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COVARIANCE OPERATOR OF FUNCTIONAL MEASURE IN $P(\varphi)_2$-QUANTUM FIELD THEORY

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1. INTRODUCTION

The functional integration method is an imprescriptible means of investigation in many branches of contemporary science [1,2]. One of the main areas of its employment is quantum field theory [3]. This method appeared to be the convenient tool as this enabled one for the first time to perform the theoretical and numerical investigation of nonperturbative characteristics in quantum gauge theory (e.g. [4]). The idea of utilizing the functional integrals in Quantum Physics expressed by R.Feynman served as a basis for the contemporary constructive quantum field theory [5]. The successive realization of constructive program has led to the mathematically rigorous construction of quantum fields in two-dimensional and some three-dimensional models of Euclidean field theory (e.g.[6]). Significant success has been achieved recently in constructing the local relativistic interacting fields in 4-dimensional space-time [7]. One of the most simple ways of assigning a specified mathematical meaning to functional integrals in quantum field theory and providing the numerical calculations is the introduction of space-time lattice. The employment of lattice regularization turns functional integrals into ordinary ones of high dimension ($\geqslant 10^5$). The Monte Carlo method is usually applied to evaluate these integrals. Many numerical results important to the physical theory have been obtained in this way [8]. When performing the lattice computations one has to extrapolate the results to the continuum limit [8,4]. This non-simple problem serves as an object of investigation for many authors [9-14]. As it has been pointed out in [9], the problem of removal of the finite-size effects and the lattice artifacts arising in Monte Carlo calculations were studied insufficiently. The attempts of reaching the continuum limit numerically by computation on lattices with decreasing spacing usually failed [9,10]. Even the computations with the record lattice sizes (up to 20^4 points) on the CRAY computer do not allow to get rid of the dependence of results on the lattice spacing [11]. Besides that, the difficulties of employing the lattice Monte Carlo method increase with decreasing the lattice spacing [11]. Some authors are engaged nowdays in searching for the improved modifications of the action functional [13], and also in the investigations directly in the continuum limit [15-18]. Employment of the lattice regularization entailes some other problems as well , among which is the loss of continuum topology on the lattice [19]. In this connection a number of authors successively develop the method of nonperturbative regularization of quantum gauge field theory in the continuum [15,19-22]. The ability of performing the numerical calculations in continuum is connected with the development of functional integrals computation method. Significant progress in this area has been achieved last years [23]. The problems of measure in functional integral [24] play an important role both for the study of continuum limit in constructive quantum field theory and for the numerical calculations. Here and below we shall denote by the functional integral (as distinct from a "path integral") just an integral with respect to a given measure in the definite functional space. The important results of constructing the functional measure in the quantum field theory have been obtained recently [18,25-29]. Particularly, the Euclidean measure for the electromagnetic field is obtained [26], the functional measure in Lagrangian gauge theories is defined [27], the Gaussian measure on extended Grassmanian algebra for fermion functional integrals is constructed [29]. One of the areas where the measure theory is the most profoundly elaborated is a two-dimensional quantum field theory with polynomial interactions of boson fields [30]. The mathematically rigorous construction of the Gaussian measure in $P(\varphi)_2$-model is given in [31]. This model enables one to study, in particular, such processes as phase transitions, critical phenomena, interaction of particles, scattering and bound states. The $P(\varphi)_2$-theory is investigated by many authors. In paper [32], e.g., the behaviour of the vacuum energy density in the infinite volume is studied in the framework of this model.

In papers [33,34] we have derived for the functional integrals with Gaussian measure some new approximation formulae exact on a class of the polynomial functionals of an arbitrary given degree. These formulae provide the way of computation of physical quantities directly in continuum limit. We used these formulae in particular case of the conditional Wiener measure in our computations of Feynman path integrals in Euclidean quantum mechanics [35-38]. As shown there, the employment of our formulae leads to the evaluation of the ordinary integrals of a low dimension, that allows one to use the deterministic methods (quadrature formulas) and gives the essential (by an order) economy of computer time and memory versus the lattice Monte Carlo computations. The use of the approach to the functional integrals that does not need the space-time discretization, enabled to perform the successful numerical study of the topological succeptibility and the θ-vacua energy [36].

