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COMPUTER CLASSIFICATION OF INTEGRABLE COUPLED KdV - LIKE SYSTEMS

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1.Introduction

At present intensive work on testing of integrability and classification of integrable nonlinear evolution systems

$$U_{t} = F(x, U, U_{1}, \dots, U_{N}), \quad U = U(x, t) = (U^{1}, \dots, U^{M})$$

$$F = (F^{1}, F^{2}, \dots, F^{M}), \quad U_{t} = D^{1}(U), \quad D = d/dx$$
(1)

is carried out. The integrability means that the system (1) can either be reduced to a linear one by differential substitutions $U = P(V, \dots, V_k)$ or can be integrated by inverse spectral transform. Prototype equations of these two groups are the Burgers and the Korteveg-de Vries (KdV) equations, respectively. In both cases the initial nonlinear problem reduces to a linear one that can be investigated and solved. Note that evolution systems of the second type are especially interesting in physics due to their multi-soliton solutions. In the present paper we use computer algebra to find all integrable coupled KdV - like evolution systems.

2.Symmetry approach

The classification problem consists in obtaining a complete list of integrable systems (1) for some fixed N and to describe the most general transforms connecting these systems. It can be achieved effectively by using the symmetry approach (see reviews (1-3)). In the framework of the symmetry approach, the integrable criterium is based on the property of (1) to have an infinite algebra of higher-order symmetries, i.e. the evolution systems of the form

$$U_{\pm}=H(x,U,U_{\pm},...,U_{\pm}), H=(H^{1},...,H^{M}), n>N,$$
 (2)

compatible with (1). The compatibility condition can be written as



$$\frac{dH}{dt} = \mathbb{P}_{*}(H)$$

where F, is the matrix differential operator

$$\mathbf{F}_{*} = \sum_{\mathbf{i}=0}^{N} \mathbf{F}_{\mathbf{i}} \mathbf{D}^{\mathbf{i}}, \quad |\mathbf{F}_{\mathbf{i}}|_{\mathbf{j}\mathbf{k}} = \frac{\partial \mathbf{F}^{\mathbf{i}}}{\partial \mathbf{U}_{\mathbf{j}}^{\mathbf{k}}}$$

It is shown in [1,2] that in the case where the evolution system (1) has an infinite algebra of symmetries, it has an infinite number of local conservation laws

$$\frac{dR_{1j}}{dt} = \frac{dQ_{1j}}{dx}, \quad 1=1,2,\ldots, \quad j=1,\ldots,M, \quad (3)$$

where the densities R_{ij} can be expressed in terms of F and Q_{ik} (k<j). The conditions (3) generate an overdetermined system of equations in F. By solving this system it is possible to find a list of concrete F's containing all the integrable cases (usually it is sufficient to use conditions (3) for i<3). This list is then checked for higher i against the condition (3) in order to remove non-integrable cases. The next step consists in finding higher-order symmetries and a Lax representation.

3.Role of Computer Algebra

The algorithms for the most tedious steps of classification, such as computing the densities $R_{i,j}$, checking the conditions (3), derivation of the overdetermined systems in F and finding the symmetries, have been suggested in [4,5]. They have been implemented using the computer algebra system FORMAC for scalar equations (M=1 in (1)) [4] and for the following wide class of systems [5]

$$U_{t} = \Lambda U_{N} + f(\mathbf{x}, \mathbf{U}, \mathbf{U}_{1}, \dots, \mathbf{U}_{N-1}), \quad \mathbf{f} = (\mathbf{f}^{1}, \dots, \mathbf{f}^{M})$$

$$(4)$$

$$\Lambda = \operatorname{diag}(\lambda_{1}, \dots, \lambda_{M}), \quad \lambda_{1} \neq 0, \quad \lambda_{4} \neq \lambda_{4} (\mathbf{1} \neq \mathbf{J}), \quad \lambda_{4} \in \mathfrak{C} \quad .$$

The step in our algorithm which remains to be done by hand is to solve the overdetermined system of differential equations in P. However, in a special case (which is very important for applications), the $F^1, \ldots F^M$ are polynomials in $U, U^1, \ldots U^N$ with certain homogeneity properties, and the overdetermined system in F reduces to an algebraic system for the coefficients of these polynomials. The general approach to solve such a system exactly is based on the well-known technique of Groebner basis construction [6] which is implemented, for example, in the last version of the computer algebra system REDUCE [7]. An example of a Groebner basis computation for one of the classification problems [8] is given in [9]. Thus the classification of the integrable evolution systems (4) with homogeneously-polynomial right hand sides can be completely automated by means of computer algebra.

