and to perform Hermitian conjugation and inversion of both sides in the last equality. ($\Longleftarrow$) We relate to $W^{(0)}$ a solution $(J,W,\hat{W}) \in \mathcal{S}$ according to Proposition 3.1. From (3.6) it follows $W^{(0)}(y+\bar{\lambda}\bar{z},z-\bar{\lambda}\bar{y},-1/\bar{\lambda})^{+} = W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)$. The k-th homogeneous term of $W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)$ in the variables x contains powers of λ not lower than $(-k+1)$ and not greater than $(k-1)$. Consequently we are allowed to multiply the equation (3.2) from the right by this expression. So we have $J(x)^{-1}W(x,\lambda) = \hat{W}(x,\lambda)W^{(0)}(y-\lambda^{-1}\bar{z},z+\lambda^{-1}\bar{y},\lambda)$. Replace λ by $-1/\bar{\lambda}$ and conjugate and invert both sides of this equality to find that $(\hat{W}(x,-1/\bar{\lambda})^{+})^{-1}$ is again a solution with the same initial condition $W^{(0)}$. By uniqueness we have

$(\hat{W}(x,-1/\bar{\lambda})^+)^{-1} = W(x,\lambda)$ and $J(x)^+(W(x,-1/\bar{\lambda})^+)^{-1} = J(x)\hat{W}(x,\lambda)$. Hence $(J,W,\hat{W}) \in \mathcal{S}_u$. Validity of (3.4) follows immediately from (3.2) and uniqueness is guaranteed by Lemma 3.2 ad(ii).

Definition 3.4. A solution $(J,W,\hat{W}) \in \mathcal{S}_u$ will be called canonical provided it fulfils (3.4). The subspace of canonical solutions will be denoted by $\mathcal{S}_c$. An initial condition $W^{(0)}$ will be called canonical provided it fulfils (3.5),(3.6). The space of canonical initial conditions will be denoted by $\mathcal{W}$.

Hence $W^{(0)} \in \mathcal{W}$ iff it holds

$$W^{(0)}(y,z,\lambda) = 1 + \sum_{k=1}^{\infty} \sum_{n=k+1}^{\infty} \sum_{j=0}^{n} W^{(0)}_{njk}\, y^j z^{n-j} \lambda^k ,$$

where $W^{(0)}_{njk} \in gl(r,\mathbb{C})$, $W^{(0)}_{njk} = (-1)^{n+j+k}\, W^{(0)+}_{n,n-j,n-k}$.

Suppose $(J,W,\hat{W}) \in \mathcal{S}_u$. We introduce a gauge transformation $\Gamma_c(x,\lambda) = \underline{\Gamma}_c(\lambda y-\bar{z},\lambda z+\bar{y},\lambda)$, where

$$\underline{\Gamma}_c(\xi,\eta,\zeta) = W(0,0,\eta,-\xi,\zeta)^{-1} J(0,0,\eta,-\xi) . \tag{3.7}$$

Having performed this gauge transformation we get another solution $(J_c,W_c,\hat{W}_c) \in \mathcal{S}_u$. From (2.6) it follows that $J_c, \hat{W}_c$ satisfy (3.4) and according to Theorem 3.3 the initial condition $W^{(0)}_c$ fulfils (3.5). Hence the relation (3.7) yields the announced explicit form of the canonical gauge transformation.

We have just constructed a projection

$$pr_c : \mathcal{S}_u \longrightarrow \mathcal{S}_c : (J,W,\hat{W}) \longmapsto (J_c,W_c,\hat{W}_c) ,$$

which, moreover induces a one-to-one mapping of the quotient $\mathcal{S}_u$/gauge transformations onto $\mathcal{S}_c$. According to Theorem 3.3 the mapping $\mathcal{S}_c \longrightarrow \mathcal{W} : (J,W,\hat{W}) \longmapsto W^{(0)}$ is one-to-one and so we have

$$\mathcal{S}_u/\text{gauge transformations} \cong \mathcal{S}_c \cong \mathcal{W} . \tag{3.8}$$

In other words, gauge equivalence classes of local self-dual solutions are parametrized by the points from the space $\mathcal{W}$.

Denote by $\mathcal{G}_c \subset \mathcal{G}_u$ the subspace of germs of those transition functions G such that $G(y,z,0,0,\lambda) \in \mathcal{W}$. By restriction we obtain the projection

$pr_c : \mathcal{G}_u \longrightarrow \mathcal{G}_c : G = (J\hat{W})^{-1} W \longmapsto G_c$, where

$$\underline{G}_c(\xi,\eta,\zeta) = W^{(0)}(\xi/\zeta,\eta/\zeta,\zeta)\big(J(\bar{\eta},-\bar{\xi},0,0)\hat{W}(\bar{\eta},-\bar{\xi},0,0,-1/\bar{\zeta})\big)^+ .$$

Again this projection induces a one-to-one mapping

$$\mathcal{G}_u/\text{gauge transformations} \cong \mathcal{G}_c . \tag{3.9}$$

We have a simple relation between the canonical transition function G_c and the canonical initial condition $W_c^{(0)}$:

$$\underline{G}_c(\xi,\eta,\zeta) = W_c^{(0)}(\xi/\zeta,\eta/\zeta,\zeta) \quad , \text{ i.e.,}$$
$$G_c(x,\lambda) = W_c^{(0)}(y-\lambda^{-1}\bar{z}, z+\lambda^{-1}\bar{y},\lambda) \quad , \tag{3.10}$$

and vice versa

$$W_c^{(0)}(y,z,\lambda) = \underline{G}_c(\lambda y,\lambda z,\lambda) = G_c(y,z,0,0,\lambda) \quad .$$

Proposition 3.5. The isomorphism class of a global framed instanton bundle F over $\mathbb{P}^3$ is unambiguously determined by its local restriction to $\pi^{-1}(\mathcal{U})$ with $\mathcal{U}$ being any neighbourhood of x_0 .

Proof. Gram matrix of the Hermitian form expressed in a holomorphic frame is real analytic. Owing to this fact the distinguished orthonormal frame over L_0 can be extended as an orthonormal real analytic trivialization $\{t_1,\dots,t_r\}$ of F over $\mathbb{P}^3 \setminus L_\infty$ in the following way. The sections t_j are defined on $H_\infty \setminus L_\infty \cong \mathbb{C}^2$ as horizontal lifts over the segments $\overline{P_\infty Q}$, and then they are extended as global holomorphic sections over each real line. Using this trivialization we get a connection A on $\mathbb{C}^2 \cong \mathbb{R}^4$ with the self-dual curvature and with the finite topological charge equal to $c_2(F)$. At the same time A depends only on the isomorphism class of F and is real analytic and hence it is unambiguously determined by its germ at the origin. Using now Uhlenbeck Theorem [9] to remove the singularity at ∞ and applying the Penrose transformation we conclude that A determines F uniquely up to isomorphism.