The following characteristics of measure, such as the covariance and the mean value, are of principal significance in studying the problems concerned the functional measure including the construction of

approximation formulae for functional integrals. In this paper we derive for the kernel of the covariance operator of the $P(\varphi)_2$-functional measure the representation in the form of the expansion over the eigenfunctions of some boundary problem for the heat equation. As an example, the two cases of the integration domains with different configurations are considered.

2. BASIC DEFINITIONS

The Lagrangian of the $P(\varphi)_2$-model is written as follows [31]:

$$\mathcal{L}(\varphi(x)) = :\tfrac{1}{2}(\nabla\varphi)^2 + \tfrac{1}{2}m^2\varphi^2(x) + \lambda P(\varphi(x)): \qquad (1)$$

Here $x \in R^2$; $\varphi(x) \in \mathcal{S}'(R^2)$ – the space of the generalized functions of moderate increase; P is a bounded from below polynomial. The space of the basic functions is a Schwarz's space of rapidly decreasing functions $\mathcal{S}(R)^2$. The value of φ at the basic function $f \in \mathcal{S}(R^2)$ is given by

$$\varphi(f) \equiv \langle \varphi, f \rangle = \int_{R^2} \varphi(x) f(x)\, dx.$$

The Wick's ordering ("Wick's colon") is defined as

$$:\varphi(x)^n: = \lim_{\varkappa\to\infty} \sum_{j=0}^{[n/2]} \frac{(-1)^i\, n!}{(n-2j)!\, j!\, 2^j}\, k_\varkappa^i(x) \left(\varphi_\varkappa(x)\right)^{n-2j}, \qquad (2)$$

where

$$\varphi_\varkappa(x) = \int_{R^2} \varphi(y)\, \delta_{\varkappa,x}(y)\, dy$$ – impulse cut-off of the field φ;

$$\delta_{\varkappa,x}(y) = \varkappa^2 h(\varkappa(x-y))$$ –the "Smeared" δ-function,

$$h \in C_0^\infty(R^2), \quad h(y) \geq 0, \quad \int_{R^2} h(y)\, dy = 1.$$

$$k_\varkappa(x) = \langle \delta_{\varkappa,x}, K\delta_{\varkappa,x} \rangle.$$

Here K is a covariance operator of the measure [23], $K(f,g) = \langle f, Kg \rangle$ (a continuous nondegenerated bilinear form on the product of spaces $\mathcal{S}(R^2) \times \mathcal{S}(R^2)$):

$$K(f,g) = \int [\langle \varphi, f \rangle - \xi(f)][\langle \varphi, g \rangle - \xi(g)]\, d\mu(\varphi),$$

where $\xi(f) = \int \langle \varphi, f \rangle\, d\mu(\varphi)$ – the mean value of measure $d\mu(\varphi)$. In the sequel we shall assume $\xi(f) = 0$ without any limitations of generality.

The measure in the space $\mathcal{S}'(R^2)$ is defined as follows [31]:

First the measure in finite volume $\Lambda \subset R^2$ is introduced

$$\partial\mu_\Lambda = Z^{-1} e^{-V(\Lambda)} d\varphi_{K_{\partial\Lambda}}, \tag{3}$$

where

$$V(\Lambda) = \int_\Lambda :P(\varphi(x)):_{K_\emptyset} dx\,; \qquad Z = Z(\Lambda) = \int e^{-V(\Lambda)} d\varphi_{K_{\partial\Lambda}}.$$

The Wick's ordering is performed here with respect to the free covariance operator $K_\emptyset$ [31]. $d\varphi_{K_{\partial\Lambda}}$ is a Gaussian measure with the covariance $K_{\partial\Lambda}$ satisfying the Dirichlet boundary conditions on $\partial\Lambda$ -the boundary of the region Λ and with the mean value $\xi(f)=0$. The covariance operator and the mean value define the Gaussian measure in the unique way [23], i.e., on the space $\mathcal{S}'(R^2)$ there exists the unique Gaussian measure $d\varphi_K$ with the covariance K and the mean equals zero.