4.Coupled KdV - Like Systems

In this paper we apply the above technique to the classification of the following systems from class (4) (coupled KdV-like systems)

$$u_{t} = a_{0}u_{3} + a_{1}uu_{1} + a_{2}vv_{1} + a_{3}uv_{1} + a_{4}vu_{1}, \quad a_{0} \neq b_{0}, \quad a_{0} \neq 0, \quad (5)$$

$$v_{t} = b_{0}v_{3} + b_{1}vv_{1} + b_{2}uu_{1} + b_{3}vu_{1} + b_{4}uv_{1}, \quad a_{1}, b_{1} \in \mathbb{C} \quad (1=0+5) \quad (5)$$

which have an infinite algebra of symmetries. Using the FORMAC program described in [5] we obtain a system of equations for the parameters a_1, b_j consisting of twelve equations of sixth degree in ten unkowns obtained from (3) for 1=1+4 and in part for 1=5

 $e_2 = (2a_3 - a_4)y_1 - b_2y_2$, $y_1 = 6a_0a_3b_2 + (a_0 - b_0)(a_1^2 + a_4b_2)$,

$$e_{r}=e_{r}=0, (k=1+6),$$
 (6

where $\hat{e}_{k} = e_{k} | a_{i} \circ b_{i}$ and

 $e_1 = a_1 (a_3 - a_4) - a_4 (b_3 - b_4)$,

 $e_{3} = a_{2}y_{1} - (2b_{3} - b_{4})y_{2}, \qquad y_{2} = 6a_{0}a_{2}b_{3} + (a_{0} - b_{0})(a_{1}a_{2} + a_{4}b_{1}),$ $e_{4} = 3a_{0}(a_{2}b_{2} + a_{3}b_{3}) + (a_{0} - b_{0})(a_{1} + b_{3})a_{4},$ $e_{5} = 2(2a_{0}^{2} + 8a_{0}b_{0} - b_{0}^{2})a_{3}b_{3} + 2(a_{0} - b_{0})(4a_{0} - b_{0})a_{3}b_{4} - 6a_{0}(a_{0} + 2b_{0})a_{2}b_{2}$ $+ (a_{0} - b_{0})^{2}(5a_{1}a_{3} - 5a_{1}a_{4} + a_{4}b_{4}) - (a_{0} - b_{0})(7a_{0} - b_{0})a_{4}b_{3},$

 $e_{6} = 3a_{0}[(a_{0}-b_{0})^{3}-3a_{0}(a_{0}+2b_{0})^{2}](a_{2}b_{2}+a_{3}b_{3})+(a_{0}-b_{0})^{3}[3a_{0}a_{1}a_{3}-2(2a_{0}+b_{0})a_{1}a_{4}] \\ +9a_{0}^{2}(a_{0}-b_{0})[(a_{0}-b_{0})a_{4}-(a_{0}+2b_{0})a_{3}]b_{4}-(a_{0}-b_{0})(2a_{0}^{3}-30a_{0}^{2}b_{0}+b_{0}^{3})a_{4}b_{3}.$

5.Solving the System (6)

In order to solve system (6) one may use the technique of Groebner basis. However, we have used instead a much more effective algorithm [10] which exploits the special structure of the system (6). Its main idea is to consider several alternative cases:

$$a_1 \neq 0, a_4 \neq 0$$
 2) $a_1 \neq 0, a_4 = 0$ 3) $a_1 = 0, a_4 \neq 0$ 4) $a_1 = a_4 = 0$

and two subcases inside each case:

a) $y_1 \neq 0$ or $y_2 \neq 0$ b) $y_1 = y_2 = 0$.

One can find simple relations connecting a_1, b_1 for each subcase and thus the system (6) can be considerably simplified. For example, in the case 1), the equations

e₁=ê₁=0

lead to two possibilities:

 $1)a_3=a_4,b_3=b_4$ $11)a_3\neq a_4$ or $b_3\neq b_4$, $a_1b_1=a_4b_4$.

