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ALGEBRAS OF OBSERVABLES OF THE CCR

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In 1982 Araki and Jurzak have introduced certain class of $*$ - algebras of unbounded operators. Among others, they formulated conditions I, I_o, I_o' for countably dominated $*$ - algebras and proved under these conditions useful properties of the commutant and double commutant. In their paper there are no examples.

There we show that the algebra of canonical commutation relations for infinite many degrees of freedom satisfies condition I_o'.

LIST OF NOTATIONS

$\mathbf{N} = \{0, 1, 2, \dots\}$ set of naturals

$n = (n_o, n_1, n_2, \dots)$ sequence of naturals

m, n, p elements of $\mathbf{N}^\infty$

ϕ, ψ elements of Hilbert space

a_i annihilation operator

a_i^* creation operator

$$a_i^{\# h} = a_i^{\ell_1} a_i^{*k_1} a_i^{\ell_2} a_i^{*k_2} \dots a_i^{\ell_r} a_i^{*k_r}$$

$$\ell_1 + \ell_2 + \dots + \ell_r = s$$

$$k_1 + k_2 + \dots + k_r = s^*, \quad s + s^* = h$$

$$\nu = \begin{pmatrix} h_1 h_2 \dots h_r \\ i_1 i_2 \dots i_r \end{pmatrix} \text{ multi-index}$$

$$A_\nu = a_{i_1}^{\# h_1} a_{i_2}^{\# h_2} \dots a_{i_r}^{\# h_r}$$

$\mathcal{Z}$ σ - algebra generated by cylindric sets

μ measure on $\mathcal{Z}$.

$U_\epsilon(n)$ ϵ - neighbourhood of the element

Z^k compact subset of $\mathbf{N}^\infty$.

1. PRELIMINARIES

In this section we recall basic definitions and introduce notations. Let H be an infinite dimensional separable Hilbert space and $D \subset H$ a dense linear submanifold. By $L^+(D)$ we denote the $*$ - algebra of linear operators (possibly unbounded) defined on D, leaving D invariant and such that the adjoint operator also leaves D invariant. A * - algebra on D is a * - subalgebra of $L^+(D)$ containing the unit.

A * - algebra $\mathfrak{A} \subset L^+(D)$ is said to be countably dominated if in $\mathfrak{A}$ exists a sequence of operators A_k such that $A_k \geq 1$ and for each $A \in \mathfrak{A}$ there exists a natural k such that

$$| \langle A \phi, \phi \rangle | \leq \lambda \langle A_k \phi, \phi \rangle,$$

for some $\lambda \geq 0$ and all $\phi \in D$. For countably dominated * - algebras Araki and Jurzak [1] introduced the following additional conditions on the dominating sequence:

Condition I. $A_k^{-1} \in \mathfrak{A}$ for all k.

Condition I_0. $A_k D = D$ for all k.

Condition I_0'. A_k^2 is essentially selfadjoint on D for all k.

Now we repeat representations of canonical commutation relations. That is a set of operators $\{a_k, a_k^*\}_{k=0}^{\infty}$ all defined on a common dense domain $D \subset H$ being there essentially selfadjoint and satisfying commutation relations on D for all $i,j = 0,1,2,\ldots$

$$[a_i, a_j^*] = \delta_{ij} 1$$

$$[a_i, a_j] = [a_i^*, a_j^*] = 0.$$

The Gårding - Wightman theorem tells us that for given representation the Hilbert space can be decomposed into a direct integral $H = \int_{N^\infty} H(n)\, d\mu(n)$ and annihilation, respectively; creation operators act on it in the following fashion

$$(a_i \phi)(n) = \sqrt{n_i + 1}\; C_i(n)\, \phi(n + \delta_i) \sqrt{\frac{d\mu(n+\delta_i)}{d\mu(n)}}, \qquad (1)$$

$$(a_i^* \phi)(n) = \sqrt{n_i}\; C_i^*(n - \delta_i)\, \phi(n - \delta_i) \sqrt{\frac{d\mu(n-\delta_i)}{d\mu(n)}}. \qquad (1')$$

