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THE GREEN'S FUNCTION FOR THE TWO-SOLENOID AHARONOV-BOHM EFFECT

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1. INTRODUCTION

Undoubtedly the Aharonov-Bohm (AB) effect [1] helped to understand more deeply the geometric nature of gauge fields. Going in this way Wu and Yang [2] generalized the problem to non-Abelian gauge groups. The AB effect with the SU(2) gauge group on the punctured plane was elaborated by Horvathy [3]. But the calculations become much more complicated for more than one sources of the gauge field (solenoids). A general scheme to attack this problem was proposed by Sundrum and Tassie [4] and by Oh, Soo and Lai [5]. The main trick is to use the universal covering space technique originally developed by Schulman [6] in connection with the Feynman path integral on multiply connected spaces.

In this paper we consider a non-relativistic quantum particle moving in an external gauge field with the flux concentrated in two infinitelly thin parallel solenoids. In the idealized setup the configuration space is the double punctured plane $M = \mathbb{R}^2 \setminus \{a,b\}$ and the gauge field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ vanishes on M. The gauge group is allowed to be non-Abelian and without lack of generality we can regard it to be $U(N)$, $N \geq 1$. The basic tool having been applied is again the universal covering space technique though the Feynman integral is not considered at all. This approach enables us to express both the Green's function and the propagator in the form of infinite series the convergence of which is much more easy to prove in the former case. But the both objects are closely related by the Laplace transformation

$$G(z) = \frac{i}{\hbar} \int_0^\infty e^{izt/\hbar} K(t)\, dt , \quad \mathrm{Im}\, z > 0 . \tag{1}$$

2. QUANTUM MECHANICS ON MULTIPLY CONNECTED SPACES

With very general assumptions it can be shown [7] that, up to equivalence, all gauge fields A_ν with the vanishing field strength $F_{\mu\nu} = 0$ on some configuration space M are in one-to-one correspondence with unitary representations U of the fundamental group $\Gamma = \pi_1(M, x_{ref})$: for each homotopic class $[\gamma] \in \Gamma$ it holds

$$U([\gamma])^{-1} = P\exp(-\int_\gamma A_\nu \, dx^\nu) ,$$

where the fixed reference point x_{ref} is the starting and the terminating point of the closed curve γ. The group multiplication in Γ is defined as follows: $[\gamma_1][\gamma_2] = [\gamma_1 * \gamma_2]$, where $\gamma_1 * \gamma_2$ means that the curve γ_2 follows the curve γ_1.

The quantum mechanical description can be done in at least three equivalent ways. The most usual one is to choose the Hamiltonian equal to $-(\hbar^2/2\mu)(\partial + A)^2$ and acting in the Hilbert space of $\mathbb{C}^N$-valued wave functions on M. The second possibility is to use the universal covering space $\widetilde{M}$. The discrete group Γ acts on $\widetilde{M}$ from the left and the quotient $\Gamma \setminus M$ coincides with the original space M. The Hamiltonian $\hat{H}_U = -(\hbar^2/2\mu)\Delta_{LB}$ is defined in the Hilbert space $\mathcal{H}_U$ consisting of $\mathbb{C}^N$-valued wave functions on $\widetilde{M}$ which are required to be U-equivariant, i.e., $\psi(g.x) = U(g)\psi(x)$, $g \in \Gamma$. The Hermitian product in $\mathcal{H}_U$ is defined