In the case 11) we can set $a_1=a_4$ and $b_1=b_4$ by taking into account

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$$v_{t} = b_{0}v_{3} + b_{1}vv_{1} + b_{2}uu_{1} + b_{3}vu_{1} + b_{4}uv_{1}, \quad a_{1}, b_{1} \in \mathbb{C} (1 = 0 + 5) ,$$

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$$e_k = \hat{e}_k = 0, \quad (k = 1 + 6), \quad (6)$$

where
$$e_{k} = e_{k} | a_{1} \circ b_{1}$$
 and
 $e_{1} = a_{1} (a_{3} - a_{4}) - a_{4} (b_{3} - b_{4}),$
 $e_{2} = (2a_{3} - a_{4})y_{1} - b_{2}y_{2},$ $y_{1} = 6a_{0}a_{3}b_{2} + (a_{0} - b_{0})(a_{1}^{2} + a_{4}b_{2})$

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 $e_3 = a_2 y_1 - (2b_3 - b_4) y_2$, $y_2 = 6a_0 a_2 b_3 + (a_0 - b_0) (a_1 a_2 + a_4 b_1)$,

 $e_4 = 3a_0(a_2b_2 + a_3b_3) + (a_0 - b_0)(a_1 + b_3)a_4$,

 $e_5 = 2(2a_0^2 + 8a_0b_0 - b_0^2)a_3b_3 + 2(a_0 - b_0)(4a_0 - b_0)a_3b_4 - 6a_0(a_0 + 2b_0)a_2b_2$

 $+(a_0^{-}b_0^{-})^2(5a_1a_3^{-}5a_1a_4^{+}a_4b_4^{-})-(a_0^{-}b_0^{-})(7a_0^{-}b_0^{-})a_4b_3^{-}$

$$\begin{split} & e_6 = 3a_0 [(a_0 - b_0)^3 - 3a_0 (a_0 + 2b_0)^2] (a_2 b_2 + a_3 b_3) + (a_0 - b_0)^3 [3a_0 a_1 a_3 - 2(2a_0 + b_0) a_1 a_4] \\ & + 9a_0^2 (a_0 - b_0) [(a_0 - b_0) a_4 - (a_0 + 2b_0) a_3] b_4 - (a_0 - b_0) (2a_0^3 - 30a_0^2 b_0 + b_0^3) a_4 b_3 . \end{split}$$

5.Solving the System (6)

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$$1)a_1 \neq 0, a_4 \neq 0$$
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$$a_3=a_4, b_3=b_4$$
 11) $a_3\neq a_4$ or $b_3\neq b_4$, $a_1b_1=a_4b_4$.

In the case ii) we can set $a_1=a_4$ and $b_1=b_4$ by taking into account

the invariance of the ten-parametric family (5) under the scales transformations $u \Rightarrow \alpha u$, $v \Rightarrow \beta v$ ($\alpha, \beta \in \mathbb{C}$). On the other hand, the subcase a) implies $a_2b_2=(2a_3-a_4)(2b_3-b_4)$, so a_2,b_2 can immediately be eliminated from the equations (6) for k=3+6, etc. Applying this method and carring out all computations in the interactive mode of the computer algebra system REDUCE, we have found all the non-trivial solutions of (6) (see ref.[10] fore more details). It should be noted that in each alternative case the problem reduces to simple gcd and resultant computations which are built-in in REDUCE. It turns out that in the most tedious case 1) $a_1\neq 0, a_4\neq 0$ the system (6) has a single non-trivial solution (up to a scale transformation)

 $a_{0} = (3\pm\sqrt{5})/6, a_{1} \neq 0, a_{2} = a_{1}(9a_{0}-7)/(12a_{0}-1), a_{3} = a_{1}/(3a_{0}), a_{4} = a_{1},$ (7) $b_{0} = (-3\pm\sqrt{5})/6 \quad b_{1} = -a_{1}/(3a_{0}), b_{2} = a_{1}(3a_{0}+1)/(9a_{0}-7), b_{3} = -a_{1}, b_{4} = -a_{1}/(3a_{0}).$

The evolution system (5) with the coefficients (7) can be transformed to the well-known integrable Drinfeld-Sokolov system [11] by appropriate linear transformation of the vector space (u,v). For the cases 2)+4), we have obtained a list of four nonlinear coupled systems of the form (5) with coefficients a_i, b_i satisfying (6) and containing four to six arbitrary constants.