It follows that having in mind (3.8),(3.9) we can relate to every isomorphism class the unique point from the space $\mathcal{W}$. In this way we get an embedding of the framed instanton moduli spaces $M(r,c_2)$ into $\mathcal{W}$. Explicit expressions will be given in Sec.5 .

4. THE GEOMETRIC INTERPRETATION

Despite of the local way of its definition the canonical initial condition will be shown to have a clear geometric interpretation in the twistor framework. The global embedding of S^4 into $\mathbb{G}_2$ and the pull-back of the instanton bundle $pr_1^* F$ on $\mathbb{F}_{1,2}$ corresponds to the local analytic extension from $\mathbb{C}^2$ to $\mathbb{C}^4$. The initial condition, i.e., the local restriction to the 2-dimensional subspace $\bar{y} = \bar{z} = 0$, has a counterpart on the global level in the restrictions from $\mathbb{G}_2$ to $\mathbb{P}^2$ and from $\mathbb{F}_{1,2}$ to $\hat{\mathbb{P}}^3$. Really, in the local coordinates $y,z,\bar{y},\bar{z}$ on $\mathbb{G}_2$ at the point x_0 introduced via the mapping $(y,z,\bar{y},\bar{z}) \longmapsto \mathbb{P}((ye_1+ze_2+e_3,-\bar{z}e_1+\bar{y}e_2+e_4))$, it holds : S^4 is locally determined by the equations $\bar{y}$ = c.c. y , $\bar{z}$ = c.c. z , and $\mathbb{P}^2$ is locally determined by the equations $\bar{y} = \bar{z} = 0$.

The following construction and its consequences are formulated in a more general setting for the n-dimensional projective spaces $\mathbb{P}^n$, $n \geq 2$, with a fixed line L_o and for the framed holomorphic rank-r vector bundles F over $\mathbb{P}^n$, $r \geq 2$, with a distinguished holomorphic trivialization on L_o. For such vector bundles the first Chern class $c_1(F)$ vanishes and according to Grothendieck Theorem the vector bundle F decomposes over every line L as $F|L = \mathcal{O}(i_1) \oplus \ldots \oplus \mathcal{O}(i_r)$, $i_1 \geq \ldots \geq i_r$, $i_1 + \ldots + i_r = 0$. Hence $i_1 \geq 0$ and $i_1 = 0$ iff F is holomorphically trivial on L. The set S_F of jumping lines, i.e., consisting of lines over which F is not holomorphically trivial, is a proper closed analytic set in $\mathbb{G}_2$ (cf. [10],Chp.I). Fix two different points $P_o, P_\infty \in L_o$ and denote by S_o, S_∞ the sets of jumping lines passing through P_o and P_∞, respectively. Let $Sing(P_o) \subset \mathbb{P}^n$ be the union of all lines belonging to S_o, analogously define $Sing(P_\infty)$. Again the sets S_o, S_∞ are proper closed analytic sets in the (n-1)-dimensional projective spaces consisting of lines in $\mathbb{P}^n$ containing P_o or P_∞, respectively. Consequently $Sing(P_o)$ and $Sing(P_\infty)$ are proper closed analytic subsets in $\mathbb{P}^n$.

More information provides the following theorem due to Barth generalized to higher orders. Denote by $\mathcal{F}$ the locally free rank-r sheaf of germs of holomorphic sections in F. We redenote the projections $p = pr_1 : \mathbb{F}_{1,2} \longrightarrow \mathbb{P}^n$ and $q = pr_2 : \mathbb{F}_{1,2} \longrightarrow \mathbb{G}_2$.

Theorem 4.1. The set S_F of jumping lines is an analytic subset in the Grassmann manifold $\mathbb{G}_2$ of codimension 1 everywhere. The sheaf $\mathcal{E} = R^1 q_* p^*(\mathcal{F}(-1))$ determines in $\mathbb{G}_2$ a divisor D_F of degree $c_2(F)$ and such that $S_F = \operatorname{supp} \mathcal{E} = \operatorname{supp} D_F$.

Proof. In [10],Chp.II there is given a proof for the case $r = \operatorname{rank} F = 2$. The main part of the proof may be reproduced almost verbatim also in the general case. We shall not do this and notice only the last part in which the degree of the divisor is computed. It is sufficient to verify the equality $\deg D_F = c_2(F)$ only for the dimension $n = 2$ and that will be assumed up to the end of the proof. In this case there exists a resolution

$$0 \longrightarrow \bigoplus_{i=1}^{s} \mathcal{O}(k_i) \longrightarrow \bigoplus_{j=1}^{t} \mathcal{O}(m_j) \longrightarrow \mathcal{F}(-1) \longrightarrow 0, \text{ with } k_i, m_j < 0 .$$

The line bundle corresponding to the divisor D_F is $[D_F] = \det E_2 \otimes \det E_1^*$, where $E_1 = \oplus R^1 q_* p^* \mathcal{O}(k_i)$, $E_2 = \oplus R^1 q_* p^* \mathcal{O}(m_j)$. Finally, $c_1(R^1 q_* p^* \mathcal{O}(k)) = -k(k+1)/2$. Now we can compute $\deg D_F$ using the Whitney formula for this resolution:

$-r = c_1(F(-1)) = \sum m_j - \sum k_i$,

$c_2(F(-1)) = \frac{1}{2}r(r-1) + c_2(F) = \sum_{i<j} m_i m_j - \sum_{i<j} k_i k_j + r\sum k_i$

$\deg D_F = c_1([D_F]) = c_1(E_2) - c_1(E_1) = \frac{1}{2}\sum(k_i^2+k_i) - \frac{1}{2}\sum(m_j^2+m_j) =$

$= c_2(F(-1)) - \frac{1}{2}r(r-1) = c_2(F)$.

Corollary 4.2. Let $\widetilde{\mathbb{P}}^n \subset \mathbb{P}^n \times \mathbb{P}^{n-1}$ be the blow-up of $\mathbb{P}^n$ at the point P_o . $\mathbb{P}^{n-1}$ is considered as a submanifold in $\mathbb{G}_2$. Then $S_o = S_F \cap \mathbb{P}^{n-1}$ is an analytic subset in $\mathbb{P}^{n-1}$ of codimension 1 everywhere and the sheaf $\mathcal{E}_o = R^1 pr_{2*} pr_1^*(\mathcal{F}(-1))$ determines in $\mathbb{P}^{n-1}$ a divisor D_o of degree $c_2(F)$ and such that $S_o = \operatorname{supp} \mathcal{E}_o =$ $\operatorname{supp} D_o$.