$$\langle \psi_1, \psi_2 \rangle = \int_D \psi_1(x)^* \psi_2(x) \, dV(x) .$$

Here Δ_{LB} designates the Laplace-Beltrami operator, $dV(x)$ designates the Riemann measure on $\widetilde{M}$ and D is an arbitrary fundamenatal domain, i.e., such an open connected domain in $\widetilde{M}$ for which the sets $g.D$, $g \in \Gamma$, do not intersect each other and their union covers $\widetilde{M}$ up to a zero measure set. We note that the Riemann metric is naturally transferred from M to $\widetilde{M}$ and the left action of Γ on $\widetilde{M}$ preserves this metric and hence the measure dV as well. The third possibility is to fix a simply connected fundamental domain $D \subset \widetilde{M}$. Then D is projected one-to-one onto an open simply connected set $M \setminus L$, with L being a cut in M, $\dim L = \dim M - 1$. Now the Hamiltonian is $\hat{H} = -(\hbar^2/2\mu)\Delta$ acting in $\mathcal{H} = L^2(M, \mathbb{C}^N, dV)$, but its definition entails also the boundary conditions on L. Clearly, the transcription from the second description to the third one is rather straightforward. The presented results will be given in the third formulation.

Suppose we know the propagator $\widetilde{K}(t)$ for the free particle on $\widetilde{M}$. It is worth to emphasize that in this case wave functions and the propagator are scalar and the integration in the Hermitian product is over the whole space $\widetilde{M}$. The Schulman's Ansatz (generalized to non-Abelian gauge groups) enables us to compute the propagator $K^U(t)$ in the second formulation ($x, x_0 \in \widetilde{M}$):

$$K_t^U(x, x_0) = \sum_{g \in \Gamma} U(g^{-1}) \, \widetilde{K}_t(g.x, x_0) . \qquad (2)$$

Manipulating formally the infinite series $K^U(t)$ is easily checked to fulfil all the basic properties. The propagator $K(t)$ in the third formulation is obtained as a restriction to the fixed fundamental domain, $K_t(x, x_0) = K_t^U(x, x_0)|\, D \times D$.

3. THE 1-SOLENOID CASE

The 1-solenoid example provides us with an inspiration opening the way to the more complicated 2-solenoid case. We shall discuss it shortly from this point of view. So $M = \mathbb{R}^2 \setminus \{0\}$, the fundamental group $\Gamma \cong \mathbb{Z}$ has one generator g_0 which is chosen to be the homotopic class of a simple positively oriented (counterclockwise) curve containing the origin in its inner. Then $U(g_0) = \exp(2\pi i\alpha)$, where α is a $N \times N$ Hermitian matrix unambiguously specified by the condition $0 \le \alpha < 1$ (i.e., all eigen-values of α obey these inequalities). The cut is chosen to be the positive x-halfaxis, $L = \{(x, 0);\ x > 0\}$. The Hamiltonian is defined $\hat{H} = -(\hbar^2/2\mu)\Delta$ on $\mathbb{R}^2 \setminus L$ together with the boundary conditions on L written in the polar coordinates ($r > 0$, $\varphi \in (-\pi, \pi)$):

$$\psi(r, \pi) = e^{2\pi i\alpha}\psi(r, -\pi) , \ \partial_\varphi \psi(r, \pi) = e^{2\pi i\alpha}\partial_\varphi \psi(r, -\pi) .$$

The universal covering space $\widetilde{M}$ is $\mathbb{R}_+ \times \mathbb{R}$, i.e., the angle φ is allowed to take any real value. We can complete $\widetilde{M}$ with one "ideal" point A for which $r = 0$ (φ is not specified). This point can be achieved from each point of $\widetilde{M}$ by a free classical particle in a finite time, i.e., it can be connected with each point from $\widetilde{M}$ by a geodesic curve of finite length. There is another peculiar property of $\widetilde{M}$. Two points $x, x_0 \in \widetilde{M}$ can be connected by a finite geodesic curve only if $|\varphi - \varphi_0| < \pi$. Put $\chi(x, x_0) = 1$ (resp. 0) provided the points x, x_0 can (resp. cannot) be connected by a finite geodesic. Then by definition $\chi(A, x) = 1$ for all $x \in \widetilde{M}$.

