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# ON THE INVERSE PROBLEM OF A DISSIPATIVE SCATTERING THEORY. III

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## 1. Introduction

In this note we want to solve the so-called inverse scattering problem of an abstract dissipative scattering theory. Such a scattering theory was created in [6,7] and can be understood as an extension of the well-known scattering theory of self-adjoint operators [2] to maximal dissipative operators. The necessity of this generalization arises from the fact that in many scattering systems we have to do with dissipation of energy. To include such dissipation effects maximal dissipative operators are often used. An extensive reasoning concerning this subject can be found in [3].

In [6,7] it is assumed that both perturbed and free evolutions are governed by maximal dissipative operators which in general are defined on different separable Hilbert spaces. The comparison of the different evolutions is established by bounded identification operators. The notion of the wave and scattering operators is introduced. Summarizing it can be said that in [6,7] the so-called direct scattering problem of maximal dissipative operators was posed and solved on an abstract operator-theoretical level.

But every direct scattering problem yields the so-called inverse scattering problem. In general this means to restore the perturbed or full evolution knowing the free evolutions and the scattering operator. But this setting of the inverse problem immediately implies three further problems.

(i ) We must know the set of possible scattering operators for a given scattering theory.

(ii ) We have to indicate a certain algorithm allowing one to restore the full evolution.

(iii) We have to describe all full evolutions which solve the inverse problem or, if it is possible, to show that the solution is unique in a certain class of admissible full evolutions.

Problems of this kind can be posed in a concrete manner, for instance, for ordinary and partial differential operators, or in a more abstract manner, for instance, formulating the problem in a certain operator-theoretical language. The inverse problems of the Lax-Phillips scattering theory with and without dissipation [1,4,10] belong to the last class, for example. We call problems of that type abstract inverse scattering problems.

For the scattering theory of selfadjoint operators [2] the abstract inverse scattering problem was solved by M. Wollenberg [2,14,15] who answered all three problems (i) - (iii). Naturally, the question arises to find a solution of the inverse problem in the scattering theory of maximal dissipative operators. Such attempts were undertaken in [8,9]. In [9] considering unitary free evolutions it was shown that every intertwining contraction of these unitary free groups obeying some obvious properties can be regarded as a scattering operator of a dissipative scattering theory. In such a way the problems (i) and (ii) were solved.

In this paper we do a furhter step allowing that free evolutions are contraction semigroups. In this setting we solve (i) and (ii). The problem (iii) is not considered. In order to use the definitions and notions of [13] we prefer contractions instead of maximal dissipative operators. This means, we replace the one-parameter contraction semigroups used in [8,9] by power semigroups of single contractions.

It is found that not every intertwining contraction of two contractions can be regarded as a scattering operator of a



dissipative scattering theory. To this end it is necessary that the intertwining contraction fulfils some additional properties. Applying this result to special free evolutions we obtain that the class of contractive Hankel operators can be viewed as scattering operators. Moreover, this implies the possibility that the scattering operator can be compact, for instance, nuclear which was forbidden for unitary free evolutions. At the end, we give an application to the so-called Lax-Phillips scattering theory with losses restoring a result of B.S.Pavlov [12] in a quite different way.

## 2. Preliminaries

Let $T$ be a contraction on the separable Hilbert space $\mathcal{H}$. By $U$ we denote the minimal unitary dilation of $T$ defined on the dilation space $\mathcal{K}$, $\mathcal{H} \subseteq \mathcal{K}$. In accordance with [13, chapter II] we can introduce the residual and dual residual subspaces $\mathcal{R}$ and $\mathcal{R}_*$ of $U$. Taking into account Proposition 3.1 of [13, chapter II] we obtain that the orthogonal projections $P_{\mathcal{R}}$ and $P_{\mathcal{R}_*}$ from $\mathcal{K}$ onto $\mathcal{R}$ and $\mathcal{R}_*$ admit the representation

$$P_{\mathcal{R}} = \operatorname*{s-lim}_{n\to+\infty} U^n P_{\mathcal{H}} U^{-n} \tag{2.1}$$

and

$$P_{\mathcal{R}_*} = \operatorname*{s-lim}_{n\to+\infty} U^{-n} P_{\mathcal{H}} U^{n} , \tag{2.2}$$

where $P_{\mathcal{H}}$ denotes the orthogonal projection from $\mathcal{K}$ onto $\mathcal{H}$.

The residual and dual residual subspaces reduce the unitary operator $U$. We denote the residual and dual residual parts of $U$ by $R$ and $R_*$, i.e. $R = U \restriction \mathcal{R}$ and $R_* = U \restriction \mathcal{R}_*$.

Remark 2.1. It is quite possible that the residual or the dual residual or both subspaces are zero. For instance, $\mathcal{R}_* = \{0\}$ if and only if $T^n \to 0$ strongly as $n \to +\infty$, i.e. $T \in C_{0.}$.

In accordance with Theorem 3.2 of [13, chapter I] every contraction can be canonically decomposed into a unitary part and a completely non-unitary part. In the following the subspace performed by the orthogonal sum of the absolutely continuous subspace of the unitary part and the completely non-unitary subspace is called the absolutely continuous one of a contraction. Obviously, the absolutely continuous subspace reduces a contraction. The corresponding part of a contraction is called the absolutely continuous one. If the absolutely continuous part of a contraction coincides with the contraction itself we call the contraction an absolutely continuous one. For instance, every completely non-unitary contraction is absolutely continuous.

This concept of absolute continuity for contractions agrees very well with that for unitary operators. So it can be shown that the minimal unitary dilation of an absolutely continuous contraction is absolutely continuous (Proposition 6.3 of [13, chapter II]). Consequently, denoting by $\mathcal{K}^{ac}(U)$ the absolutely continuous subspace of the minimal unitary dilation U of T we obtain

$$(2.3) \qquad \mathcal{H}^{ac}(T) = \mathcal{H} \cap \mathcal{K}^{ac}(U) \subseteq \mathcal{K}^{ac}(U),$$

where $\mathcal{H}^{ac}(T)$ is the absolutely continuous subspace of T. The relation (2.3) yields that the absolutely continuous part $U^{ac}$ of U is a minimal unitary dilation of the absolutely continuous part $T^{ac}$ of T.



Furthermore, we can introduce the absolutely continuous residual and dual residual subspaces and parts of a minimal unitary dilation. Obviously, these subspaces can be regarded as the residual and dual residual subspaces of the minimal unitary dilation of the absolutely continuous part $T^{ac}$ of T.