Proof. The proof follows from the standard application of the base--change theorem (cf.[11],§9).

Owing to the distinguished frame on L_o we have a fixed basis in the fiber over P_∞ . Extend this basis as a holomorphic trivialization over each line containing P_∞ and not belonging to S_∞ . We get a holomorphic frame $\{s_1,\dots,s_r\}$ over the open set $\mathcal{U}_o =$ $\mathbb{P}^n \setminus \operatorname{Sing}(P_\infty) \ni P_o$. Construct analogously the holomorphic frame $\{\hat{s}_1,\dots,\hat{s}_r\}$ over $\mathcal{U}_\infty = \mathbb{P}^n \setminus \operatorname{Sing}(P_o) \ni P_\infty$. The corresponding transition function defined on $\mathcal{U}_o \cap \mathcal{U}_\infty = \mathbb{P}^n \setminus (\operatorname{Sing}(P_o) \cup \operatorname{Sing}(P_\infty))$ will be denoted by $G^F = (G^F_{jk})$, $s_k = \sum \hat{s}_j G^F_{jk}$.

Owing to Chow Theorem all the sets S_o, S_∞, $\operatorname{Sing}(P_o)$, $\operatorname{Sing}(P_\infty)$ are projective varieties in the corresponding projective spaces, and owing to a generalization of the same theorem, F is an algebraic vector bundle on $\mathbb{P}^n$ (cf.[12],§1.3). According to the same principles, G^F is a matrix of rational functions on $\mathbb{P}^n$. Corollary 4.2 suggests that in the homogeneous coordinates G^F has the form $G^F(z) = 1 + S(z)/(p_o(z)p_\infty(z))$, where $S(z)$ is a matrix of homogeneous polynomials with degrees equal to $2c$ ($c = c_2(F)$) and such that $S(z) \equiv 0$ on the line L_o , p_o, p_∞ are homogeneous polynomials with degrees equal to c and such that the projective set $p_o(z) = 0$ (resp. $p_\infty(z) = 0$) coincides with $\operatorname{Sing}(P_o)$ (resp. $\operatorname{Sing}(P_\infty)$) . By the construction G^F depends only on the isomorphism class of the framed vector bundle F .

Theorem 4.3. The isomorphism class of a framed holomorphic vector bundle F on $\mathbb{P}^n$ is unambiguously determined by the matrix function G^F .

Proof. G^F considered as transition function determines the framed vector bundle F over the open set $\mathcal{U}_o \cup \mathcal{U}_\infty =$ $\mathbb{P}^n \setminus (\operatorname{Sing}(P_o) \cap \operatorname{Sing}(P_\infty))$, up to isomorphism. The analytic sets $\operatorname{Sing}(P_o)$, $\operatorname{Sing}(P_\infty)$ have dimensions equal at most to $(n-1)$ and their intersection has dimension equal at most to $(n-2)$. Really, provided dimension of the intersection equals to $(n-1)$, the analytic sets $\mathrm{Sing}(P_0)$, $\mathrm{Sing}(P_\infty)$, $\mathrm{Sing}(P_0)\cap\mathrm{Sing}(P_\infty)$ are of the same dimension and have at least one common irreducible component T again of dimension $(n-1)$ (cf.[13],§1.3). The irreducible components of the analytic set $\mathrm{Sing}(P_0)$ are in one-to-one correspondence with the irreducible components of the analytic set S_0 and each of them contains the point P_0. Consequently $P_0 \in T \subset \mathrm{Sing}(P_\infty)$ and we get a contradiction. Let $\iota: \mathcal{U}_0\cup\mathcal{U}_\infty \hookrightarrow \mathbb{P}^n$ be the embedding. Owing to a consequence of Hartogs Theorem which guarantees removability of singularities of a complex analytic function provided the singular points are contained in an analytic set of codimension at least 2, we have a natural isomorphism $\iota_*\iota^*\mathcal{F} \cong \mathcal{F}$. Hence F is determined by its restriction $F|(\mathcal{U}_0\cup\mathcal{U}_\infty)$ in the unique way.

<u>Proposition 4.4</u>. $\det G^F = 1$.

<u>Proof</u>. The line bundle $\det F$ on $\mathbb{P}^n$ is holomorphically trivial. The choice of a frame of F over L_0 induces a holomorphic frame of $\det F$ over L_0 which extends to a holomorphic trivialization over $\mathbb{P}^n$ in the unique way. From the construction of G^F as a transition function it follows immediately that $\det G^F$ is a transition function of $\det F$ identically equal to 1 on $\mathcal{U}_0\cap\mathcal{U}_\infty$ and hence everywhere on $\mathbb{P}^n$.

Let us now specify the construction for the framed instanton bundles on $\mathbb{P}^3$. In this case L_0 is a real line, $\tau(P_0) = P_\infty$ and the distinguished frame is orthonormal. Moreover, $\tau(\mathrm{Sing}(P_0)) = \mathrm{Sing}(P_\infty)$ and the frame $\{\hat{s}_1,\dots,\hat{s}_r\}$ is related to the dual of the frame $\{s_1,\dots,s_r\}$ by the isomorphism σ. It holds

$$G^F(\tau(Q)) = G^F(Q)^+ \quad \text{on } \mathcal{U}_0\cap\mathcal{U}_\infty , \tag{4.1}$$

and hence the germ of G^F belongs to $\mathcal{G}_{\mathfrak{u}}$.

Denote by $\tilde{P}_0 = (P_0, L_0)$ the point from the exceptional divisor in the blow-up $\tilde{\mathbb{P}}^3$. The pull-back of the matrix function $\mathrm{pr}_1^* G^F$ is holomorphic at the point $\tilde{P}_0$. Identify $\mathbb{P}^2$ with the plane $H_\infty \subset \mathbb{P}^3$ and introduce local coordinates on $\tilde{\mathbb{P}}^3$ via the mapping $(y,z,\lambda) \longmapsto \mathrm{span}(\lambda y e_1+\lambda z e_2+\lambda e_3+e_4)$, $\mathrm{span}(y e_1+z e_2+e_3)$; the values in $\tilde{P}_0$ are $y = z = \lambda = 0$. Denote $\mathrm{pr}_1^* G^F$ expressed in these local coordinates by $W_c^{(0)}(y,z,\lambda)$. Clearly, $W_c^{(0)}(y,z,0) = W_c^{(0)}(0,0,\lambda) = 1$. From (4.1) it follows that equation $W_c^{(0)}(\bar\lambda\bar z,-\bar\lambda\bar y,-1/\bar\lambda)^+ = W_c^{(0)}(y,z,\lambda)$ holds on an open set $\mathcal{U}\times\mathcal{V}$, where $\mathcal{U}$ and $\mathcal{V}$ are neighbourhoods of the origin in $\mathbb{C}^2$ and of the unit circle in $\mathbb{C}$, respectively. Looking at the Lorain λ-expansion in this equality we find the condition (3.5) to be valid as well as the reality condition (3.6). Hence $W_c^{(0)}$ is indeed the canonical initial condition corresponding to the framed instanton bundle F . G^F expressed in the local real analytic coordinates $(x,\lambda) \equiv (y,z,\bar{y},\bar{z},\lambda)$ satisfies $G^F(x,\lambda) = W_c^{(0)}(y-\lambda^{-1}\bar{z}, z+\lambda^{-1}\bar{y},\lambda)$. Comparing this identity with (3.10) we conclude that G^F coincides with the canonical transition function G_c .