Here $\phi(n)$ is a vector function in the direct integral and $C_i(n)$ is an isometric operator mapping the space $H(n)$ onto $H(n + \delta_i)$ and making the following diagramm commutative

$$\begin{array}{ccc} H(n+\delta_i) & \xrightarrow{\;C_i(n)\;} & H(n) \\ {\scriptstyle C_j(n+\delta_i)} \uparrow & & \uparrow {\scriptstyle C_j(n)} \\ H(n+\delta_i+\delta_j) & \xrightarrow{\;C_i(n+\delta_j)\;} & H(n+\delta_j). \end{array}$$

Moreover, the measure μ in the direct integral decomposition is quasi-invariant. Lets describe in more detail this property (see [2]). By N^∞ we denote the space of occupation numbers, i.e. the set of all sequences $n = (n_0, n_1, n_2, \ldots)$ of non-negative integral numbers. By δ_i we denote the sequence such that $n_i = 1$ and $n_j = 0$ for all $j \neq i$. Further, for fixed i and j Z_i^j denotes the cylindric set of all sequences n such that $n_i = j$. The σ-algebra generated by all cylindric sets is denoted by $\mathcal{Z}$ and μ is a measure on this σ-algebra. So $(N^\infty, \mathcal{Z}, \mu)$ becomes a measure space. For each non-negative integral i we define a measure μ_i on $\mathcal{Z}$ by setting

$$\mu_i(M) = \mu(M + \delta_i).$$

If for each i the measure μ_i is absolutely continuous with respect to μ then the measure μ is called quasi-invariant, and $\frac{d\mu(n+\delta_i)}{d\mu(n)}$ denotes the Radon - Nikodym derivative. This is a positive integrable function unique up to a subset of measure zero. The natural domain of definition for operators a_i and a_i^* is the set of such $\phi \in H$ that

$$\int_{N^\infty} n_i \| \phi(n) \|^2 \, d\mu(n) < +\infty.$$

The intersection for all $i = 0,1,2,\ldots$ of these domains we denote by D_0. In space N^∞ we have a partial order $m \leq n$, the meaning is componentwise.

2. THE RESULT

In the following we assume to be given a representation of canonical commutation relations and the Hilbert space assume to be decomposed into a direct integral as described in Sec. 1. We construct in Sec. 3 a dense linear submanifold of the direct integral such that annihilation and creation operators on it generate a *-algebra satisfying condition I_0'.

Theorem. For each representation of canonical commutation relations of countable many degrees of freedom there exists a dense linear submanifold such that the *-algebra generated by annihilation and creation operators is countably dominated and satisfies condition I_0'.

Remark. The theorem holds, of course, for finite many degrees of freedom. By using particle number operators the proof becomes much more simple than in the infinite case.

Proof. We have $m_t = \begin{cases} n_t & \text{if } t \neq i \\ n_t - 1 & \text{if } t = i \end{cases}$. The point m does not belong to Z^k if and only if there exists ℓ such that $m_\ell > a(k,\ell)$. By the same argument there exists j such that $n_j > a(k+1,j)$. In case $j \neq i$ we have $m_j = n_j > a(k+1,j) > a(k,j)$, consequently, $m \bar{\in} Z^k$. If $j = i$, then $m_j = n_j - 1 > a(k+1,j) - 1 \geq a(k,j)$ and again $m \bar{\in} Z^k$. The lemma is proved.