In order to consider a scattering theory we introduce two further contractions $T_-$ and $T_+$ defined on the separable Hilbert spaces $\mathcal{H}_-$ and $\mathcal{H}_+$ which we call the past and future free evolutions, respectively. For simplicity we assume throughout this note that these contractions are absolutely continuous.

Further we assume the existence of two bounded linear operators $J_\pm: \mathcal{H}_\pm \to \mathcal{H}$ which we call the identification operators. We define the wave operators $W_\pm: \mathcal{H}_\pm \to \mathcal{H}$ by

$$(2.4) \qquad W_- = \operatorname*{s-lim}_{n\to+\infty} T^{*n} J_- T_-^n$$

and

$$(2.5) \qquad W_+ = \operatorname*{s-lim}_{n\to+\infty} T^n J_+ T_+^{*n} .$$

We introduce the minimal unitary dilations $U_\pm$ of $T_\pm$ defined on the dilation spaces $\mathcal{K}_\pm$. Extending the identification operators $J_\pm$ to operators $\bar{J}_\pm$ acting from $\mathcal{K}_\pm$ into $\mathcal{K}$ by $\bar{J}_\pm f = J_\pm P_{\mathcal{H}_\pm} f \in \mathcal{K}$, $f \in \mathcal{K}_\pm$, we are able to consider the dilation wave operators $\Omega_\pm$,

$$(2.6) \qquad \Omega_\pm = \operatorname*{s-lim}_{n\to\pm\infty} U^n \bar{J}_\pm U_\pm^{-n} .$$

It can be shown that if the dilation wave operator $\Omega_+$ ($\Omega_-$) exists, then the operator actually acts only from the (dual) residual subspace of $U_+$ ($U_-$) into the absolutely continuous (dual) residual space of U, i.e. $\ker(\Omega_+) \supseteq \mathcal{H}_+ \ominus \mathcal{R}(U_+)$, $\mathrm{ima}(\Omega_+) \subseteq \mathcal{R}^{ac}$ (i.e. $\ker(\Omega_-) \supseteq \mathcal{H}_- \ominus \mathcal{R}_*(U_-)$, $\mathrm{ima}(\Omega_-) \subseteq \subseteq \mathcal{R}_*^{ac}$).

Definition 2.2. The wave operator $W_+$ ($W_-$) is called complete if

(i ) the dilation wave operator $\Omega_+$ ($\Omega_-$) exists and

(ii) $\Omega_+$ ($\Omega_-$) is a partial isometry from the(dual) residual subspace of $U_+$ ($U_-$) into the absolutely continuous (dual) residual subspace of U, i.e $\ker(\Omega_+) = \mathcal{H}_+ \ominus \mathcal{R}(U_+)$ and $\mathrm{ima}(\Omega_+) = \mathcal{R}^{ac}$ (i.e. $\ker(\Omega_-) = \mathcal{H}_- \ominus \mathcal{R}_*(U_-)$ and $\mathrm{ima}(\Omega_-) = \mathcal{R}_*^{ac}$).

We note that the completeness of $W_\pm$ does not mean in general the completeness of the dilation wave operators. This is the case only if all involed contractions are unitary operators.

Furthermore, it is interesting to remark that the completeness of $W_+$ ($W_-$) yields that the (dual) residual part of $U_+$ ($U_-$) and the absolutely continuous (dual) residual part of U are unitarily equivalent.

Now we say the 5-tuple $\mathcal{A} = \{T; T_+, T_-; J_+, J_-\}$ forms a complete scattering system if the wave operators $W_\pm$ exist and are complete.

With every complete scattering system $\mathcal{A}$ we associate a scattering operator S defined by

(2.7 ) $$S = W_+^* W_-$$

and a dilation scattering operator $\Sigma$ defined by

(2.8 ) $$\Sigma = \Omega_+^* \Omega_- .$$



Obviously, the dilation scattering operator intertwines the minimal unitary dilations $U_+$ and $U_-$, i.e.

(2.9 ) $$U_+ \Sigma = \Sigma U_- .$$

Taking into account Definition 2.2 the dilation scattering operator is contraction which actually acts from the dual residual subspace of $U_-$ into the residual subspace of $U_+$, i.e.

(2.10) $$\ker(\Sigma) \supseteq \mathcal{H}_- \ominus \mathcal{R}_*(U_-)$$

and

(2.11) $$\mathrm{ima}(\Sigma) \subseteq \mathcal{R}(U_+).$$

Further it is useful to note that the scattering operator is the compression of the dilation scattering operator, i.e.

(2.12) $$S = \mathrm{pr}(\Sigma) = P_{\mathcal{H}_+} \Sigma \upharpoonright \mathcal{H}_- .$$

From this representation it immediately follows that the scattering operator is also a contraction. Taking into account (2.7) we obtain that S is an intertwining contraction of $T_+$ and $T_-$, i.e.

(2.13) $$ST_+ = T_- S.$$

Since we have two scattering operators we obtain two inverse scattering problems which can be formulated as follows: Assume that the identification operators $J_\pm$ and the free evolutions $T_\pm$ are given.

A) Let $S: \mathcal{H}_- \longrightarrow \mathcal{H}_+$ be an intertwining contraction of $T_+$ and $T_-$. Does there exist a contraction $T$ on $\mathcal{H}$ such that $\mathcal{S} = \{T;T_+,T_-;J_+,J_-\}$ forms a complete scattering system whose scattering operator coincides with $S$?

B) Let $\Sigma: \mathcal{K}_- \longrightarrow \mathcal{K}_+$ be an intertwining contraction of $U_+$ and $U_-$ obeying (2.10) and (2.11). Does there exist a contraction $T$ on $\mathcal{H}$ such that $\mathcal{S} = \{T;T_+,T_-;J_+,J_-\}$ forms a complete scattering system whose dilation scattering operator coincides with $\Sigma$ ?

First of all we note that a solution of the proposed problems can be expected only if the identification operators satisfy certain conditions.

Definition 2.3. We say the identification operators $J_+$ and $J_-$ are admissible with respect to $T_+$ and $T_-$ if there are two isometries $F_\pm: \mathcal{H}_\pm \longrightarrow \mathcal{H}$ such that

(i ) $F_+^* F_- = 0$,

(ii ) $\underset{n\to+\infty}{\text{s-lim}}\, (F_- - J_-)T_-^n = 0$,

(iii) $\underset{n\to+\infty}{\text{s-lim}}\, (F_+ - J_+)T_+^{*n} = 0$.