Note that the same construction of the canonical initial condition is applicable also to the local case.

5. EXPLICIT EXPRESSIONS

For a quadruple $(\alpha_1, \alpha_2, a, b) \in \mathbb{C}^{c,c}\times\mathbb{C}^{c,c}\times\mathbb{C}^{r,c}\times\mathbb{C}^{c,r}$ and $\mathfrak{z} = (z_j) \in \mathbb{C}^4$ set

$$R(\mathfrak{z}) = \begin{pmatrix} \alpha_1 z_1+\alpha_2^{+}z_2-z_3 \\ \alpha_2 z_1-\alpha_1^{+}z_2+z_4 \\ az_1-b^{+}z_2 \end{pmatrix} , \quad \Delta(\mathfrak{z}) = R(\mathfrak{z})^{+}R(\mathfrak{z}) .$$

The moduli space $M(r,c_2)$ of framed instanton bundles on $\mathbb{P}^3$ is the quotient of the set of matrices (α_1,α_2,a,b) satisfying

(i) $R(\mathfrak{z})$ is of rank $c = c_2$ for all $\mathfrak{z} \neq 0$,

(ii) $R(\tau\mathfrak{z})^{+}R(\mathfrak{z}) = 0$ for all $\mathfrak{z}$, or equivalently,

$$\Delta(\tau\mathfrak{z}) = \Delta(\mathfrak{z}) \quad \text{for all } \mathfrak{z} , \tag{5.1}$$

by the action of $U(c)$: $R(\mathfrak{z}) \longmapsto \mathrm{diag}(g,g,1)\,R(\mathfrak{z})\,g^{-1}$, $g \in U(c)$ (cf.[1]). Let $Z \in \mathbb{P}^3$ be a point with homogeneous coordinates $\mathfrak{z} = (z_1,\ldots,z_4)$. Then the fibre F_Z of the instanton bundle in the ADHM construction is given by (cf.[14,15]) $F_Z = \ker R(\tau\mathfrak{z})^{+}/\mathrm{im}\,R(\mathfrak{z})$. The Hermitian product in F_Z is given by $([s],[t]) = s^{+}(1 - R(\mathfrak{z})\,\Delta(\mathfrak{z})^{-1}R(\mathfrak{z})^{+})t$.

<u>Lemma 5.1</u>. Let L be a line in $\mathbb{P}^3$ and $Y_o, Z_o \in L$ be two different points with homogeneous coordinates $\mathfrak{y}_o$, $\mathfrak{z}_o$, respectively. Then $F|L$ is holomorphically trivial if and only if the matrix $R(\tau\mathfrak{y}_o)^{+}R(\mathfrak{z}_o)$ is invertible.

<u>Proof</u>. ($\Rightarrow$) Fix a basis in the fibre over Z_o via the choice of a $(2c+r)\times r$ matrix N satisfying: $R(\tau\mathfrak{z}_o)^{+}N = 0$, $\mathrm{rank}(N,R(\mathfrak{z}_o)) = r+c$. Then the matrix function $S(Y) = [1 - R(\mathfrak{z}_o)(R(\tau\mathfrak{y}_o)^{+}R(\mathfrak{z}_o))^{-1}R(\tau\mathfrak{y}_o)^{+}]N$ modulo $\mathrm{im}\,R(\mathfrak{y})$, $Y = \mathrm{span}\,\mathfrak{y} \in L$, is in fact a holomorphic trivialization of F over L .

($\Leftarrow$) Suppose $R(\tau\mathfrak{y}_o)^{+}R(\mathfrak{z}_o)$ is singular. We shall show that $F|L$ has a nontrivial section with at least one zero. Denote $K = \ker R(\tau\mathfrak{y}_o)^{+}R(\mathfrak{z}_o) \subset \mathbb{C}^c$. Let Λ be a projector onto $R(\mathfrak{z}_o)K \subset \mathbb{C}^{2c+r}$ according to any direct summand. Since the restriction

$R(\mathfrak{z}_0): K \longrightarrow R(\mathfrak{z}_0)K$ is an isomorphism we can define $A = R(\mathfrak{z}_0)^{-1}\circ\Lambda\circ R(y_0) : K \longrightarrow K$. Choose an eigenvalue $-\varkappa$ of the endomorphism A and a vector $f \in K\setminus \mathrm{im}(A+\varkappa 1)$. Hence $R(\mathfrak{z}_0)f \neq (R(y_0) + \varkappa R(\mathfrak{z}_0))g$ for all $g \in K$. The section $s(Y) = R(\mathfrak{z}_0)f$ modulo $\mathrm{im}\, R(y)$, $Y = \mathrm{span}\, y \in L$, is holomorphic and $s(Z_0) = 0$. Now it suffices to verify that $s(Y) \neq 0$, where $Y \in L$ is the point with homogeneous coordinates $y = y_0 + \varkappa \mathfrak{z}_0$. Really, provided $R(\mathfrak{z}_0)f = R(y)g = (R(y_0) + \varkappa R(\mathfrak{z}_0))g$ we have $0 = R(\tau\mathfrak{z}_0)^{+}R(\mathfrak{z}_0)f = R(\tau\mathfrak{z}_0)^{+}R(y_0)g = -R(\tau y_0)^{+}R(\mathfrak{z}_0)g$, hence $g \in K$ and we get a contradiction.