For $\phi \in D$ we put $\psi = a_i^* \phi$. It is sufficient to show that $\mu\{n \in N^\infty\ Z^{k+1}: \|\psi(n)\| \neq 0\} = 0$, where k is such that $\mu\{n \in N^\infty\ Z^k : \|\phi(n)\| \neq 0\} = 0$. Since $\phi \in D$ the k exists. In the first place we remark that both sets $\{n \in N^\infty\ Z^{k+1}: \|\psi(n)\| \neq 0\}$ and Z_i^0 have empty intersection. This follows immediatly from the expression

$$\psi(n) = \sqrt{n_i}\, C_i^*(n-\delta_i)\,\phi(n-\delta_i)\sqrt{\frac{d\mu(n-\delta_i)}{d\mu(n)}} .$$

Secondly we remark that

$$\{n \in N^\infty\ Z^{k+1}: \|\psi(n)\| \neq 0\} = \{n \in N^\infty\ Z^{k+1}: \|\phi(n-\delta_i)\| \neq 0\}.$$

This follows from expression of $\psi(n)$ and from the fact that the Randon - Nikodym derivative is a positive function. Thirdly we show the inclusion

$$\{n \in N^\infty\ Z^{k+1}: \|\phi(n-\delta_i)\| \neq 0\} \subset \{m \in N^\infty\ Z^k : \|\phi(m)\| \neq 0\} + \delta_i .$$

To do this, we consider an arbitrary $n \bar{\in} Z^{k+1}$ such that $\|\phi(n-\delta_i)\| \neq 0$. Put $m = n - \delta_i$. By the previous lemma $m \bar{\in} Z^k$ since $n \bar{\in} Z_i^0$. Thus, $n = m + \delta_i$ and $\|\phi(m)\| \neq 0$. This shows the inclusion. Finally, using quasi-invariance of the measure μ we get

$$\mu\{n \in N^\infty\ Z^{k+1}: \|\phi(n)\| \neq 0\} < \mu(\{m \in N^\infty\ Z^k : \|\phi(m)\| \neq 0\} + \delta_i) = 0.$$

Proposition 3 is proved.

Owing to this proposition we have right to restrict annihilation and creation operators to the domain D and consider the $*$-algebra $\mathfrak{A}$ generated by all these restricted operators. All what follows is a study of the pair $(\mathfrak{A}, D)$.

4. ACTION OF THE OPERATORS ON THE DOMAIN

In this section we study how act operators of the $*$-algebra $\mathfrak{A}$ on direct integral. We obtain some formulas for this ac-

tion and shall use them for estimations. Always we assume that ϕ belongs to D constructed in the previous section. We have for an arbitrary natural i the formulas (1) and (1'). Applying these formulas successively we get for naturals ℓ,k the following formulas (3). Their structure says us that only the factor standing before depends on the order of a_i and a_i^* .

$$(a_i^\ell\, a_i^{*k} \phi)(n) = B_i^{\ell,k}(n)\, U_i^{\ell-k}(n)\, \phi(n+(\ell-k)\delta_i)\sqrt{\frac{d\mu(n+(\ell-k)\delta_i)}{d\mu(n)}} \qquad (2)$$

$$(a_i^{*k} a_i^\ell \phi)(n) = A_i^{k,\ell}(n)\, U_i^{\ell-k}(n)\, \phi(n+(\ell-k)\delta_i)\sqrt{\frac{d\mu(n+(\ell-k)\delta_i)}{d\mu(n)}}, \qquad (3)$$

where

$$U_i^{\ell-k}(n) = \begin{cases} C_i(n)C_i(n+\delta)\dots C_i(n+(\ell-k-1)\delta_i) & \text{for } \ell > k \\ 1_{H(n)} & \text{for } \ell = k \\ C_i^*(n-\delta)C_i^*(n-2\delta_i)\dots C_i^*(n-(k-\ell)\delta_i) & \text{for } \ell < k \end{cases} \qquad (4)$$

and functions $B_i^{\ell,k}(n)$ are computed by the following series of formulas (here $\ell \vee k$ denotes the maximum of numbers ℓ and k)

$$B_i^{\ell,k}(n) = \prod_{j=1}^{\ell \vee k} \beta_j(n), \qquad (5)$$

where

$$\beta_j(n) = \begin{cases} \sqrt{n_i + j} & \text{for } j = 1,2,\dots,\ell-k \text{ and } \ell > k \\ n_i + j & \text{for } j = \ell-k+1,\dots,\ell \text{ and } \ell \geq k \\ \sqrt{n_i + j} & \text{for } j = 1,2,\dots,\ell \text{ and } \ell < k \\ \sqrt{n_i + 1 + \ell - j} & \text{for } j = \ell+1, \ell+2,\dots,k \text{ and } \ell < k. \end{cases} \qquad (6)$$