Put

$$Z_t(x,x_0) = \vartheta(t)\ \chi(x,x_0)\ \frac{\mu}{2\pi i\hbar t} \exp\left[\frac{i\mu}{2\hbar t} \mathrm{dist}^2(x,x_0)\right] \ , \qquad (3)$$

where $\vartheta(t)$ is the Heaviside step function. Since we know a complete set of normalized generalized eigen-functions for a free particle on $\widetilde{M}$, $\{(\sqrt{2\pi}\,\hbar)^{-1} J_{|\nu|}(pr/\hbar)\, e^{i\nu\varphi}\ ;\ \nu\in\mathbb{R},\ p>0\}$, we can compute for $t>0$

$$\widetilde{K}_t(x,x_0) = Z_t(x,x_0) + \frac{\mu}{2\pi i\hbar t}\int_{-\infty}^{\infty}\frac{ds}{2\pi}\left(\frac{1}{\Phi-\pi+is} - \frac{1}{\Phi+\pi+is}\right)\times$$

$$\times\ e^{i\mu R^2(s)/2\hbar t} \ , \qquad (4)$$

where $\Phi = \varphi - \varphi_0$, $R^2(s) = r^2 + r_0^{\,2} + 2\, r\, r_0\, \mathrm{ch}(s)$.

For three points $x_1, x_2, x_3 \in \widetilde{M}\cup\{A\}$ such that $\chi(x_1,x_2) = \chi(x_2,x_3) = 1$ and for two positive times $t_1,\ t_2$ we put

$$V\begin{pmatrix} x_3, x_2, x_1 \\ t_2, t_1 \end{pmatrix} = \frac{i\hbar}{\mu}\left(\frac{1}{\theta-\pi+iu} - \frac{1}{\theta+\pi+iu}\right) \ , \qquad (5)$$

where $\theta = \sphericalangle\, x_1 x_2 x_3$ (the angle is oriented, i.e., θ is defined including the sign), $u = \ln(t_2 r_1/t_1 r_2)$ and $r_1 = \mathrm{dist}(x_1,x_2)$, $r_2 = \mathrm{dist}(x_2,x_3)$. Using the substitution

$$s = \ln(t_1 r_0/t_0 r)\ ,\ ds = t\ \delta(t_1+t_0-t)\ \vartheta(t_1)\ \vartheta(t_0)\ (t_1 t_0)^{-1}\ dt_1 dt_0 ,$$

we can rewrite

$$\widetilde{K}_t(x,x_0) = Z_t(x,x_0) + \iint dt_1 dt_0\ \delta(t_1+t_0-t)\ V\begin{pmatrix} x, A, x_0 \\ t_1, t_0 \end{pmatrix}\times$$

$$\times\ Z_{t_1}(x,A)\ Z_{t_0}(A,x_0) \ . \qquad (6)$$

The Schulman's Ansatz leads to the correct result in the third formulation

$$K_t(x,x_0) = \begin{Bmatrix} 1 \\ \exp(2\pi i\alpha) \\ \exp(-2\pi i\alpha) \end{Bmatrix} \frac{\mu}{2\pi i\hbar t}\ e^{i\mu|x-x_0|^2/2\hbar t} -$$

$$- \frac{\sin\pi\alpha}{\pi}\int_{-\infty}^{\infty} ds\ \frac{\mu}{2\pi i\hbar t}\ e^{i\mu R^2(s)/2\hbar t}\ \frac{e^{-\alpha(s-i\Phi)}}{1+e^{-s+i\Phi}} \ , \qquad (7)$$

where the value in the composite brackets depends on whether $\Phi = \varphi - \varphi_0$ belongs to the interval $(-\pi,\pi)$ or $(\pi,2\pi)$ or $(-2\pi,-\pi)$. Having performed the Laplace transformation (1) we get the Green's function ($z\notin\mathbb{R}_+$, $K_\nu(x)$ is the Macdonald function)

$$G_z(x,x_0) = \begin{Bmatrix} 1 \\ \exp(2\pi i\alpha) \\ \exp(-2\pi i\alpha) \end{Bmatrix} \frac{\mu}{\pi\hbar^2}\ K_0(w\,|x-x_0|) -$$