Set $R_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $R_\infty = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $N_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, where the dimensions of the matrices are $(2c+r)\times c$, $(2c+r)\times c$ and $(2c+r)\times r$, respectively. The columns of N_0 determine the distinguished frame over L_0 . Follow now the construction of Sec.4. The frames $\{s_1,\dots,s_r\}$ and $\{\hat{s}_1,\dots,\hat{s}_r\}$ are determined by the matrix functions $S(Z)$, $\hat{S}(Z)$ modulo $\mathrm{im}\, R(\mathfrak{z})$, respectively , where $(Z = \mathrm{span}\,\mathfrak{z})$, $S(Z) = [1 - R_\infty(R(\tau\mathfrak{z})^{+}R_\infty)^{-1}R(\tau\mathfrak{z})^{+}]N_0$, $\hat{S}(Z) = [1 - R_0(R(\tau\mathfrak{z})^{+}R_0)^{-1}R(\tau\mathfrak{z})^{+}]N_0$. At the same time, as it should be, $\hat{S}(\tau Z)^{+}S(Z) = 1$. The canonical transition function is given by $G_c(Z) = S(\tau Z)^{+}S(Z)$, $G_c(Z)^{-1} = \hat{S}(\tau Z)^{+}\hat{S}(Z)$. Hence $G_c(Z) = 1 - N_0^{+}R(\mathfrak{z})(R(\tau\mathfrak{z})^{+}R_0)^{-1}(R_0^{+}R(\mathfrak{z}))^{-1}R(\tau\mathfrak{z})^{+}N_0$. Taking (3.10) into account we get the canonical initial condition

$$W_c^{(0)}(y,z,\lambda) = 1 + \lambda(ay-b^{+}z)(1-\alpha_1 y-\alpha_2^{+}z)^{-1}(1+\lambda(\alpha_2 y-\alpha_1^{+}z))^{-1}(by+a^{+}z) , \tag{5.2}$$

<u>Lemma 5.2</u>. Let F be a local instanton bundle and let $\{t_1,\dots,t_r\}$ be a holomorphic frame in a neighbourhood of the point P_0 $(y=z=\lambda=0)$. Then the Gram matrix $h = ((t_j,t_k))$ has a form

$$h(x,\lambda,\bar{\lambda}) = W(x,\lambda)^{+}J(x)^{-1}W(x,\lambda) ,$$

where $J(x) = J(x)^{+}$, $W(x,0) = 1$ and J,W solve (2.3) .

<u>Proof</u>. The assertion follows immediately from the form of the Birkhoff decomposition of the transition function (2.8).

Now we apply this lemma. In coordinates (x,λ) we have $\mathfrak{z} = (\lambda y-\bar{z},\lambda z+\bar{y},\lambda,1)$, and so

$$J(x)^{-1}W(x,\lambda) = S(\mathfrak{z})^{+}(1 - R(\mathfrak{z})\Delta(\mathfrak{z})^{-1}R(\mathfrak{z})^{+})S(\mathfrak{z})\big|_{\bar{\lambda}=0} ,$$

$$J(x) = \hat{S}(\tau\mathfrak{z})^{+}(1 - R(\tau\mathfrak{z})\Delta(\mathfrak{z})^{-1}R(\tau\mathfrak{z})^{+})\hat{S}(\tau\mathfrak{z})\big|_{\lambda=\bar{\lambda}=0} ,$$

To get the explicit formulas we set

$$\Psi = ay - b^{+}z \ , \quad \Psi_\sigma = by + a^{+}z \ ,$$
$$\Phi = \alpha_2 y - \alpha_1{}^{+}z \ , \ \Phi_\sigma = \alpha_1 y + \alpha_2{}^{+}z \ , \tag{5.3}$$

and hereinafter

$$\Delta = (1-\Phi_\sigma{}^{+})(1-\Phi_\sigma) + \Phi^{+}\Phi + \Psi^{+}\Psi = (1-\Phi_\sigma)(1-\Phi_\sigma{}^{+}) + \Phi\Phi^{+} + \Psi_\sigma\Psi_\sigma^{+} \ . \tag{5.4}$$

In this notation (5.1) is equivalent to $\Psi_\sigma\Psi = [\Phi,\Phi_\sigma]$ and (5.2) may be rewritten

$$W_c^{(0)}(y,z,\lambda) = 1 + \lambda\Psi(1-\Phi_\sigma)^{-1}(1+\lambda\Phi)^{-1}\Psi_\sigma \ .$$

Theorem 5.3. The following formulas together with (5.3),(5.4) provide the full transcription of the ADHM construction into the J,W-formalism of the inverse scattering approach.

$$W(x,\lambda) = 1 + \lambda(\Psi + \Psi_\sigma{}^{+}(1-\Phi_\sigma{}^{+})^{-1}\Phi)\Delta^{-1}(1 + \lambda\Phi(1-\Phi_\sigma{}^{+})^{-1})^{-1} \times$$
$$\times(\Psi_\sigma - \Phi(1-\Phi_\sigma{}^{+})^{-1}\Psi^{+}) \ ,$$

$$J(x) = 1 + \Psi_\sigma{}^{+}(1-\Phi_\sigma{}^{+})^{-1}(1-\Phi_\sigma)^{-1}\Psi_\sigma -$$
$$- (\Psi + \Psi_\sigma{}^{+}(1-\Phi_\sigma{}^{+})^{-1}\Phi)\Delta^{-1}(\Psi^{+} + \Phi^{+}(1-\Phi_\sigma)^{-1}\Psi_\sigma) \ .$$

If we put $\hat W(x,\lambda) = (W(x,-1/\bar\lambda)^{+})^{-1}$, then $(J,W,\hat W) \in \mathcal{G}_c$ is the distinguished canonical solution defined in Sec.3.

The most familiar self-dual solutions are the 't Hooft's instantons with the gauge group SU(2). In this case $a = \begin{pmatrix}\rho\\0\end{pmatrix}$, $b = (0,-\rho^{t})$, $\rho = (\rho_1,\dots,\rho_c)$, $-\alpha_1 = \mathrm{diag}(\bar u_1,\dots,\bar u_c)$, $-\alpha_2 = \mathrm{diag}(v_1,\dots,v_c)$, where the constants ρ_j are positive and the points $(u_j,v_j) \in \mathbb{C}^2$, $1 \le j \le c$, are mutually different. The canonical initial condition takes the form

$$W_c^{(0)}(y,z,\lambda) = 1 + \lambda\sum_{j=1}^{c}\frac{\rho_j{}^2}{(1+\bar u_j y+\bar v_j z)(1-\lambda(v_j y-u_j z))}\begin{pmatrix} yz & -y^2\\ z^2 & -yz\end{pmatrix} .$$

6. CONSEQUENCES

Owing to the Donaldson's result we can consider the moduli space $\mathcal{O}M(r,c_2)$ of framed holomorphic vector bundles on $\mathbb{P}^2$ instead of $M(r,c_2)$. The description of $\mathcal{O}M(r,c_2)$ is similar to that of $M(r,c)$ (cf.[1]). Theorem 4.3 asserts that the points of $\mathcal{O}M(r,c)$ are in one-to-one correspondence with the restricted canonical initial conditions $W_{res}^{(0)} = W_c^{(0)}\big|_{z=0}$. We introduce a new coordinate $w = \lambda y$ and define $W_{res}^{(0)}(y,w) = W_c^{(0)}(y,0,w/y)$. From (5.2) it follows

$$W_{res}^{(0)}(y,w) = 1 + yw\, a(1-\alpha_1 y)^{-1}(1+\alpha_2 w)^{-1}b \ .$$

The Taylor expansion at the origin shows that a point from $\mathcal{O}M(r,c)$ corresponding to the equivalence class $[(\alpha_1,\alpha_2,a,b)]$ is unambiguously determined by the infinite double indexed sequence of matrices $a\alpha_1^j\alpha_2^k b$, $j,k = 0,1,\ldots$ •.