Analogously are computed the functions $A_i^{k,\ell}(n)$.
Properties of function $B_i(n)$.

$$B_i^{\ell,k}(n) \geq 0 \qquad \text{for all } \ell, k \text{ and } n \in N^\infty, \qquad (7)$$

$$B_i^{\ell,k}(n) > 0 \qquad \text{if } \ell \geq k, \qquad (8)$$

$$B_i^{\ell,k}(n) = 0 \qquad \text{if } \ell < k \text{ and } n_i \le k - \ell + 1, \tag{9}$$

$$B_i^{\ell,k}(n) < B_i^{\ell+1,k}(n), \tag{10}$$

$$B_i^{\ell,k}(n) < B_i^{\ell,k+1}(n), \tag{11}$$

$$B_i^{\ell,k}(n) < B_i^{\ell+1,k-1}(n), \tag{12}$$

$$B_i^{\ell,k}(n) < (n_i + \ell)^{\ell \vee k}. \tag{13}$$

Product formula

$$B_i^{\ell_1,k_1}(n)\, B_i^{\ell_2,k_2}(n + (\ell_1 - k_1)\delta_i) = \prod_{j=1}^{k_1} \frac{n_i + \ell_1 - k_1 + j}{n_i + \ell_1 + \ell_2 - k_1 + j}\, B_i^{\ell_1+\ell_2,k_1+k_2}(n), \tag{14}$$

$$B_i^{\ell_1,k_1}(n)\, B_i^{\ell_2,k_2}(n + (\ell_1 - k_1)\delta_i) < B_i^{\ell_1+\ell_2,k_1+k_2}(n), \tag{15}$$

$$B_i^{\ell,k}(n + t\delta_j) = B_i^{\ell,k}(n) \quad \text{for } i \ne j,\ t \in \boldsymbol{N}. \tag{16}$$

The proof of each property listed below follows from elementary computation ,inclusively, the proof of product formula is a straight forward calculation looking on separate all possible combinations of cases $\ell_j \ge k_j$ or $\ell_j < k_j$, $j = 1,2$, and considering that $\ell_1 + \ell_2 > k_1 + k_2$ or not. Property (15) follows immediatly from (14).

Properties of operator valued function $U_i(n)$. $U_i^{\ell-k}(n)$ is an isometric operator mapping the space

$$H(n + (\ell - k)\delta_i) \qquad \text{onto } H(n), \tag{17}$$

$$(U_i^{\ell-k}(n))^* = U_i^{k-\ell}(n - (k-\ell)\delta_i), \tag{18}$$

$$U_i^{\ell_1-k_1}(n)\, U_i^{\ell_2-k_2}(n + (\ell_1 - k_1)\delta_i) = U_i^{\ell_1+\ell_2-k_1-k_2}(n). \tag{19}$$

Next we introduce the following notation

$$a_i^{\#h} = a_i^{\ell_1} a_i^{*k_1} a_i^{\ell_2} a_i^{*k_2} \dots a_i^{\ell_r} a_i^{*k_r},$$

where

$\ell_1 \ge 0$, $\ell_2, \ell_3, \dots, \ell_r \ge 1$,

$k_1, k_2, \dots, k_r \ge 1$, $k_r \ge 0$,

$s = \ell_1 + \ell_2 + \dots + \ell_r$,

$s^* = k_1 + k_2 + \dots + k_r$ and $h = s + s^*$.