The important result obtained by Glimm and Jaffe [31] is the proof of the existence of the measure in infinite volume. This measure is constructed as a limit of the considered above measures in the finite volumes. Namely, it has been proved that under certain conditions on P and if $f \in C_0^\infty$ the sequence of characteristic functionals

$$S_\Lambda\{f\} = \int e^{i\varphi(f)} d\mu_\Lambda$$

of finite volume measures has the limit

$$S\{f\} = \lim_{\Lambda \uparrow R^2} S_\Lambda\{f\}$$

and the limiting functional $S\{f\}$ satisfies the Euclidean axioms of analyticity, regularity, invariance with respect to shifts, rotations, and reflections that is necessary to construct quantum field [31].

Thus, the operator observables defined as the averages over the vacuum state of interacting fields Ω can be obtained by the evaluation of the functional integral

$$\langle\Omega|F(\varphi)|\Omega\rangle = \lim_{\Lambda\uparrow R^2} \frac{\int_{\mathcal{S}'(\Lambda)} \exp\{-\lambda \int_\Lambda :P(\varphi(x)):_{K_\emptyset} dx\} F(\varphi)\, d\varphi_{K_{\partial\Lambda}}}{\int_{\mathcal{S}'(\Lambda)} \exp\{-\lambda \int_\Lambda :P(\varphi(x))_{K_\emptyset} dx\}\, d\varphi_{K_{\partial\Lambda}}}.$$

It is essential that under $d=2$ the renormalizations in $P(\varphi)_d$ -model are reduced to the subtraction connected with the Wick's ordering (2), i.e., the divergensies in the presented expression for the observables do not arise.

3. THE KERNEL OF THE COVARIANCE OPERATOR OF $P(\varphi)_2$ -MEASURE

In many cases including the constructing of the approximation formulae for the functional integrals it is necessary to have the explicit expression for the covariance operator K. Writing $K(f,g)$ in the form

$$K(f,g) = \int_{R^2\times R^2} \mathcal{K}(x,y)\, f(x)\, g(y)\, dx\, dy$$

consider its integral kernel $\mathcal{K}(x,y)$, $x,y \in R^2$:

$$\mathcal{K}(x,y) = \int_{\mathcal{S}'(R^2)} \varphi(x)\,\varphi(y)\, d\varphi_K. \tag{4}$$

For the measure covariance operator $K_{\partial\Lambda}$ with the Dirichlet boundary conditions on $\partial\Lambda$ - the boundary of arbitrary region $\Lambda \subset R^2$ there exists the representation for the kernel through the integral with respect to the conditional Wiener measure [31]:

$$\mathcal{K}_{\partial\Lambda}(x,y) = \int_0^\infty dt\, e^{-m^2 t} \int_{C_{x,y}[0,t]} \chi_{\partial\Lambda}(\omega)\, d_W\omega. \tag{5}$$

The functional integration in (5) is performed over the set $C_{x,y}[0,t]$ of continuous functions $\omega(\tau)$, $\tau \in [0,t]$, satisfying the condition $\omega(0)=x$; $\omega(t)=y$. Here $\chi_{\partial\Lambda}(\omega)$ is the characteristic function of the paths that do not have points of intersection with $\partial\Lambda$, i.e.,

$$\chi_{\partial\Lambda}(\omega) = \begin{cases} 0, & \text{if } \exists\, \tau_0 \in [0,t] : \omega(\tau_0) \in \partial\Lambda \\ 1, & \text{otherwise}. \end{cases}$$