Lemma 6.1. Let $f(t)\in \mathcal{O}_{0,\mathfrak{f}}$ be a rational function in one variable over any field $\mathfrak{f}$ of characteristic zero and assume f to be regular at the origin. If f can be expressed as a quotient $f = p/q$ with p,q being polynomials of degrees less or equal to n then f is unambiguously determined by $2n$ numbers $\frac{1}{j!}\frac{d^j}{dt^j}f(0)$, $0\leq j\leq 2n-1$.

Proof. The proof is simple and fully algebraic. We omit it.

Proposition 6.2. The holomorphic mapping $\mathcal{O}M(r,c)\longrightarrow \mathbb{C}^{4c^2r^2}$: $[(\alpha_1,\alpha_2,a,b)]\longrightarrow (a\alpha_1^j\alpha_2^k b\ ;\ 0\leq j,k\leq 2c-1)$ is injective.

Proof. Use the Cramer rule and apply twice Lemma 6.1. First putting $\mathfrak{f} = \mathbb{C}$ we reconstruct $a(1-\alpha_1 y)^{-1}\alpha_2^k b$, $0\leq k\leq 2c-1$. Then putting $\mathfrak{f} = \mathbb{C}(y)$ - the field of complex rational functions in the variable y - we reconstruct $a(1-\alpha_1 y)^{-1}(1+\alpha_2 w)^{-1}b$.

Let us now consider another holomorphic injective and maybe more fruitful mapping. The Cramer rule implies

$$W^{(0)}_{res}(y,w) = 1 + (1+\sum_{j=1}^{c} s_j\, y^j)^{-1}(1+\sum_{j=1}^{c} t_j\, w^j)^{-1}\sum_{j,k=1}^{c} S_{jk}\, y^j w^k \ , \tag{6.1}$$

where $s_j,\ t_j\in\mathbb{C}$. $S_{jk}\in gl(r,\mathbb{C})$. The mapping

$$\mathcal{O}M(r,c)\longrightarrow \mathbb{C}^{2c+r^2c^2} : [(\alpha_1,\alpha_2,a,b)]\longrightarrow (s_j,t_j,S_{jk};\ 1\leq j,k\leq c\ )$$

is indeed holomorphic and injective. Moreover, the interpretation of $W^{(0)}_{res}$ as a transition function shows that this mapping is a homeomorphism onto its image. By Remmert Theorem (cf.[13],§IV.7) the image of $\mathcal{O}M(r,c)$ is a (locally) analytic set , irreducible and of dimension equal to $\dim_{\mathbb{C}}\mathcal{O}M(r,c)$ everywhere. Remind that the moduli spaces are known to be connected (cf.[10],Chp.II).

The 1-instantons provide the most simple example. The moduli space $\mathcal{O}M(2,1)$ is biholomorphically equivalent to $\mathcal{A}^{(1)}_{reg}$, the algebraic set $\mathcal{A}^{(1)}\subset\mathbb{C}^6$ is determined by the equations: $\operatorname{tr} S_{11} = \det S_{11} = 0$ ($\mathcal{A}^{(1)}_{sing}$ consists of points for which $S_{11} = 0$). Proposition 4.4 characterizes partially the image in the general case. Substitute (6.1) into $\det W^{(0)}_{res} = 1$ to get

$$\operatorname{tr} S_{11} = \operatorname{tr} S_{1j} = \operatorname{tr} S_{j,1} = 0\ ,\quad 2\leq j\leq c\ , \tag{6.2a}$$

$$(1+\sum_{j=1}^{c} s_j\, y^j)(1+\sum_{j=1}^{c} t_j\, w^j)\sum_{j,k=2}^{c} \operatorname{tr} S_{jk}\; y^{j-2}w^{k-2} +$$

$$+\det\Big(\sum_{j,k=1}^{c} S_{jk}\, y^{j-1}w^{k-1}\Big) = 0 \quad . \quad \bullet \tag{6.2b}$$

Note that for rank-2 vector bundles (6.2a,b) represent $2c(2c-1)$ equations and if we subtract this number from the number of independent variables we obtain exactly $4c = \dim_{\mathbb{C}} \mathcal{G}M(2,c)$. On the other hand, the algebraic set $\mathcal{A}^{(c)} \subset \mathbb{C}^{2c(2c+1)}$ determined by the equations (6.2) contains the $(c+1)^2$ - dimensional algebraic subset: $\operatorname{tr} S_{jk} = \operatorname{tr}(S_{jk}\, S_{j'k'}) = 0$ for all j,k,j',k' .

7. ON THE LOOP GROUP ACTION

The embedding of the space $\mathcal{S}$ into the vector space $gl(r,\mathbb{C}[[x]])\times gl(r,\mathbb{C}[[x,\lambda]])\times gl(r,\mathbb{C}[[x,\lambda^{-1}]])$ enables us to treat formally the Lie algebra $\mathfrak{X}(\mathcal{S})$ of vector fields on $\mathcal{S}$. Denote by $gl(r,\mathbb{C}[\lambda,\lambda^{-1}])$ the Lie algebra of Lorain polynomials in λ with coefficients from the Lie algebra $gl(r,\mathbb{C})$. Dolan [16] and Chau, Ge, Sinha and Wu [17] discovered an infinitesimal action, i.e., a Lie algebra homomorphism $\delta : gl(r,\mathbb{C}[\lambda,\lambda^{-1}]) \longrightarrow \mathfrak{X}(\mathcal{S}) : T\lambda^{-k} \longmapsto \delta_k(T)$, where $k\in\mathbb{Z}$, $T\in gl(r,\mathbb{C})$. The components of $\delta_k(T)$ can be expressed with the help of $gl(r,\mathbb{C}[[x]])$ - valued functions $d_k(T)$, $\hat{d}_k(T)$, $k\in\mathbb{N}_0$, defined on $\mathcal{S}$ and linearly depending on T , introduced by the relations $W(\lambda)TW(\lambda)^{-1} = \sum_{k=0}^{\infty} d_k(T)\,\lambda^k$, $\hat{W}(\lambda)T\hat{W}(\lambda)^{-1} = \sum_{k=0}^{\infty} \hat{d}_k(T)\,(-\lambda)^{-k}$.