Using the structure of formulas (2) and (4) and property (19) we get

$$(a_i^{\#h}\phi)(n) = \prod_{j=1}^{r} B_i^{\ell_j,k_j}\Big(n + \big(\sum_{t=1}^{j-1} \ell_t - k_t\big)\delta_i\Big) U_i^{s-s^*}(n) \times \tag{20}$$

$$\times\, \phi(n + (s - s^*)\delta_i) \sqrt{\frac{d\mu(n + (s - s^*)\delta_i)}{d\mu(n)}}.$$

For a product of different operators of kind $a_i^{\#h}$ we then obtain

$$(a_{i_1}^{\#h_1} a_{i_2}^{\#h_2} \dots a_{i_r}^{\#h_r}\phi)(n) = \prod_{j=1}^{r} B_{i_j}^{(s_j,s_j^*)}(n) \prod_{j=1}^{r} U_{i_j}^{s_j-s_j^*}\Big(n + \sum_{t=1}^{j-1}(s_t - s_t^*)\delta_{i_t}\Big)\, \phi\Big(n + \sum_{j=1}^{r}(s_j - s_j^*)\delta_{i_j}\Big) \sqrt{\frac{d\mu(n + \sum_{j=1}^{r}(s_j - s_j^*)\delta_{i_j})}{d\mu(n)}}. \tag{21}$$

In this formula we have used abbreviation

$$B_{i_1}^{(s_1,s_1^*)}(n) = \prod_{j=1}^{r_1} B_{i_1}^{\ell_j,k_j}\Big(n + \sum_{t=1}^{j-1} \ell_t - k_t\Big)\delta_{i_1}), \tag{22}$$

and have used the invariance property (16). In Sec. 5 we shall use the following estimation

$$B_i^{(s,s^*)}(n) < B_i^{h,h}(n), \quad h = s + s^*. \tag{23}$$

Estimation (23) is a consequence of the property

$$B_i^{(s,s^*)}(n) < B_i^{s,s^*}(n), \tag{24}$$

and (24) follows from (15). We remark also

$$B_i^{h,h}(n) = (n_i + 1)(n_i + 2)\dots(n_i + h). \tag{25}$$

5. PROOF OF THE THEOREM

There we prove for the pair $(\mathfrak{A}, D)$ the following statements:

I. The *-algebra is countably dominated.

II. The *-algebra satisfies condition I_0'.

I. Each $A \in \mathfrak{A}$ can be written as a finite sum

$$A = \sum_{(i_1, i_2, \dots, i_r)} \lambda_{i_1 i_2 \dots i_r}^{h_1 h_2 \dots h_r} a_{i_1}^{\#h_1} a_{i_2}^{\#h_2} \dots a_{i_r}^{\#h_r}. \tag{26}$$

We put λ equal to the maximum of the absolute values of each coefficient in the sum (26). For arbitrary but fixed $A \in \mathcal{A}$ as written in (26) we introduce the following notation which is slightly different from notation before. Let J be the set of r-tupels $i = (i_1, i_2, \dots, i_r) = \sum_{j=1}^{r} \delta_{i_j}$. In the finite set J there exists the greatest element denoted by α. For each $i \in J$ consider the corresponding $h = (h_1, h_2, \dots, h_r) = \sum_{j=1}^{r} h_j \delta_{i_j}$. In the finite set $\mathcal{H} = \{h\}$ there exists the greatest element denoted by β. The statement I will be proved if we show for all $\phi \in D$ the following estimation for each $i \in J$ and $h \in \mathcal{H}$:

$$|\langle a_{i_1}^{\#h_1} a_{i_2}^{\#h_2} \dots a_{i_r}^{\#h_r}\phi,\phi\rangle| \le \langle a_{i_1}^{h_1} a_{i_1}^{*h_1}\dots a_{i_r}^{h_r} a_{i_r}^{*h_r}\phi,\phi\rangle \qquad (27)$$

$$\langle a_{i_1}^{h_1} a_{i_1}^{*h_1} a_{i_2}^{h_2} a_{i_2}^{*h_2}\dots a_{i_r}^{h_r}a_{i_r}^{*h_r}\phi,\phi\rangle \le \langle a_{\alpha_1}^{\beta_1}a_{\alpha_1}^{*\beta_1}\dots a_{\alpha_t}^{\beta_t}a_{\alpha_t}^{*\beta_t}\phi,\phi\rangle \qquad (28)$$