As we are interested in $\lim_{\Lambda\uparrow R^2}$, we can assume without the limitation of generality that $x,y \in \Lambda \setminus \partial\Lambda$; in the case $x \in \partial\Lambda$ and/or $y \in \partial\Lambda$ it is obvious that $\mathcal{K}(x,y)=0$. If we denote

$$C^\Lambda_{x,y}[0,t] = \{\omega(\tau) \in C[0,t];\ \omega(0)=x,\ \omega(t)=y,\ \omega(\tau) \in \Lambda\setminus\partial\Lambda\ \forall\,\tau \in [0,t]\}, \tag{6}$$

then

$$\mathcal{K}_{\partial\Lambda}(x,y) = \int_0^\infty dt\, e^{-m^2 t} \int_{C^\Lambda_{x,y}[0,t]} d_W\omega = \int_0^\infty M(x,y,t)\, e^{-m^2 t}\, dt, \tag{7}$$

where $M(x,y,t) = \operatorname{mes} C^\Lambda_{x,y}[0,t]$ is the Wiener volume of the set $C^\Lambda_{x,y}[0,t]$. Thus, in order to obtain $\mathcal{K}_{\partial\Lambda}(x,y)$ it is sufficient to determine the functional volume of the set of two-dimensional continuous on the segment functions with fixed values at the ends of the segment which take only the values of the interior of the given region Λ. The main result of this paper is formulated in the following

Theorem

For an arbitrary bounded connected region $\Lambda \subset R^2$ with the piecewise smooth boundary $\partial\Lambda$ the Wiener volume $M = mes\, C^{\Lambda}_{x,y}[0,t]$ is the solution of the following boundary problem :

$$\begin{cases} \dfrac{\partial M}{\partial t} = \dfrac{1}{2}\Delta_y M\,; & x,y \in \Lambda\setminus\partial\Lambda,\ t>0 \\ M(x,y,0) = \delta(y-x), & \\ M(x,y,t)\big|_{y\in\partial\Lambda} = 0, & \end{cases} \tag{8}$$

where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$; $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$; $\Delta_y = \dfrac{\partial^2}{\partial y_1^2} + \dfrac{\partial^2}{\partial y_2^2}$.

Proof.

As it is known [39], the conditional Wiener integral

$$Z(x,y,t) = \int\limits_{C_{x,y}[0,t]} e^{-\int_0^t U[\omega(\tau)]\,d\tau}\, d_w\omega, \qquad x,y\in R^2 \tag{9}$$

is the solution of the problem

$$\begin{cases} \dfrac{\partial Z}{\partial t} = \dfrac{1}{2}\Delta_y Z - U(y)Z, & t>0 \\ Z(x,y,0) = \delta(y-x), & \\ Z(x,y,t) \xrightarrow[|y|\to\infty]{} 0. & \end{cases} \tag{10}$$