It was also recognized by Chau et al. that to get a well defined action on the subspace $\mathcal{S}_u \subset \mathcal{S}$ we must restrict $gl(r,\mathbb{C}[\lambda,\lambda^{-1}])$ to the subalgebra consisting of those elements $\sum T_k\lambda^{-k}$ which satisfy $T_{-k} = (-1)^{k+1}\, T_k^{+}$.

Let Ω be the loop group of holomorphic mappings from $\mathbb{C}$ to $GL(r,\mathbb{C})$ defined on a neighbourhood of the unit circle. Crane discovered [5] that one is able to exponentiate the infinitesimal action if we replace $\mathcal{S}$ by its subspace of convergent power series $\mathcal{G}$. Let $G = (J\hat{W})^{-1}W\in\mathcal{G}$, $\mathcal{T} = \sum T_k\lambda^{-k} \in gl(r,\mathbb{C}[\lambda,\lambda^{-1}])$, $(J',W',\hat{W}') = (J,W,\hat{W}) + \varepsilon\,\delta(\mathcal{T})$, ε - infinitesimal. Then we get after some algebra $G' := (J'\hat{W}')^{-1}W' = (1+\varepsilon\mathcal{T})G(1-\varepsilon\mathcal{T})$ (mod ε^2) . Hence the global action of Ω on $\mathcal{G}$ should be defined as:

$$(g\cdot G)(x,\lambda) = g(\lambda)\,G(x,\lambda)\,g(\lambda)^{-1} \quad ,$$

where $g\in\Omega$, $G\in\mathcal{G}$. This definition is easily seen to be correct. Crane defined the action of Ω on $\mathcal{G}_u$ as follows
$(g\cdot G)(x,\lambda) = g(\lambda)\,G(x,\lambda)\,g(-1/\bar{\lambda})^{\dagger}$.

Clearly, this result requires an additional specification. The global action will be correctly defined on the subspace $\mathcal{G}_u \subset \mathcal{G}$ provided we restrict the group Ω to its subgroup Ω_u consisting of those elements $g(\lambda)$ which satisfy $g(-1/\bar{\lambda})^{+} = g(\lambda)^{-1}$.

The loop group action is gauge dependant and the moduli spaces $M(r,c)$ embedded into $\mathcal{Y}_c$ are not invariant with respect to this action. Let us notice more closely the infinitesimal action on the 1-instanton moduli space $M(2,1)$. The action of Ω_u includes as a special case the $ad(U(2))$ - transformations corresponding to the choice of the distinguished frame and there is no necessity to persue them further. Because $d_k(1) = \hat{d}_k(1) = 0$ for $k>0$, it is enough to consider only the vector fields $\delta(\mathcal{T})$, where $\mathcal{T} = \sum_{k\geq 1} (T_k \lambda^{-k} + (-1)^{k+1} T_k^{+} \lambda^{k})$, $T_k \in sl(2,\mathbb{C})$. Denote $\tilde{\delta}_k(T) = \delta_k(T\lambda^{-k} + (-1)^{k+1} T^{+}\lambda^{k})$, and $\delta_k^{(0)}(T) = pr_{c*}\tilde{\delta}_k(T)$, $k\geq 1$. Since $\mathcal{Y}_c \cong \mathcal{W}$ we can regard the vector field $\delta_k^{(0)}(T)$ as being defined on $\mathcal{W}$. We shall try to answer the following question: Which vector fields $\tilde{\delta}(\mathcal{T}) = \sum_{k\geq 1} \tilde{\delta}_k(T_k)$ having been restricted to $M(2,1)$ are tangent to $pr_c^{-1}(M(2,1)) \subset \mathcal{Y}_u$, i.e., which vector fields $\delta^{(0)}(\mathcal{T})$ are tangent to $M(2,1) \subset \mathcal{W}$? Roughly speaking, we ask when the loop group action preserves **infinitesimally the topological charge** finite and equal to 1 .

To this end we derive some necessary formulas. Let $(J,W,\hat{W}) \in \mathcal{Y}_c$ and $W_c^{(0)} \in \mathcal{W}$ be the corresponding canonical initial condition. Then it holds: $\hat{d}_k(x;\, T^{+}) = d_k(x;\, T)^{+}$ and $\hat{d}_k(y,z,0,0;\, T) = 0$ for $k\geq 1$. Use these relations to get

$$\delta_k^{(0)}(T) = -\sum_{j=1}^{\infty} d_{k+j}^{(0)}(y,z;T)\, \lambda^{j}\, W^{(0)}(y,z,\lambda) -$$

$$- W^{(0)}(y,z,\lambda) \sum_{j=1}^{\infty} d_{k+j}^{(0)}(\bar{\lambda}\bar{z},-\bar{\lambda}\bar{y};T)^{+}\, (-\lambda)^{j} \quad .$$

where by definition $d_k^{(0)}(y,z;T) = d_k(y,z,0,0;T)$. The infinitesimal canonical gauge transformation $(J,W,\hat{W}) + \varepsilon\,\tilde{\delta}_k(T) \longmapsto (J,W,\hat{W}) + \varepsilon\,\delta_k^{(0)}(T)$, ε - infinitesimal, takes the form $\Gamma_c(\xi,\eta,\zeta) = 1 + \varepsilon \sum_{j=0}^{k-1} d_{k-j}^{(0)}(\bar{\eta},-\bar{\xi};T)^{+}\, (-\zeta)^{j} \pmod{\varepsilon^2}$. The 1-instanton canonical initial conditions depend on four parameters $u,v,a,b \in \mathbb{C}$, $(a,b) \neq (0,0)$, $W_c^{(0)}(y,z,\lambda) = 1 + \lambda(1+\phi_\sigma)^{-1}(1-\lambda\phi)^{-1}\, X$, where $X = \begin{pmatrix} ay - \bar{b}z \\ by + \bar{a}z \end{pmatrix} (by + \bar{a}z, -ay + \bar{b}z)$, $\phi = vy - uz$, $\phi_\sigma = \bar{u}y + \bar{v}z$. We obtain $(k\geq 1)$

$$d_k^{(0)}(T) = \frac{\phi^{k-1}}{1+\phi_\sigma}\ [\mathbf{X},T] - (k-1)\ \frac{\phi^{k-2}}{(1+\phi_\sigma)^2}\ \mathbf{XTX}\quad , \text{ and}$$

$$\delta_k^{(0)}(T) = -\frac{\lambda\phi^{k-1}}{(1+\phi_\sigma)^2(1-\lambda\phi)}\ \{\phi(1+\phi_\sigma)[\mathbf{X},T] - k\ \mathbf{XTX}\} + $$
$$+ \frac{\lambda^{k+1}\ \phi_\sigma^{\ k-1}}{(1+\phi_\sigma)(1-\lambda\phi)^2}\ \{\phi_\sigma(1-\lambda\phi)[\mathbf{X},T^+] - \lambda k\ \mathbf{X}T^+\mathbf{X}\}\ .$$