Then follows the desired estimation

$$|\langle A\phi,\phi\rangle| \le \lambda\langle A_\nu\phi,\phi\rangle \quad \phi\in D, \qquad (29)$$

where ν is a multi-index

$$\nu = \begin{pmatrix}\beta_1 & \beta_2 & \dots & \beta_t\\ \alpha_1 & \alpha_2 & \dots & \alpha_t\end{pmatrix}$$

and

$$A_\nu = a_{\alpha_1}^{\beta_1} a_{\alpha_1}^{*\beta_1} a_{\alpha_2}^{\beta_2} a_{\alpha_2}^{*\beta_2}\dots a_{\alpha_t}^{\beta_t}a_{\alpha_t}^{*\beta_t}$$

Thus, the countable subset of all operators of the kind A_ν does the job of dominating property for the *-algebra $\mathcal{A}$.

Proof of estimation (27). We have

$$\langle a_{i_1}^{\#h_1} a_{i_2}^{\#h_2}\dots a_{i_r}^{\#h_r}\phi,\phi\rangle = \int_{N^\infty} \langle (a_{i_1}^{\#h_1} a_{i_2}^{\#h_2}\dots a_{i_r}^{\#h_r}\phi)(n), \phi(n)\rangle\, d\mu(n).$$

Using formula (21) we get for the last integral the expression

$$\int_{N^\infty} \prod_{j=1}^{r} B_{i_j}^{(s_j,s_j^*)}(n)\langle U\phi(n+\sum_{j=1}^{r} (s_j - s_j^*)\delta_{i_j}),\phi(n)\rangle \sqrt{\quad}\, d\mu(n),$$

where U is the isometric operator

$$U = \prod_{j=1}^{r} U_{i_j}^{s_j - s_j^*}(n + \sum_{t=1}^{j-1}(s_t - s_t^*)\delta_{i_t}),$$

and under the square root stands Randon - Nikodym derivative

$$\frac{d\mu(n+\sum_{j=1}^{r} (s_j - s_j^*)\delta_{i_j})}{d\mu(n)}.$$

If we apply to the product $\prod_{j=1}^{r} B_{i_j}^{(s_j,s_j^*)}(n)$ estimation (23) and to the scalar product under the last integral apply Schwarz' inequality then we obtain the following

$$|\langle a_{i_1}^{\#h_1} a_{i_2}^{\#h_2}\dots a_{i_r}^{\#h_r}\phi,\phi\rangle| \le$$

$$\int_{N^\infty}\prod_{j=1}^{r} B_{i_j}^{h_j,h_j}(n)\|\phi(n+\sum_{j=1}^{r}(s_j-s_j^*)\delta_{i_j})\|\,\|\phi(n)\|\sqrt{\quad}\,d\mu(n).$$

Applying again Schwarz' inequality we get

$$\int_{N^\infty}\|\phi(n+\sum_{j=1}^{r} (s_j-s_j^*)\delta_{i_j})\|\,\|\phi(n)\|\sqrt{\quad}\,d\mu(n) \le$$

$$\sqrt{\int_{N^\infty}\|\phi(n+\sum_{j=1}^{r}(s_j-s_j^*)\delta_{i_j})\|^2 d\mu(n+\sum(s_j-s_j^*)\delta_{i_j})}\sqrt{\int_{N^\infty}\|\phi(n)\|^2 d\mu(n)} \le$$

$$\sqrt{\int_{N^\infty}\|\phi(n')\|^2 d\mu(n')}\sqrt{\int_{N^\infty}\|\phi(n)\|^2d\mu(n)} = \|\phi\|^2.$$