For further applications we make the following

Remark 2.4. Let $T_+ = T_- \equiv T_0$, $\mathcal{H}_+ = \mathcal{H}_- \equiv \mathcal{H}_0$ and $J_+ = J_- \equiv J_0$. Then $J_0$ and $J_0$ are admissible with respect to $T_0$ and $T_0$ if and only if there is an isometry $F_0: \mathcal{H}_0 \longrightarrow \mathcal{H}$ such that

$\underset{n\to+\infty}{\text{s-lim}}\, (F_0 - J_0)T_0^n = 0$ and $\underset{n\to+\infty}{\text{s-lim}}\, (F_0 - J_0)T_0^{*n} = 0$.

It can be shown that $J_+$ and $J_-$ are admissible with respect to $T_+$ and $T_-$ if $\mathcal{S} = \{T;T_+,T_-;J_+,J_-\}$ is a complete scattering system.

Immediately from Theorem 2.4 of [9] we obtain the following

Theorem 2.5. Let $T_\pm$ be two absolutely continuous unitary operators on $\mathcal{H}_\pm$ and let $J_+$ and $J_-$ be two identification operators



which are admissible with respect to $T_+$ and $T_-$. If $S$ is an intertwining contraction of $T_+$ and $T_-$, $T_+S = ST_-$, then there is a contraction $T$ such that $\mathcal{S} = \{T;T_+,T_-;J_+,J_-\}$ forms a complete scattering system whose scattering operator coincides with $S$.

Proof. We apply Theorem 2.3 of [9] to $L_\pm = \int_0^{2\pi} \lambda \; dE_\pm(\lambda)$ and $F_\pm$, where $E_\pm(.)$ are the spectral measures of $T_\pm$. Obviously, $S$ intertwines $L_+$ and $L_-$. Moreover, $F_+$ and $F_-$ are admissible with respect to $L_+$ and $L_-$. Consequently, there is a maximal dissipative operator $H$ on $\mathcal{H}$ such that $\mathcal{S}' = \{H;L_+,L_-;F_+,F_-\}$ forms a complete scattering system whose scattering operator equals $S$. We set $T = e^{iH^*}$. Now it is not hard to show that the 5-tuple $\mathcal{S} = \{T;T_+,T_-;J_+,J_-\}$ solves the problem. ■

Corollary 2.6. If in addition $S$ fulfils $\ker(S) = \{0\}$ and $(\text{ima}(S))^- = \mathcal{H}_+$, then $T$ can be chosen from $C_{11}$. Moreover, if $S$ is an isometry from $\mathcal{H}_-$ onto $\mathcal{H}_+$, then $T$ can be taken from the unitary operators on $\mathcal{H}$.

Corollary 2.6 is a consequence of the Remarks 2.5 and 2.6 of [9]. We note that the additional conditions of Corollary 2.6 are necessary if we assume that the scattering operator arises from a scattering system with a full evolution of the indicated classes.

In the following the results will be essentially based on Theorem 2.5 and Corollary 2.6.

## 3. Inverse problem

In this section we try to extend Theorem 2.5 to the case that $T_+$ and $T_-$ are arbitrary absolutely continuous contractions. To this end we remark that in distinction from Theorem 2.5

every intertwining contraction cannot be regarded as a scattering operator.

Example 3.1. We consider the Hardy spaces $\mathcal{H}_{\pm} = H^2(\mathbb{T}, \mathcal{N}_{\pm})$ [13, chapter V], where $\mathbb{T}$ is the unit circle, i.e. $\mathbb{T} = \{z \in \mathbb{C}: |z| = 1\}$, and $\mathcal{N}_{\pm}$ are separable Hilbert spaces. We view $\mathcal{H}_{\pm}$ as subspaces of $L^2(\mathbb{T}, \mathcal{N}_{\pm})$. On $\mathcal{H}_{\pm}$ we introduce the shift operators $T_{\pm}$ defined by $(T_{\pm}f)(z) = zf(z)$, $f \in \mathcal{H}_{\pm}$, $z \in \mathbb{T}$. The minimal unitary dilations of $T_{\pm}$ obviously coincide with the shift operators $U_{\pm}$ on $\mathcal{K}_{\pm} = L^2(\mathbb{T}, \mathcal{N}_{\pm})$ given by $(U_{\pm}f)(z) = zf(z)$, $f \in \mathcal{K}_{\pm}$, $z \in \mathbb{T}$. Taking into account Lemma 3.2 of [13, chapter V] the condition

$$(3.1)\qquad T_+S = ST_-$$

yields the existence of a contractive analytic function $\{\mathcal{N}_-, \mathcal{N}_+; \theta(z)\}$ such that the representation

$$(3.2)\qquad (Sf)(z) = \theta(z)f(z),$$

$f \in \mathcal{H}_-$, holds. Hence there are contractions $S \neq 0$ obeying (3.1) However, this contraction $S \neq 0$ cannot be the scattering operator of a complete scattering system with the free evolutions $T_+$ and $T_-$. To this end we remark that the residual subspace of $U_+$ is zero. Consequently, the dilation scattering operator must be zero. Taking into account (2.11) the scattering operator must be zero which contradicts $S \neq 0$.

The condradiction of Example 3.1 was obtained by taking into account the condition (2.12). In the following we want to clarify the meaning of this condition.

We introduce the limits $D_+$,



$$(3.3)\qquad D_+ = (\text{s-lim}_{n\to+\infty} T_+^n T_+^{*n})^{1/2},$$

and $D_-$,

$$(3.4)\qquad D_- = (\text{s-lim}_{n\to+\infty} T_-^{*n} T_-^n)^{1/2},$$

which exist. Let $\mathcal{D}_{\pm} = (D_{\pm}\mathcal{H}_{\pm})^- \subseteq \mathcal{H}_{\pm}$. By

$$(3.5)\qquad G_+^*D_+f = D_+T_+^*f,$$

$f \in \mathcal{H}_+$, and

$$(3.6)\qquad G_-D_-f = D_-T_-f,$$

$f \in \mathcal{H}_-$, we associate two isometries $G_+^*$ and $G_-$ with $T_+$ and $T_-$. We call $G_+$ the associated co-isometry of $T_+$ and $G_-$ the associated isometry of $T_-$. Further, by $I(.,.)$ we denote the set of intertwining contractions of two bounded operators.