6.List of Integrable Systems.Conclusion

Using our FORMAC program [5] we have checked the list obtained whether or not conditions (3) for 1=5+8 are satisfied. We have found that only three evolution systems satisfy these conditions, namely

$$u_{t} = u_{3} + uu_{1} + vv_{1}, v_{t} = -2v_{3} - uv_{1}$$
 (8)

$$u_{t} = u_{3} + uu_{1}, \quad v_{t} = 4v_{3} + uv_{1} + 1/2 \quad u_{1}v$$
 (9)

 $u_{t}=u_{3}+uu_{1}, v_{t}=-2v_{3}-vu_{1}-v_{1}$ (10)

The system (8) is the well-known Hirota-Satsuma system [12] with an

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infinite algebra of symmetries. The system (9) was firstly considered in our paper [5]. Based on the generally accepted conjecture (see, for example, [3]) that the existance of higher-order symmetries implies integrability, we may conclude that the system (9) also has an infinite algebra of symmetries. In [5] we have found the following 5-order symmetry for the system (9)

 $u_{+}=u_{5}+5/3$ $u_{3}+10/3$ $u_{1}u_{2}+5/6$ $u^{2}u_{1}$,

 $v_{1}=16v_{1}+20/3 uv_{1}+5/2vu_{1}+10u_{1}v_{2}+25/3 v_{1}u_{2}+5/6 u^{2}v_{1}+5/6 vuu_{1}$

We believe that the system (10) is also integrable. It has the 5-order symmetry with the same first equation in accordance with the structure of (9) and (10)

 $u_{+}=u_{5}+5/3$ $u_{13}+10/3$ $u_{1}u_{2}+5/6$ $u^{2}u_{1}$,

 $v_{+}=-4v_{5}-10/3 uv_{3}-5/3 vu_{3}-20/3 u_{1}v_{2}-5v_{1}u_{2}-5/18 u^{2}v_{1}-5/9 uu_{1}$.

We may conclude from the above computations that computer algebra is a powerful tool for investigating nonlinear evolution equations. It allows to make a complete classification of the integrable coupled systems from the ten-parametric family (5). All integrable cases are exhausted by the four systems (7)+(10).

Authors are thankful to K.S.Kölbig and S.I.Svinolupov for useful discussions.

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 $a_{0} = (3 \pm \sqrt{5})/6, a_{1} \neq 0, a_{2} = a_{1}(9a_{0} - 7)/(12a_{0} - 1), a_{3} = a_{1}/(3a_{0}), a_{4} = a_{1}, (7)$ $b_{0} = (-3 \pm \sqrt{5})/6 \quad b_{1} = -a_{1}/(3a_{0}), b_{2} = a_{1}(3a_{0} + 1)/(9a_{0} - 7), b_{3} = -a_{1}, b_{4} = -a_{1}/(3a_{0}).$

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- $u_t = u_3 + uu_1, \quad v_t = 4v_3 + uv_1 + 1/2 \quad u_1 v$ (9)
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The system (8) is the well-known Hirota-Satsuma system [12] with an

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We believe that the system (10) is also integrable. It has the 5-order symmetry with the same first equation in accordance with the structure of (9) and (10)

 $u_t = u_5 + 5/3 u_3 + 10/3 u_1 u_2 + 5/6 u^2 u_1$, $v_+ = -4v_c - 10/3 u_{v_2} - 5/3 v_{u_3} - 20/3 u_1 v_2 - 5v_1 u_2 - 5/18 u^2 v_1 - 5/9 u_1$.

We may conclude from the above computations that computer algebra is a powerful tool for investigating nonlinear evolution equations. It allows to make a complete classification of the integrable coupled systems from the ten-parametric family (5). All integrable cases are exhausted by the four systems (7)+(10).

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Гердт В.П., Жарков А.Ю. Н Классификация на ЭВМ интегрируемых нелинейных эволюционных систем типа связанного уравнения Кортевега – де Вриза

Кратко описаны основы симметрийного подхода к классификации интегрируемых нелинейных эволюционных систем. В рамках этого подхода исследовано 10-параметрическое семейство нелинейных эволюционных систем третьего порядка типа связанного уравнения КдВ. Получены необходимые условия интегрируемости таких систем, сводящиеся к переопределенной системе нелинейных алгебраических уравнений на параметры исходного семейства. Использован эффективный метод решения полученной системы, опирающийся на ее структуру. Это позволило получить полный список интегрируемых систем рассматриваемого типа. Все вычисления были выполнены с помощью систем аналитических вычислений FORMAC и REDUCE.

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Gerdt V.P., Zharkov A.Yu. E5-89-232 Computer Classification of Integrable Coupled KdV - Like Systems

The foundations of the symmetry approach to the classification problem of integrable nonlinear evolution systems are briefly described. Within the framework of symmetry approach the ten-parametric family of the third order nonlinear evolution coupled KdV - like systems is investigated. The necessary integrability conditions lead to the overdetermined nonlinear algebraic system. To solve that system the effective method based on its structure had been used. This allows us to obtain the complete list of integrable systems of a given type. All computations has been completed on the basis of computer algebra systems FORMAC and REDUCE.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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