Therefore we have

$$\int_{N^\infty}\prod_{j=1}^{r} B_{i_j}^{h_j,h_j}(n)\|\phi(n+\sum_{j=1}^{r}(s_j-s_j^*)\delta_{i_j}\|\,\|\phi(n)\|\sqrt{\quad}\,d\mu(n) =$$

$$= \sum_{m_1,\dots,m_r}\prod_{j=1}^{r} (m_j+1)(m_j+2)\dots(m_j+h_j)\int_{Z^{m_1 m_2\dots m_r}_{i_1 i_2\dots i_r}}\|\phi(n+\sum_{j=1}^{r}(s_j-s_j^*)\delta_{i_j})\|\times$$

$$\times\|\phi(n)\|\sqrt{\quad}\,d\mu(n) \le \sum_{m_1,m_2,\dots,m_r}\prod_{j=1}^{r}(m_j+1)(m_j+2)\dots(m_j+h_j)\int_{Z^{m_1m_2\dots m_r}_{i_1i_2\dots i_r}}\|\phi(n)\|^2d\mu(n).$$

$$= \int_{N^\infty}\prod_{j=1}^{r} B_{i_j}^{h_j,h_j}(n)\|\phi(n)\|^2\, d\mu(n) =$$

$$= \langle a_{i_1}^{h_1}a_{i_1}^{*h_1}a_{i_2}^{h_2}a_{i_2}^{*h_2}\dots a_{i_r}^{h_r}a_{i_r}^{*h_r}\phi,\phi\rangle.$$

There we remark that the last summation is finite since $\phi \in D$. Estimation (27) is proved. When δ is the maximum of the finite set $\mathcal{H}$ then estimation (28) follows from the formula (25). Thus, statement I is proved.

II. For each multi-index

$$\nu = \begin{pmatrix} h_1 & h_2 & \dots & h_r \\ i_1 & i_2 & \dots & i_r \end{pmatrix}$$

we consider the operator

$$A_\nu = a_{i_1}^{h_1} a_{i_1}^{*h_1} a_{i_2}^{h_2} a_{i_2}^{*h_2} \dots a_{i_r}^{h_r} a_{i_r}^{*h_r}.$$

We show that the operator A_ν^2 is essentially selfadjoint on D. To do this, we consider Nelsons criterion and prove that D is a set of analytic vectors for A_ν^2. Let

$$\Pi_\nu(n) = \prod_{j=1}^{r} (n_{i_j} + 1)(n_{i_j} + 2) \dots (n_{i_j} + h_j).$$

Then $(A_\nu \phi)(n) = \Pi_\nu(n)\,\phi(n)$. It follows $A_\nu \geq 1$, and A_ν, A_ν^2 are symmetric operators. Further, we denote by

$$C_\nu(k) = \prod_{j=1}^{r} (a(k, i_j) + 1)(a(k, i_j) + 2) \dots (a(k, i_j) + h_j).$$

Hence $\Pi_\nu(n) \leq C_\nu(k)$ for all $n \in Z^k$. Let $\phi \in D$ then exists k such that $\|\phi(n)\| = 0$ for μ-almost every $n \bar{\in} Z^k$. Therefore, we have

$$\|A_\nu^{2\ell} \phi\| < C_\nu^{2\ell}(k) \|\phi\|, \quad \ell = 0, 1, 2, \dots$$

Consequently,

$$\sum_{\ell=0}^{\infty} \frac{\|A_\nu^{2\ell}\phi\|}{\ell!} t^\ell \leq e^{tC_\nu^2(k)} \|\phi\|.$$

Thus, each vector of D is an analytic vector for A_ν^2. The proof of the theorem is complete.

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Зенф П. E5-88-700

Алгебры наблюдаемых канонических соотношений коммутации

На плотной области изучается $*$-алгебра канонических коммутационных отношений с бесконечным числом степеней свободы. Показывается, что она удовлетворяет условию I'_0.

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Algebras of Observables of the CCR

The $*$-algebra of canonical commutation relations with infinite many degrees of freedom is studied on a dense domain. It is shown that it satisfies condition I'_0.

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