Lemma 3.2. Let $T_+$ and $T_-$ be two contractions on $\mathcal{H}_+$ and $\mathcal{H}_-$, respectively, and let $S$ be a contraction acting from $\mathcal{H}_-$ into $\mathcal{H}_+$. Then the following conditions are equivalent:

(i) $\exists\, \Sigma \in I(U_+, U_-)$ obeying $\ker(\Sigma) \supseteq \mathcal{H}_- \ominus \mathcal{R}_*(U_-)$ and $\operatorname{ima}(\Sigma) \subseteq \mathcal{R}(U_+)$ such that $S = pr(\Sigma)$.

(ii) $\exists\, \Gamma \in I(G_+, G_-)$ such that $S = D_+\Gamma D_-$.

(iii) $S \in I(T_+, T_-)$ and

$$(3.7)\qquad 2\mathrm{Re}(\sqrt{I-D_+^2}\, f, Sg) + \|Sg\|^2 \leq \|D_+f\|^2 + \|D_-g\|^2$$

for every $f \in \mathcal{H}_+$ and $g \in \mathcal{H}_-$.

(iv ) $S\in I(T_+,T_-)$ and

$$(3.8)\qquad 2\mathrm{Re}(S^*f,\sqrt{I-D_-^2}\,g)+\|S^*f\|^2\le\|D_+f\|^2+\|D_-g\|^2$$

for every $f\in\mathcal{H}_+$ and $g\in\mathcal{H}_-$.

Proof. We use the following proof scheme (i)⇔(ii)⇔{(iii), (iv)}.

(i)⇒(ii): We consider the linear operators $B_\pm:\mathcal{H}_\pm\longrightarrow\mathcal{K}_\pm$ defined by $B_+=P_{\mathcal{R}(U_+)}P_{\mathcal{H}_+}$ and $B_-=P_{\mathcal{R}_*(U_-)}P_{\mathcal{H}_-}$. Taking into account (2.1) and (2.2) and considering the polar decompositions of $B_\pm$ we get $B_\pm=V_\pm D_\pm$, where $V_+$ and $V_-$ are partial isometries, ranges of which are subspaces of the residual and dual residual subspaces of $U_+$ and $U_-$, respectively. Setting $\Gamma=V_+^*\Sigma V_-:\mathcal{D}_-\longrightarrow\mathcal{D}_+$ we find a contraction $\Gamma$ such that $S=D_+\Gamma D_-$. Because of $U_+^*B_+=B_+T_+^*=V_+G_+^*D_+$ and $U_-B_-=B_-T_-=V_-G_-D_-$ we immediately obtain $\Gamma\in I(G_+,G_-)$.

(ii)⇐(i): By $\overline{G}_\pm$ we denote the minimal unitary dilations of $G_\pm$ defined on the dilation spaces $\overline{\mathcal{D}}_\pm$, $\mathcal{D}_\pm\subseteq\overline{\mathcal{D}}_\pm$. Applying Proposition 2.2 of [13,chapter II] there is a contraction $\overline{\Gamma}\in I(\overline{G}_+,\overline{G}_-)$ such that $\Gamma=\mathrm{pr}(\overline{\Gamma})$. By $\overline{V}_+=\operatorname*{s-lim}_{n\to+\infty}U_+^nV_+P_{\mathcal{D}_+}\overline{G}_+^{-n}$, $\overline{V}_-=\operatorname*{s-lim}_{n\to+\infty}U_-^{-n}V_-P_{\mathcal{D}_-}\overline{G}_-^{n}$ we define isometrical extensions of $V_\pm$, $\overline{V}_\pm\restriction\mathcal{D}_\pm=V_\pm$, ranges of which are subspaces of the residual and dual residual subspaces of $U_+$ and $U_-$, respectively. Obviously, we have $U_+^*\overline{V}_+=\overline{V}_+\overline{G}_+^*$ and $U_-\overline{V}_-=\overline{V}_-\overline{G}_-$. Setting $\Sigma=\overline{V}_+\overline{\Gamma}\,\overline{V}_-^*:\mathcal{K}_-\longrightarrow\mathcal{K}_+$ we get a contraction belonging to $I(U_+,U_-)$ and obeying $\ker(\Sigma)\supseteq\mathcal{K}_-\ominus\mathcal{R}_*(U_-)$ and $\mathrm{ima}(\Sigma)\subseteq\mathcal{R}(U_+)$. The simple calculation $S=D_+\Gamma D_-=D_+\overline{\Gamma}D_-=D_+V_+^*\overline{V}_+\overline{\Gamma}\,\overline{V}_-^*V_-D_-=P_{\mathcal{H}_+}\Sigma\restriction\mathcal{H}_-=\mathrm{pr}(\Sigma)$ completes this part of the proof.

(ii)⇒(iii): Because of $\Gamma\in I(G_+,G_-)$, (3.5) and (3.6) we ob-



tain $S\in I(T_+,T_-)$. Further we estimate

$$2\mathrm{Re}(\sqrt{I-D_+^2}\,f,Sg)+\|Sg\|^2=$$

$$(3.9)\qquad =2\mathrm{Re}(D_+f,\sqrt{I-D_+^2}\,\Gamma D_-g)+\|D_+\Gamma D_-g\|^2\le$$

$$\le\|D_+f\|^2+\|\Gamma D_-g\|^2\le\|D_+f\|^2+\|D_-g\|^2,$$

$f\in\mathcal{H}_+$, $g\in\mathcal{H}_-$.

(iii)⇒(ii): Setting $f=0$ we find $\|Sg\|^2\le\|D_-g\|^2$, $g\in\mathcal{H}_-$. Hence there is a contraction $X:\mathcal{D}_-\longrightarrow\mathcal{H}_+$ such that $S=XD_-$ (Corollary 7-2 of [5, p.125]) holds. We get

$$(3.10)\qquad 2\mathrm{Re}(\sqrt{I-D_+^2}\,f,Xh)+\|Xh\|^2\le\|D_+f\|^2+\|h\|^2,$$

$h\in\mathcal{D}_-$. Let $f=\sqrt{I-D_+^2}\,f'$, $f'\in\mathcal{H}_+$. From (3.10) we obtain

$$(3.11)\qquad \|D_+^2f'-Xh\|^2\le\|D_+f'\|^2-2\mathrm{Re}(X^*f',h)+\|h\|^2$$

which yields the estimate

$$(3.12)\qquad \|X^*f'\|^2\le\|D_+f'\|^2+\|X^*f'-h\|^2,$$

$f'\in\mathcal{H}_+$, $h\in\mathcal{D}_-$. Choosing $h=X^*f'$ we find $\|X^*f'\|\le\|D_+f'\|$, $f\in\mathcal{H}_+$. Using again Corollary 7-2 of [5,p.125] there is a unique contraction $Y:\mathcal{D}_-\longrightarrow\mathcal{D}_+$ such that $X^*=Y^*D_+$. Hence we obtain the representation $S=D_+YD_-$. It remains to show $Y\in I(G_+,G_-)$. But this follows from $ST_-=D_+YG_-D_-=T_+S=D_+G_+YD_-$.