$$- \frac{\sin\pi\alpha}{\pi}\int_{-\infty}^{\infty} ds\ \frac{\mu}{\pi\hbar^2}\ K_0[w\ R(s)]\ \frac{e^{-\alpha(s-i\Phi)}}{1+e^{-s+i\Phi}} \ , \qquad (8)$$

$$w = \sqrt{-2\mu z}\,/\,\hbar \ ,\ \mathrm{Re}\ w > 0 \ . \qquad (9)$$

4. THE 2-SOLENOID CASE

Let us now turn to the 2-solenoid case. So $M = \mathbb{R}^2\setminus\{a,b\}$. We choose the coordinate axes such that $a = (0,0)$, $b = (\rho,0)$, $\rho = |a-b| > 0$, and the cut to be a union $L = L_a \cup L_b$ of two halflines lying on the x-axis: $L_a = \{(x,0)\ ;\ x<0\}$, $L_b = \{(x,0)\ ;\ x>\rho\}$. We shall need two polar coordinate systems with respect to the centres a and b . The angles are again counterclockwise oriented and $\varphi_a,\ \varphi_b \in (-\pi,\pi)$. The values $\varphi_a = \pm\pi$ (resp. $\varphi_b = \pm\pi$) correspond to two sides of the cut L_a (resp. L_b). Γ is the free group with two generators $g_a,\ g_b$ corresponding to two simple positively oriented curves winding round the point a (resp. b) and with the point b (resp. a) lying in the outside. The universal covering space $\widetilde{M}$ results from the infinite process of patching together countably many copies of the typical sheet (fundamental domain) $D = \mathbb{R}^2\setminus L$. The boundary ∂D consists of four halflines (two sides of L_a and two sides of L_b) and of four points: a, b and two times ∞ (reached from the upper and from the lower halfplane). This fact will be emphasized in the notation: $D = D(a,\infty,b,\infty)$. Each sheet is patched together along four halflines with four other sheets. $\widetilde{M}$ is again completed with "ideal" points which now constitute a union $\mathcal{A}\cup\mathcal{B}$. The countable set $\mathcal{A}$ (resp. $\mathcal{B}$) is projected in the point a (resp. b).

The function $\chi(.,.)$ retains its meaning. But it holds never more that $\chi(C,x)=1$ for all $x\in\widetilde{M}$ provided $C\in\mathcal{A}\cup\mathcal{B}$. For example, let $A\in\mathcal{A}$. Then the set of points which can be connected with A by a finite geodesic is a union $\cup D_j$, $D_j = D(A,\infty,B_j,\infty)$, $j=\ldots,-1,0,1,2,\ldots$, and $B_j\in\mathcal{B}$, $\mathrm{dist}(A,B_j)=\rho$. The domains D_j are arranged in "spiral stairs" centred at the point A, D_j is patched together with D_{j-1} and D_{j+1}.

Put $U(g_a)=\exp(2\pi i\alpha)$, $U(g_b)=\exp(2\pi i\beta)$, $0\le\alpha,\beta<1$. The Hermitian matrices α,β are not constrained with any other condition. The Hamiltonian is defined $\hat{H}=-(\hbar^2/2\mu)\Delta$ on $\mathbb{R}^2\setminus L$ together with the boundary conditions on the halflines L_a, L_b:

$$\psi|_{\varphi_a=\pi}=e^{2\pi i\alpha}\,\psi|_{\varphi_a=-\pi},\quad \partial_{\varphi_a}\psi\,|_{\varphi_a=\pi}=e^{2\pi i\alpha}\,\partial_{\varphi_a}\psi\,|_{\varphi_a=-\pi},$$
$$\psi|_{\varphi_b=\pi}=e^{2\pi i\beta}\,\psi|_{\varphi_b=-\pi},\quad \partial_{\varphi_b}\psi\,|_{\varphi_b=\pi}=e^{2\pi i\beta}\,\partial_{\varphi_b}\psi\,|_{\varphi_b=-\pi}. \tag{10}$$