In order to reduce the integration domain in (9) to the set of the paths which are completely contained in the given region $\Lambda \subset R^2$, it is sufficient to let $U(y)$ be equal to infinity everywhere outside Λ and on its boundary $\partial\Lambda$. Simultaneously, the zero boundary conditions on $\partial\Lambda$ should be imposed to $Z(x,y,t)$. Furthermore, if we set $U=0$ inside Λ we shall evidently obtain $Z(x,y,t) = M(x,y,t)$ for $x,y\in\Lambda$. Indeed, for all paths $\omega(\tau)$ from x to y if ω does not have points of intersection with $\partial\Lambda$ the value of the integrand in (9) is equal to the unity. On the other hand, if for some ω_0 there exists the point τ_0 where $\omega_0(\tau_0)\in\partial\Lambda$, then $U[\omega_0(\tau_0)] = \infty$. In order to complete the proof it is sufficient to show that such the paths ω_0 do not contribute to $Z(x,y,t)$. Generally speaking, it does not follow from $U[\omega_0(\tau_0)] = \infty$ that $\int_0^t U[\omega_0(\tau)]\,d\tau = \infty$, so the value of the functional under the integral sign in (9) may be different from zero for the path ω_0. However, as shown in [40], the trajectories that touch the boundary without crossing it form the set of measure zero with respect to the conditional Wiener measure in the space of continuous functions. This circumstance is connected with the fact that the conditional Wiener measure is concentrated on the set of the Hölder continuous functions with the index $\alpha < 1/2$ (i.e., on the nondifferentiable functions) and not on all the functions continuous on segment. These phenomena themselves are of particular interest and should be studied elsewhere. This result, as applied to the problem under consideration, means that the trajectories ω_0 touching $\partial\Lambda$ in a countable number of points do not contribute to integral (9), i.e., the equality $Z(x,y,t) = M(x,y,t)$ holds. Thus the proof of the theorem is complete.

Corollary

The kernel of the covariance operator of $P(\varphi)_2$-measure can be expressed in the form :

$$\mathcal{K}(x,y) = \sum_n \frac{1}{E_n+m^2}\,\eta_n(x)\,\eta_n(y), \qquad x,y\in\Lambda\setminus\partial\Lambda, \tag{11}$$

where E_n and η_n are the eigenvalues and eigenfunctions of the problem

$$\begin{cases} -\dfrac{1}{2}\Delta\eta(x) = E\,\eta(x), & x\in\Lambda\setminus\partial\Lambda \\ \eta(x) = 0, & x\in\partial\Lambda\,. \end{cases} \tag{12}$$

Indeed, if expanding the solution $M(x,y,t)$ of the problem (8) over the eigenfunctions of the boundary problem (12) [41], we obtain

$$M(x,y,t) = \sum_n e^{-E_n t}\,\eta_n(x)\,\eta_n(y).$$

Performing the integration over t in expr.(5), we directly obtain (11).

The concrete expression for $\mathcal{K}(x,y)$ depends on the shape of the region Λ. In the next section we consider the two examples of Λ of the simple shape when the problem (12) can be solved explicitly.

4. EXAMPLES

Consider the region Λ of the rectangular shape: $\Lambda = [-a,a]\times[-b,b]$.

Applying the method of separation of variables we get [41]

$$M(x,y,t) = \sum_{n_1,n_2=1}^{\infty} e^{-E_{n_1n_2}t}\, \eta_{n_1n_2}(x)\, \eta_{n_1n_2}(y),$$

where

$$E_{n_1n_2} = \frac{\pi^2}{2}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}\right)$$

$$\eta_{n_1n_2}(y) = \frac{1}{\sqrt{ab}} \sin\frac{n_1\pi}{a}y_1 \cdot \sin\frac{n_2\pi}{b}y_2 .$$

After integrating over t we obtain for $\mathcal{K}(x,y)$

$$\mathcal{K}_{\partial\Lambda}(x,y) = \frac{1}{ab}\sum_{n_1,n_2=1}^{\infty}\left(m^2+\frac{\pi^2 n_1^2}{2a^2}+\frac{\pi^2 n_2^2}{2b^2}\right)^{-1} \sin\frac{n_1\pi}{a}x_1 \sin\frac{n_2\pi}{b}x_2 \sin\frac{n_1\pi}{a}y_1 \sin\frac{n_2\pi}{b}y_2 .$$

Let us consider now the case when Λ is a circle, $r \leq r_0$. Introducing the polar coordinates and supposing $\psi = \psi(r,\vartheta,t)$, we have

$$\begin{cases} \dfrac{\partial\psi}{\partial t} = \dfrac{1}{2}\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\psi}{\partial r}\right)+\dfrac{1}{r^2}\dfrac{\partial^2\psi}{\partial\vartheta^2}\right], & r\in[0,r_0) \\ \psi(r,\vartheta,0) = \delta(y-x), & \vartheta\in[0,2\pi] \\ \psi(r_0,\vartheta,t) = 0. & t>0 \end{cases}$$