(ii)⇔(iv): We establish this part of the proof applying the previous considerations to $S^* = D_- \Gamma^* D_+$. ∎

Corollary 3.3.

(v ) If in addition $\ker(T_-^*) = \{0\}$, then (i) - (iv) are equivalent to $\|Sg\| \leq \|D_-g\|$, $g \in \mathcal{H}_-$.

(vi) If in addition $\ker(T_+) = \{0\}$, then (i) - (iv) are equivalent to $\|S^*f\| \leq \|D_+f\|$, $f \in \mathcal{H}_+$.

Proof. (v)$\Rightarrow$(ii): Since $\ker(T_-^*) = \{0\}$ we have $(\mathrm{ima}(T_-))^- = \mathcal{H}_-$. Hence $G_-$ is a unitary operator on $\mathcal{H}_-$. Using the representation $S = XD_-$ we obtain $X \in I(T_+, G_-)$. Consequently, we find

$$(3.13) \qquad \|X^*f\| = \|G_-^{*n}X f\| = \|XT_+^{*n}f\| \leq \|T_+^{*n}f\|,$$

$n = 0,1,2,\ldots$ which implies $\|X^*f\| \leq \|D_+f\|$, $f \in \mathcal{H}_+$. Now we repeat the considerations of (iii)$\Rightarrow$(ii).

(ii)$\Rightarrow$(v): This part of the proof is obvious.

(vi)$\Leftrightarrow$(ii): We replace S by $S^*$. ∎

Remark 3.4. If S can be represented in accordance with (i) of Lemma 3.2, then S possesses a contractive intertwining dilation S [13] such that its unique extension to an intertwining contraction of the minimal unitary dilations $U_+$ and $U_-$ coincides with $\Sigma$. Consequently, conditions (iii) - (vi) describe a certain class of intertwining dilations with certain extension properties.

Remark 3.5. In general the representation $S = \mathrm{pr}(\Sigma)$ of (i) is not unique. Uniqueness can be obtained if $\ker(T_-^*) = \{0\}$ or $\ker(T_+) = \{0\}$.

Considering the inverse problem we have to answer the question A and B assuming that the identification operators $J_+$ and $J_-$ are admissible with respect to $T_+$ and $T_-$. Because of Example 3.1 the answer to A is in general not affirmative.



On account of Lemma 3.2 it is necessary to restrict the class of intertwining contractions by the condition (3.7) or (3.8). The problem B has in every case a solution.

Theorem 3.6. Let $T_\pm$ be two absolutely continuous contractions on $\mathcal{H}_\pm$ and let $J_+$ and $J_-$ be two identification operators which are admissible with respect to $T_+$ and $T_-$.

A) If $S \in I(T_+, T_-)$ obeys either (3.7) or (3.8), then there is a contraction T on $\mathcal{H}$ such that $\mathcal{A} = \{T; T_+, T_-; J_+, J_-\}$ is a complete scattering system whose scattering operator coincides with S.

B) If $\Sigma \in I(U_+, U_-)$ obeys $\ker(\Sigma) \supseteq \mathcal{K}_- \ominus \mathcal{R}_*(U_-)$ and $\mathrm{ima}(\Sigma) \subseteq \mathcal{R}(U_+)$, then there is a contraction T on $\mathcal{H}$ such that $\mathcal{A} = \{T; T_+, T_-; J_+, J_-\}$ is a complete scattering system, dilation scattering operator of which coincides with $\Sigma$.

Proof. To prove B we note that if $J_+$ and $J_-$ are admissible with respect to $T_+$ and $T_-$, then there are isometries $F_\pm: \mathcal{H}_\pm \longrightarrow \mathcal{H}$ which in addition to the conditions (i) - (iii) of Definition 3.2 fulfil

$$(3.14) \qquad \dim(\mathcal{H} \ominus (F_+\mathcal{H}_+ \oplus F_-\mathcal{H}_-)) = +\infty.$$

The proof of this refinement follows from the fact that for every absolutely continuous contraction $T_0$ there is a projection $P_0$ with $\dim(\mathrm{ima}(P_0)) = +\infty$ such that $\text{s-}\lim_{n\to+\infty} P_0T_0^n = 0$. We leave the proof of this assertion to the reader.

On account of (3.14) the isometries $F_\pm \restriction \mathcal{D}_\pm$ can be extended to isometries $\mathring{F}_\pm: \overline{\mathcal{D}}_\pm \longrightarrow \mathcal{H}$, $\mathring{F}_\pm \restriction \mathcal{D}_\pm = F_\pm \restriction \mathcal{D}_\pm$, ranges of which are orthogonal, i.e. $\mathring{F}_+^*\mathring{F}_- = 0$. Here and in the following we use the notation of Lemma 3.2.

Further we remark that the ranges of $\overline{V}_+$ and $\overline{V}_-$ are not only subspaces of the residual and dual residual subspaces of $U_+$ and $U_-$, respectively, but coincide with those. To prove this for $\overline{V}_-$ let $f \in \mathcal{R}_*(U_-) \ominus V_-\mathcal{D}_-$. We find

$$(3.15)\quad \begin{aligned} \|f\| &= \lim_{n\to+\infty} \|P_{\mathcal{K}_-} U_-^n f\| = \lim_{n\to+\infty} \|D_-V_-^*U_-^n f\| = \\ &= \lim_{n\to+\infty} \|D_-\overline{V}_-^*U_-^n f\| = \lim_{n\to+\infty} \|D_-\overline{G}_-^n\overline{V}_-^* f\| \leq \|\overline{V}_-^* f\| = 0 \end{aligned}$$

which implies $f = 0$.

We introduce the identification operators $\Pi_-: \mathcal{R}_*(U_-) \to \mathcal{H}$ and $\Pi_+: \mathcal{R}(U_+) \to \mathcal{H}$ defined by $\Pi_\pm = \mathring{F}_\pm\overline{V}_\pm^*$, which are isometries satisfying the condition $\Pi_+^*\Pi_- = 0$. Denoting by $R_+$ and $R_-$ the residual and dual residual parts of $U_+$ and $U_-$, $R_+ = U_+ \upharpoonright \mathcal{R}(U_+)$ and $R_- = U_- \upharpoonright \mathcal{R}_*(U_-)$, we can apply Theorem 2.5 to $R_+$, $R_-$, $\Pi_+$ and $\Pi_-$. Hence there is a contraction $T$ on $\mathcal{H}$ such that $\mathcal{A}' = \{T; R_+, R_-; \Pi_+, \Pi_-\}$ forms a complete scattering system, whose scattering operator coincides with $\Sigma$ regarded as a contraction acting from $\mathcal{R}_*(U_-)$ into $\mathcal{R}(U_+)$.