Now we can guess the form of $\widetilde{K}(t)$. The symbols $Z(t)$, $V(..)$ defined in (3), (5) retain their meaning. For $t>0$

$$\widetilde{K}_t(x,x_0)=\sum_{\gamma,n\ge0} W_\gamma(t;x,x_0),$$

$$W_\gamma(t;x,x_0)=\int_{\mathbb{R}^{n+1}} dt_n\ldots dt_0\;\delta(t_n+\ldots+t_0-t)\;V\begin{pmatrix}\gamma\\ t_n,\ldots,t_0\end{pmatrix}\times$$
$$\times Z_{t_n}(x,C_n)\,Z_{t_{n-1}}(C_n,C_{n-1})\ldots Z_{t_0}(C_1,x_0), \tag{11}$$

$$V\begin{pmatrix}\gamma\\ t_n,\ldots,t_0\end{pmatrix}=V\begin{pmatrix}x,C_n,C_{n-1}\\ t_n,t_{n-1}\end{pmatrix}V\begin{pmatrix}C_n,C_{n-1},C_{n-2}\\ t_{n-1},t_{n-2}\end{pmatrix}\ldots$$
$$\ldots V\begin{pmatrix}C_2,C_1,x_0\\ t_1,t_0\end{pmatrix}, \tag{12}$$

$V=1$ for $n=0$ and the sum is over all piecewise geodesic curves $\gamma: x\leftarrow C_n\leftarrow\ldots\leftarrow C_1\leftarrow x_0$, with the inner vertices $C_j\in\mathcal{A}\cup\mathcal{B}$, $1\le j\le n$, such that $\mathrm{dist}(C_j,C_{j+1})=\rho$, $1\le j\le n-1$.

To simplify notation we put where necessary $C_0=x_0$, $C_{n+1}=x$.

Treating formally the infinite (countable) sum we can verify the equality (x_0 - fixed)

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2\mu}\Delta_{LB}\right)\vartheta(t)\,\widetilde{K}_t(x,x_0)=i\hbar\,\delta(t)\,\delta_{\widetilde{M}}(x,x_0), \tag{13}$$

where the Dirac-type generalized function $\delta_{\widetilde{M}}$ is defined by the relation $\int_{\widetilde{M}}\delta_{\widetilde{M}}(x,\xi)\,\varphi(\xi)\,dV(\xi)=\varphi(x)$. Indeed, the equality (13) results from the following relations:

$$\lim_{t\downarrow0}Z_t(x,x_0)=\delta_{\widetilde{M}}(x,x_0),\quad \lim_{t\downarrow0}W_\gamma(t;x,x_0)=0\quad\text{for } n\ge1, \tag{14}$$

and for $t>0$, x_0 - fixed,

$$\left(i\frac{2\mu}{\hbar}\frac{\partial}{\partial t}+\Delta_{LB}\right)W_\gamma(t;x,x_0)=-\frac{\partial}{\partial\vec{n}}\left[W_\gamma(t)\,\delta_{\partial D(C_n)}\right]-$$
$$-\left[\frac{\partial}{\partial\vec{n}}W_\gamma(t)\right]\delta_{\partial D(C_n)}+\frac{\partial}{\partial\vec{n}}\left[W_{\gamma'}(t)\,\delta_{\partial D(C_{n-1};C_n)}\right]+$$
$$+\left[\frac{\partial}{\partial\vec{n}}W_{\gamma'}(t)\right]\delta_{\partial D(C_{n-1};C_n)}, \tag{15}$$

where $\gamma': x\leftarrow C_{n-1}\leftarrow\ldots\leftarrow C_1\leftarrow x_0$, $D(C_n)=\{x\in\widetilde{M};\ \chi(C_n,x)=1\}$, $\partial D(C_{n-1};C_n)$ is that part of the boundary $\partial D(C_{n-1})$ which consists of two halflines with the common vertex C_n and $\partial/\partial\vec{n}$ is the normal outer derivation.