The employment of the method of separation of variables

$$\psi(r,\vartheta,t) = R(r)\,\Phi(\vartheta)\,T(t)$$

gives in this case [41] :

$$M(x,y,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\left\{\frac{\pi r_0^2}{2}\varepsilon_n\left[J_n'(\rho_k^{(n)})\right]^2\right\}^{-1} e^{-\frac{1}{2}\left(\frac{\rho_k^{(n)}}{r_0}\right)^2 t} \times$$

$$\times\left[J_n\left(\frac{\rho_k^{(n)}}{r_0}|x|\right)\cdot\cos n\vartheta_x \cos n\vartheta_y + J_n\left(\frac{\rho_k^{(n)}}{r_0}|x|\right)\sin n\vartheta_x \sin n\vartheta_y\right]\cdot J_n\left(\frac{\rho_k^{(n)}}{r_0}|y|\right),$$

where

$$\varepsilon_n = \begin{cases}1, & n\neq 0\\ 2, & n=0\end{cases};\qquad \vartheta_x = \operatorname{arctg}\frac{x_2}{x_1},\qquad \vartheta_y = \operatorname{arctg}\frac{y_2}{y_1}.$$

$J_n(\rho)$ is the Bessel function of n-th order; $\rho_k^{(n)}$ is the k-th root of equation $J_n(\rho) = 0$.

After integrating over t we find

$$\mathcal{K}_{\partial\Lambda}(x,y) = \sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\left\{\frac{\pi r_0^2}{2}\varepsilon_n\left[J_n'(\rho_k^{(n)})\right]^2\cdot\left[m^2+\frac{1}{2}\left(\frac{\rho_k^{(n)}}{r_0}\right)^2\right]\right\}^{-1}\times$$

$$\times J_n\left(\frac{\rho_k^{(n)}}{r_0}|x|\right) J_n\left(\frac{\rho_k^{(n)}}{r_0}|y|\right)\cdot\left(\cos n\vartheta_x \cos n\vartheta_y + \sin n\vartheta_x \sin n\vartheta_y\right).$$

The derived expressions for $\mathcal{K}_{\partial\Lambda}(x,y)$ are the basis for the construction of approximation formulae for the functional integration in the space $\mathcal{S}'(\Lambda)$.

5. CONCLUDING REMARKS

In particular, it follows from the brief review of the literature given in Introduction that among the trends of employing the functional integration method in quantum field theory the following two approaches take an important part. On the one hand, there is a development of the methods which use the lattice regularization scheme including the search for the new modifications of the action functional with improved continuum properties; the perfecting of the lattice computation algorithms which employ the Monte Carlo method [42], especially with the application of parallel computations [43], is also in progress. On the other hand, the approach based on the continuum nonperturbative regularization is being successively developed. As it has been pointed out in [19,15], this approach appears to be attractive as this enables one to study the interesting problems, such as continuum confinement and the general nonperturbative properties of quantum field theory. We consider this approach to be perspective for the numerical calculations because the problems of the finite-size effects and the continuum limit do not appear in it due to the absence of space-time discretization. The favourable possibility of the numerical studying of singularities like phase transitions in the framework of this approach is in one's disposal. The development of this approach is closely connected with the development of an idea of the functional integrals as the mathematical objects on the base of the rigorous definitions of measure in functional spaces. The increasing attention to that is being paid nowdays [3]. The investigation of the covariance operator of the $P(\varphi)_2$ functional measure that has been performed in the present paper, is a step towards the construction of the methods for computation of the physical characteristics in continuum in the framework of the model under consideration. The creation of suitable approximation formulas for the functional integrals will become the subject of our forthcoming works.

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