Denoting by $\overline{\Pi}_\pm: \mathcal{K}_\pm \to \mathcal{H}$ extensions of $\Pi_\pm$ given by $\overline{\Pi}_+ f = \Pi_+ P_{\mathcal{R}(U_+)} f$, $f \in \mathcal{K}_+$, and $\overline{\Pi}_- f = \Pi_- P_{\mathcal{R}_*(U_-)} f$, $f \in \mathcal{K}_-$. We show that the identification operators $\overline{\Pi}_\pm$ and $\overline{J}_\pm$ are equivalent with respect to $U_\pm$, i.e. $\operatorname{s-lim}_{n\to\pm\infty} (\overline{\Pi}_\pm - \overline{J}_\pm)U_\pm^{-n} = 0$. To verify this equivalence it is enough to establish $\lim_{n\to\pm\infty} (\overline{\Pi}_\pm - \overline{J}_\pm)U_\pm^{-n} f = 0$ for every $f \in \mathcal{H}_\pm$. We get

$$(3.16)\quad \begin{aligned} \lim_{n\to+\infty} (\overline{\Pi}_- - \overline{J}_-)U_-^n f &= \lim_{n\to+\infty} (\mathring{F}_-D_- - J_-)T_-^n f = \\ &= \lim_{n\to+\infty} \{F_-(D_- - I_{\mathcal{H}_-})T_-^n f + (F_- - J_-)T_-^n f\} = 0, \end{aligned}$$

$f \in \mathcal{H}_-$. Analogously we find $\lim_{n\to+\infty} (\overline{\Pi}_+ - \overline{J}_+)U_+^{*n} = 0$.



Using this fact and the completeness of $\mathcal{A}' = \{T; R_+, R_-; \Pi_+, \Pi_-\}$ we find that the dilation wave operators $\Omega_\pm = \operatorname{s-lim}_{n\to\pm\infty} U^n \overline{J}_\pm U_\pm^{-n}$ exist and are partial isometries from the residual or dual residual subspaces of $U_+$ and $U_-$ into the absolutely continuous residual or dual residual subspaces of $U$. But the existence of the dilation wave operators yields the existence of the wave operators $W_\pm$. Hence we find that $\mathcal{A} = \{T; T_+, T_-; J_+, J_-\}$ is a complete scattering system whose dilation scattering operator coincides with $\Sigma$.

To prove A we take into consideration Lemma 3.2 and B. ∎

Corollary 3.7. If in addition $\Sigma$ satisfies the conditions $\ker(\Sigma) = \mathcal{K}_- \ominus \mathcal{R}_*(U_-)$ and $(\operatorname{ima}(\Sigma))^- = \mathcal{R}(U_+)$, then $T$ can be chosen from $C_{11}$.

If $\Sigma$ is a partial isometry from $\mathcal{R}_*(U_-)$ onto $\mathcal{R}(U_+)$, then $T$ can be chosen from the unitary operators on $\mathcal{H}$.

Corollary 3.7 follows from Corollary 2.6. Unfortunately, it seems to be impossible to find a simple characterization of those scattering operators which arise from scattering systems full evolution of which belongs to the class $C_{11}$ or to the class of unitary operators. The obvious conditions $\ker(S) = \mathcal{H}_- \ominus \mathcal{D}_-$ and $(\operatorname{ima}(S))^- = \mathcal{D}_+$ seem to be neither necessary nor sufficient for the solution of this problem.

Example 3.8. We consider the Hardy spaces $\mathcal{H}_- = H^2(\mathbb{T}, \mathcal{N}_-)$ and $\mathcal{H}_+ = L^2(\mathbb{T}, \mathcal{N}_+) \ominus H^2(\mathbb{T}, \mathcal{N}_+)$. On $\mathcal{K}_\pm = L^2(\mathbb{T}, \mathcal{N}_\pm)$ we introduce the multiplication operators $U_\pm$ given by $(U_\pm f)(z) = zf(z)$, $f \in \mathcal{K}_\pm$, $z \in \mathbb{T}$. We set $T_- = U_- \upharpoonright \mathcal{H}_-$ and $T_+ = P_+U_+ \upharpoonright \mathcal{H}_+$. Obviously, the minimal unitary dilations of $T_\pm$ coincide with $U_\pm$. Taking into account (2.1) and (2.2) we see that the residual and dual residual subspaces of $U_+$ and $U_-$

coincide with $\mathcal{K}_+$ and $\mathcal{K}_-$. Consequently, choosing some infinite dimensional Hilbert space and some isometries $F_\pm : \mathcal{H}_\pm \longrightarrow \mathcal{K}$ obeying $F_+^* F_- = 0$ and using Theorem 3.6 we find that every intertwining contraction $\Sigma$ of $U_+$ and $U_-$ can be regarded as a dilation scattering operator of some complete scattering system. But $\Sigma \in I(U_+, U_-)$ implies the existence of a measurable contraction-valued function $\{\mathcal{N}_-, \mathcal{N}_+; \theta(z)\}$ such that $\Sigma$ can be represented by

$$(3.17) \qquad (\Sigma f)(z) = \theta(z) f(z),$$

$z \in \mathbb{T}$, $f \in \mathcal{H}_-$. The scattering operator is now the compression of the dilation scattering operator. Denoting by $\theta$ the multiplication operator induced by $\{\mathcal{N}_-, \mathcal{N}_+; \theta(z)\}$ we get that every operator S of the form

$$(3.18) \qquad S = P_{\mathcal{H}_+} \theta \restriction \mathcal{H}_-$$

can be viewed as a scattering operator. But operators of the form (3.18) are usually called generalized Hankel operators [11]. Since every generalized Hankel operator with norm less than one admits the representation (3.18) we have found that every contractive generalized Hankel operator is a scattering operator of some natural associated scattering system.

The generalized Hankel operators reduce to the usual Hankel operators setting $\mathcal{N}_+ = \mathcal{N}_- = \mathbb{C}$. Hence, every contractive Hankel operator is a scattering operator. But a Hankel operator can be compact. Therefore, it is quite possible that a scattering operator is compact or even nuclear or finite dimensional. This effect is new and cannot occur in the case of unitary free evolutions.