Let us now apply the Schulman's Ansatz. Let $D=\mathbb{R}^2\setminus L\subset\widetilde{M}$ be the fixed domain, $x_0,x\in D$. We have to sum the contributions of all piecewise geodesics $g.x\leftarrow C_n\leftarrow\ldots\leftarrow C_1\leftarrow x_0$, $g\in\Gamma$, with the weight $U(g^{-1})$. The possible values of the angles θ_j at the inner vertices C_j are $\theta_1=-\varphi_0+2\pi k_1$, $\theta_2=2\pi k_2,\ldots$ $\ldots,\theta_{n-1}=2\pi k_{n-1}$, $\theta_n=\varphi+2\pi k_n$, $k_1,\ldots,k_n\in\mathbb{Z}$, and

$$g=g_1^{k_1}\ldots g_n^{k_n},$$

where $g_j=g_a$ (resp. g_b) provided $C_j\in\mathcal{A}$ (resp. $\mathcal{B}$). Let $K^a(t)$ designate the 1-solenoid propagator (7) centred at the point a with the cut L_a, $U(g_a)=\exp(2\pi i\alpha)$. $K^b(t)$ is defined similarly, $K^0(t)$ is the free propagator on the plane. Then for $t>0$

$$K_t(x,x_0) = K_t^a(x,x_0) + K_t^b(x,x_0) - K_t^0(x,x_0) +$$

$$+ \frac{\mu}{2\pi i\hbar} \sum_{\gamma, n\geq 2} (-1)^n \int_0^\infty \frac{dt_n}{t_n} \dots \int_0^\infty \frac{dt_0}{t_0}\, \delta(t_n+\dots t_0-t) \times$$

$$\times \exp\left\{ \frac{i\mu}{2\hbar}\left(\frac{r_n^2}{t_n} + \dots + \frac{r_0^2}{t_0} \right)\right\} \quad S_\gamma(s;\varphi,\varphi_0) \quad , \qquad (16)$$

$$S_\gamma(s;\varphi,\varphi_0) = \frac{\sin\pi\sigma_n}{\pi} \frac{e^{-\sigma_n(s_n-i\varphi)}}{1+e^{-s_n+i\varphi}} \frac{\sin\pi\sigma_{n-1}}{\pi} \frac{e^{-\sigma_{n-1}s_{n-1}}}{1+e^{-s_{n-1}}} \dots$$

$$\dots \frac{\sin\pi\sigma_2}{\pi} \frac{e^{-\sigma_2 s_2}}{1+e^{-s_2}} \frac{\sin\pi\sigma_1}{\pi} \frac{e^{-\sigma_1(s_1+i\varphi_0)}}{1+e^{-s_1-i\varphi_0}} \quad , \qquad (17)$$

$$s_j = \ln(t_j r_{j-1}/t_{j-1} r_j) \quad , \quad 1 \leq j \leq n \quad , \qquad (18)$$

where $r_j = |c_{j+1} - c_j|$, $0 \leq j \leq n$, and (r_n,φ) (resp. (r_0,φ_0)) are the polar coordinates of x (resp x_0) with respect to the center c_n (resp. c_1) ; the sum is over all finite sequences $\gamma = (c_n,\dots,c_1)$, $c_j \in \{a,b\}$, $c_j \neq c_{j+1}$ and $\sigma_j = \alpha$ (resp. β) provided $c_j = a$ (resp. b) . Again $c_0 = x_0$, $c_{n+1} = x$. The substitution (18) together with the relation

$$d^n s = t\, \delta(t_n+\dots+t_0-t)\, \vartheta(t_n) \dots \vartheta(t_0)\, (t_n \dots t_0)^{-1}\, dt_n \dots dt_0$$

enables us to rewrite

$$K_t(x,x_0) = K_t^a(x,x_0) + K_t^b(x,x_0) - K_t^0(x,x_0) +$$

$$+ \frac{\mu}{2\pi i\hbar t} \sum_{\gamma, n\geq 2} (-1)^n \int_{\mathbb{R}^n} d^n s\; e^{i\mu R_\gamma^2(s)/2\hbar t}\; S_\gamma(s;\varphi,\varphi_0) \quad , \qquad (19)$$

where

$$R_\gamma^2(s) = (r_0 + r_1 e^{s_1} + \dots + r_n e^{s_1+\dots+s_n}) \times$$

$$\times (r_0 + r_1 e^{-s_1} + \dots + r_n e^{-s_1-\dots-s_n}) \quad . \qquad (20)$$