Since $\delta_k^{(0)}(T) \in sl(2,(y,z)^{k+2})$, the infinite series $\sum_{k=1}^{\infty} \delta_k^{(0)}(T_k) = \delta^{(0)}(\mathcal{T})$ makes sense in the realm of formal power series. The 4 - dimensional tangent space at the point $W_c^{(0)} \in M(2,1) \subset \mathcal{W}$ can be obtained by variation of the parameters u,v,a,b. Comparing homogeneous terms of the formal power series, after rather straightforward considerations we arrive at the following conclusion. Provided $\mathcal{T}$ is nonzero the vector field $\delta^{(0)}(\mathcal{T})$ is tangent to the manifold $M(2,1)$ only in the points corresponding to the parameters $u = v = 0$, a, b - arbitrary, and only in the case $T_1 = 0$. The vector fields $\delta_k^{(0)}(T)$. $k \geq 2$, vanish in these points and hence the vector fields $\tilde{\delta}_k(T)$, $k \geq 2$, are tangent to the fibres of the projection pr_c . The corresponding infinitesimal canonical transformation is $\underline{T}_c(\xi,\eta,\zeta)$ =

$$1 - \varepsilon\{[\mathbf{Y},T^+]\zeta + \mathbf{Y}T^+\mathbf{Y}\}(-\zeta)^{k-2}\quad (\mathrm{mod}\ \varepsilon^2)\ ,\quad \mathbf{Y} = \mathbf{X}|_{y=\xi,z=\eta}\ .$$

8. CONCLUSION

Further investigation of the restricted canonical initial condition may render a new description of the moduli spaces $\mathcal{O}M(r,c_2) \cong M(r,c_2)$. We choose the homogeneous coordinates (z_1,z_3,z_4) on $H_o \cong \mathbb{P}^2$. The restricted canonical transition function G_{res} =

$$1 + (z_3^{\ c} + \sum_{j=1}^{c} s_j\ z_1^{\ j}\ z_3^{\ c-j})^{-1}(z_4^{\ c} + \sum_{j=1}^{c} t_j\ z_1^{\ j}\ z_4^{\ c-j})^{-1} \times$$
$$\times \sum_{j,k=1}^{c} S_{jk}\ z_1^{\ j+k}\ z_3^{\ c-j}\ z_4^{\ c-k}$$

defines a holomorphic vector bundle F over $\mathbb{P}^2 \setminus \mathcal{M}$, with $\mathcal{M}$ being the discrete set

$$z_3^{\ c} + \sum s_j\ z_1^{\ j}\ z_3^{\ c-j} = z_4^{\ c} + \sum t_j\ z_1^{\ j}\ z_4^{\ c-j} = 0\ .$$

The first question naturally arising is: For which values of the parameters (s_j,t_j,S_{jk}) are the singular points removable in the sense that $\iota_*\mathcal{F}$ is a locally free rank-r sheaf on $\mathbb{P}^2$ with

$\iota : \mathbb{P}^2 \setminus \mathcal{M} \hookrightarrow \mathbb{P}^2$ being the embedding and $\mathcal{F}$ being the sheaf of germs of holomorphic sections in F ? For such values it is further necessary to compute the second Chern class $c_2(F)$. Let $\tilde{\mathbb{P}}^2 \subset \mathbb{P}^2 \times \mathbb{P}^1$ be the blow-up of $\mathbb{P}^2$ at the point $z_1 = z_3 = 0$. Then $R^1 pr_{2*} pr_1^* \mathcal{F}(-1)$ is a sheaf on $\mathbb{P}^1 = \mathbb{P}(\langle e_1, e_3 \rangle)$ with the support contained in the discrete set $z_3^c + \sum s_j z_1^j z_3^{c-j} = 0$ and the sum of dimensions of the stalks over these points equals to $c_2(F)$ (cf. [10],Chp.II). We ask for which values of (s_j, t_j, S_{jk}) the inequality $c_2(F) \leq c$ holds. Having performed this program we hopefully obtain a description of the union $\bigcup_{k \leq c} \mathcal{O}M(r,k)$ as a stratified algebraic set.

REFERENCES

1. Donaldson S.K., Comm.Math.Phys., 1984, v.93, pp.453-460
2. Takasaki K., Comm.Math.Phys., 1984, v.94, pp.35-59
3. Corrigan E.F., Fairlie D.B., Yates R.G., Goddard P., Comm.Math.Phys., 1978, v.58, pp.223-240
4. Pohlmeyer K., Comm.Math.Phys., 1980, v.72, pp.35-47
5. Crane L., Comm.Math.Phys., 1987, v.110, pp.391-414
6. Atiyah M.F., Hitchin N.J., Singer I., Proc.R.Soc.Lond. A, 1978, v.362, pp.420-461
7. Chau L.L., Prasad M.K., Sinha A., Phys.Rev. D, 1981, v.24, p.1574
8. Ward R.S., Comm.Math.Phys., 1981, v. 80, pp.563-574
9. Uhlenbeck K., Bull.Am.Math.Soc., 1979, v.1, pp.579-581
10. Okonek C., Schneider M., Spindler H., Vector bundles on complex projective spaces, Birkhauser, Boston 1980
11. Hartshorne R., Algebraic geometry, Springer-Verlag, New-York 1977
12. Griffiths P., Harris J., Principles of algebraic geometry, John Wiley&Sons, New-York 1978
13. Hervé M., Several complex variables, Oxford University Press, Bombay 1963
14. Atiyah M.F., Drinfeld V.G., Hitchin N.J., Manin Yu.I., Phys.Lett. A, 1978, v.65, p.185
15. Christ N.H, Weinberg E.J., Stanton N.K., Phys.Rev D, 1978, v.18, pp.2013-2025
16. Dolan L., Phys.Lett. B, 1982, v.113, pp.387-390
17. Chau L.L., Ge M.L., Sinha A., Wu Y.S., Phys.Lett. B, 1983, v.121, p.391

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