Furthermore, we remark that in general the representation (3.18) is not unique. This means, if there is a contractive analytic function $\{\mathcal{N}_-, \mathcal{N}_+; \theta_0(.)\}$ such that $\{\mathcal{N}_-, \mathcal{N}_+; \theta(.) + \theta_0(.)\}$ is a contractive-valued function too, then we have $S = P_{\mathcal{H}_+}(\theta + \theta_0) \restriction \mathcal{H}_- = P_{\mathcal{H}_+} \theta \restriction \mathcal{H}_-$. Hence, we obtain different dilation scattering operators for one and the same scattering operator. Since the inverse scattering problem has been solved by using the dilation scattering operator we naturally get different solutions of the inverse problem. But this is only one of the sources of nonuniqueness of the inverse problem.

## 4. Lax-Phillips scattering theory with losses

Considering a Lax-Phillips scattering theory, which does not fulfil the so-called completeness condition, we are confronted with the fact that the time evolution of certain scattering states cannot be described by the free evolution. We call these scattering states the lost states. The problem now is to find an orthogonal extension of the free evolution by a unitary operator such that the extended free evolution describes the time evolution of all scattering states including the lost states.

Under certain assumptions concerning the Lax-Phillips scattering theory this extension was constructed in [12]. An essential tool in order to solve this problem was a certain symmetrized variant of the Foias-Sz.Nagy functional model of contraction. It was assumed that the associated contraction has no unitary and isometrical parts and that the spectrum of this contraction consists only of isolated eigenvalues in the interior of the unit circle. Furthermore, it was assumed that every triangulation of the associated contraction does not contain any $C_{01}$ or $C_{10}$ parts.

In the following we give a new proof of this result omitting all the additional conditions made in [12]. The proof will be based on the solution of the inverse scattering problem and therefore different from [12]. Moreover, we give the answer in a necessary and sufficient manner.

In order to facilitate the comparison with [12] we use the notation of [12] so far as possible. Let $\{V^n\}_{n\in\mathbb{N}}$ and $\{V_0^n\}_{n\in\mathbb{N}}$ be two unitary groups acting on the separable Hilbert spaces $\mathcal{L}$ and $\mathcal{H}_0$, $\mathcal{H}_0 \subseteq \mathcal{L}$, respectively. It is assumed that $\mathcal{H}_0$ can be decomposed into two subspaces $\mathcal{D}_-$ and $\mathcal{D}_+$, $\mathcal{H}_0 = \mathcal{D}_- \oplus \mathcal{D}_+$, such that the following conditions are fulfilled:

(i ) $V_0\mathcal{D}_+ \subseteq \mathcal{D}_+$, $V_0^*\mathcal{D}_- \subseteq \mathcal{D}_-$

(ii ) $V\restriction\mathcal{D}_+ = V_0\restriction\mathcal{D}_-$, $V^*\restriction\mathcal{D}_- = V_0^*\restriction\mathcal{D}_-$

(iii) $\bigvee_{n\in\mathbb{N}} V_0^n\mathcal{D}_+ = \bigvee_{n\in\mathbb{N}} V_0^n\mathcal{D}_- = \mathcal{H}_0$

(iv ) $\bigcap_{n\in\mathbb{N}} V_0^n\mathcal{D}_+ = \bigcap_{n\in\mathbb{N}} V_0^n\mathcal{D}_- = \{0\}$

(v ) $\bigvee_{n\in\mathbb{N}} V^n\mathcal{H}_0 = \mathcal{H}$.

The completeness condition $\bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_+ = \bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_- = \mathcal{L}$ is not assumed. By $J_0$: $\mathcal{H}_0 \longrightarrow \mathcal{L}$ we denote the natural embedding operator of $\mathcal{H}_0$ into $\mathcal{L}$. It can be shown that the conditions (i) - - (vi) imply the existence of the wave operators $W_\pm(V,V_0)$,

$$W_\pm(V,V_0) = \operatorname*{s-lim}_{n\to\pm\infty} V^n J_0 V_0^{-n}, \tag{4.1}$$

where the projection onto the absolutely continuous subspace of $V_0$ can be omitted because $V_0$ is absolutely continuous ((i),(iii),(iv)). Taking into account (ii) and (v) the same holds for V. Consequently, we can identify the subspace of scattering states of V with the whole space $\mathcal{H}$. Now it is not hard to



see that the ranges of the wave operators $W_\pm(V,V_0)$ coincide with the subspaces $\bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_\mp \subseteq \mathcal{L}$. Since $\bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_+ \neq \mathcal{L}$ or $\bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_- \neq \mathcal{L}$ at least one of the residual subspaces $\mathcal{R} = \mathcal{L} \ominus \bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_-$ and $\mathcal{R}_* = \mathcal{L} \ominus \bigvee_{n\in\mathbb{N}} V^n\mathcal{D}_+$, elements of which we have called the lost states, is different from zero.

The problem now is to find a unitary operator $V_1$ on $\mathcal{H} = \mathcal{L} \ominus \mathcal{H}_0$ such that the wave operators $W_\pm(V,V_0 \oplus V_1) = \operatorname*{s-lim}_{n\to\pm\infty} V^n(V_0^{-n} \oplus V_1^{-n})P^{ac}(V_0 \oplus V_1)$ exist and are complete, i.e. $\operatorname{ima}(W_\pm(V,V_0 \oplus V_1)) = \mathcal{L}$.

To solve this problem we introduce the associated contraction T on $\mathcal{H}$ defined by $T = P^{\mathcal{L}}_{\mathcal{H}} V\restriction\mathcal{H}$. It is easily seen that V is a unitary dilation of T but in general not a minimal unitary dilation. A minimal unitary dilation U can be obtained by introducing the subspace $\mathcal{K} = \bigvee_{n\in\mathbb{N}} V^n\mathcal{H}$ and setting $U = V\restriction\mathcal{K}$. Nevertheless it can be shown that the residual subspaces $\mathcal{R}$ and $\mathcal{R}_*$ coincide with the residual and dual residual subspaces of U. Taking into account this fact the following lemma can be proved.

<u>Lemma 4.1.</u> The wave operators $W_\pm(V,V_0 \oplus V_1)$ exist and are complete if and only if the wave operators $W_+ = \operatorname*{s-lim}_{n\to+\infty} T^n V_1^{-n} P^{ac}(V_1)$ and $W_- = \operatorname*{s-lim}_{n\to+\infty} T^{*n} V_1^{n} P^{ac}(V_1)$ exist and are complete.