The Laplace transformation applied to (19) gives the Green's function

$$G_z(x,x_0) = G_z^a(x,x_0) + G_z^b(x,x_0) - G_z^0(x,x_0) +$$

$$+ \frac{\mu}{\pi\hbar^2} \sum_{\gamma, n\geq 2} (-1)^n \int_{\mathbb{R}^n} d^n s\; K_0(w\, R_\gamma(s))\; S_\gamma(s;\varphi,\varphi_0) \quad . \qquad (21)$$

Starting from (16) we get another form

$$G_z(x,x_0) = G_z^a(x,x_0) + G_z^b(x,x_0) - G_z^0(x,x_0) +$$

$$+ \frac{\mu}{\pi\hbar^2} \sum_{\gamma, n\geq 2} (-1)^n \int_{\mathbb{R}^n} d^n\tau\; K_{i\tau_n}(wr_n)\, K_{i(\tau_{n-1}-\tau_n)}(w\rho) \dots$$

$$\dots K_{i(\tau_1-\tau_2)}(w\rho)\, K_{-i\tau_1}(wr_0)\; T_\gamma(\tau;\varphi,\varphi_0) \quad , (22)$$

where

$$T_\gamma(\tau;\varphi,\varphi_0) = \frac{\sin(\pi\sigma_n)\, e^{\varphi\tau_n}}{\pi \sin[\pi(\sigma_n+i\tau_n)]} \frac{\sin\pi\sigma_{n-1}}{\sin[\pi(\sigma_{n-1}+i\tau_{n-1})]} \dots$$

$$\dots \frac{\sin\pi\sigma_2}{\pi\sin[\pi(\sigma_2+i\tau_2)]} \frac{\sin(\pi\sigma_1)\, e^{-\varphi_0\tau_1}}{\pi\sin[\pi(\sigma_1+i\tau_1)]} \quad . \qquad (23)$$

5. CONCLUDING REMARK

One can verify directly the basic properties of the Green's function ($x_0 \in \mathbb{R}^2 \setminus L$ fixed) :

(i) $\left((\hbar^2/2\mu)\Delta + z \right) G_z(x,x_0) = -\delta(x - x_0)$ on $\mathbb{R}^2 \setminus L$,

(ii) $\psi(x) = G_z(x,x_0)$ satisfies the boundary condition (10) on L ,

(iii) $G_z(x,x_0)^* = G_{\bar z}(x_0,x)$.

This can be done owing to the asymptotics

$$K_\nu(z) = \sqrt{\pi/2z}\; e^{-z} \left[1 + O(z^{-1}) \right] \quad \text{for} \quad z \to \infty,$$

$|\arg z| < 3\pi/2$. In the Abelian case one can use the expression (21), the identity

$$\int_{-\infty}^{\infty} e^{-\sigma s}\, (1 + e^{-s})^{-1}\, ds = \pi/\sin\pi\sigma \quad , \quad 0 < \sigma < 1 \quad ,$$

and the estimate

$$R_\gamma(s) \geq r_0 + r_1 + \dots + r_n = r_0 + r_n + (n-1)\rho \quad .$$

In the non-Abelian case one can use the expression (22) and the estimate

$$\| \sin \pi\sigma / \sin[\pi(\sigma+i\tau)] \|_{\mathbb{C}^{N,N}} \leq 1/\mathrm{ch}\, \pi\tau \quad .$$

But now the properties (i - iii) are proved only for $|z|$ large enough. In this case the order of multipliers in (17), (23) is essential.

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