<u>Proof.</u> Denoting by $J_1$ the embedding operator of $\mathcal{H}$ into $\mathcal{K}$ we see that the existence of $W_\pm(V,V_0 \oplus V_1)$ yields the existence of $\Omega_\pm = \operatorname*{s-lim}_{n\to\pm\infty} U^n J_1 V_1^{-n} P^{ac}(V_1) = \operatorname*{s-lim}_{n\to\pm\infty} V^n J_1 V_1^{-n} P^{ac}(V_1) = W_\pm(V,V_0 \oplus V_1)\restriction\mathcal{H}$. But the existence of $\Omega_\pm$ implies the existence of $W_\pm$. Hence $\Omega_\pm$ is the dilation wave operator of $W_\pm$. Since $W_\pm(V,V_0 \oplus V_1)$ are complete, the dilation wave operators $\Omega_+$ and $\Omega_-$ are partial isometries from the absolutely

continuous subspace of $V_1$ onto the residual and dual residual subspaces of U.

Conversely, if the wave operators $W_{\pm}$ exist and are complete, then the dilation wave operators $\Omega_+$ and $\Omega_-$ exist and are partial isometries from the absolutely continuous subspace of $V_1$ onto the residual and dual residual subspaces $\mathcal{R}$ and $\mathcal{R}_*$ of U. Using this it is an easy exercise to conclude the existence and completeness of $W_{\pm}(V, V_o \oplus V_1)$. ∎

Theorem 4.2. Let $\{V^n\}_{n\in\mathbb{N}}$ and $\{V_o^n\}_{n\in\mathbb{N}}$ be unitary groups defined on the separable Hilbert spaces $\mathcal{L}$ and $\mathcal{H}_o$, respectively, such that the conditions (i) - (v) are fulfilled. There is a unitary operator $V_1$ on $\mathcal{H} = \mathcal{L} \ominus \mathcal{H}_o$ such that the wave operators $W_{\pm}(V, V_o \oplus V_1)$ exist and are complete if and only if the unitary operators $R = V \upharpoonright \mathcal{R}$ and $R_* = V \upharpoonright \mathcal{R}_*$ are unitarily equivalent.

Proof. The necessity of Theorem 4.2 follows from Lemma 4.1. To prove the converse we note that R and $R_*$ coincide with the residual and dual residual parts of the minimal unitary dilation U of the associated contraction T. The associated contraction T is absolutely continuous. Let $\Sigma$ be a partial isometry acting from $\mathcal{R}_*$ onto $\mathcal{R}$ and establishing the unitary equivalence of $R_*$ and R. Owing to Corollary 3.7 and Remark 2.4 there is a unitary operator $V_1$ on $\mathcal{H}$ such that $\mathcal{A} = \{V_1; T, T; I_{\mathcal{H}}, I_{\mathcal{H}}\}$ forms a complete scattering system the dilation scattering operator of which coincides with $\Sigma$. Now transforming the considerations of [6] to our discrete case the wave operators $\widetilde{W}_+ = \operatorname{s-lim}_{n\to+\infty} V_1^{-n}T^n$ and $\widetilde{W}_- = \operatorname{s-lim}_{n\to+\infty} V_1^{n}T^{*n}$ are complete if and only if the wave operators $W_+ = \operatorname{s-lim}_{n\to+\infty} T^{*n}V_1^{n}P^{ac}(V_1)$ and $W_- = \operatorname{s-lim}_{n\to+\infty} T^{n}V_1^{-n}P^{ac}(V_1)$ exist and are complete. Using Lemma 4.1 we complete the proof. ∎



Corollary 4.3. Let $\{V^n\}_{n\in\mathbb{N}}$ and $\{V_o^n\}_{n\in\mathbb{N}}$ be as before and let T be the associated contraction of V. If there are no subspaces $\mathcal{H}_{01}$ and $\mathcal{H}_{10}$ different from zero and invariant for T and $T^*$, respectively, such that $T\upharpoonright\mathcal{H}_{01} \in C_{01}$ and $T\upharpoonright\mathcal{H}_{10} \in C_{01}$, then there is a unitary operator $V_1$ on $\mathcal{H} = \mathcal{L} \ominus \mathcal{H}_o$ such that the wave operators $W_{\pm}(V, V_o \oplus V_1)$ exist and are complete.

Proof. Using Theorem 4.1 of [13, chapter II] we find that a triangulation of type (b) of T reduces to the form

$$(4.2)\qquad \begin{bmatrix} C_{00} & * & * \\ 0 & C_{11} & * \\ 0 & 0 & C_{00} \end{bmatrix}.$$

Let $U_{11}$ be the minimal unitary dilation of the $C_{11}$ part of the triangulation (4.2). Taking into account the special form of (4.2) it is not hard to see that the residual and dual residual parts of U and $U_{11}$ coincide. Owing to Proposition 3.5 (c) of [13, chapter II] the residual and dual residual parts of $U_{11}$ are unitarily equivalent. Applying Theorem 4.2 we complete the proof. ∎

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Найдхард Х. E5-88-726
Об обратной задаче диссипативной теории рассеяния. III

При рассмотрении теории рассеяния в классе сжимающих операторов на гильбертовых пространствах решается обратная задача в операторно-теоретическом смысле. Решение находим при очень общих предположениях, допуская, что свободные эволюции различны для различных направлений времени и что не только возмущенная или полная эволюция, но также и свободные эволюции заданы при сжимающих операторах. Доказано, что класс сжимающих операторов Ганкеля можно рассматривать как множество операторов рассеяния. Отсюда появляется возможность того, что оператор рассеяния будет компактен. Далее, результат применялся к так называемой теории рассеяния Лакса - Филлипса с потерями при восстановлении совершенно другим путем результата Б.С.Павлова о пополнении этой теории.

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Neidhart H. E5-88-726
On the Inverse Problem of a Dissipative Scattering Theory. III

Considering a scattering theory in the class of contractions on Hilbert spaces one solves the inverse problem in an operator-theoretical manner. The solution is obtained under the very general assumptions that the free evolutions are different for different time directions and that not only the perturbed or full evolutions but also the free evolutions are given by contractions. It is shown that the class of contractive Hankel operators can be viewed as a set of scattering operators. This implies the possibility that the scattering operator can be compact. Moreover, the result is applied to the so-called Lax-Phillips scattering theory with losses restoring a result of B.S.Pavlov on the completion of this theory in a quite different manner.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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