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V.G.Makhankov, R.Myrzakulov*, O.K.Pashaev

GAUGE EQUIVALENCE, SUPERSYMMETRY AND CLASSICAL SOLUTIONS OF OSPU (1, 1/1) HEISENBERG MODEL AND NONLINEAR SCHRÖDINGER EQUATION

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I. Introduction

During the last few years, nonlinear σ-models with noncompact symmetry group and their supersymmetric extensions have attracted considerable interest [1]. They arise in gravity theory [2], extended supergravity [3], in theory of Anderson localisation [4], the Kaluza-Klein theory [5], in theory of strings [6] and superstrings [7]. The simplest version of the nonlinear σ-model is a continuous classical spin Heisenberg model (HM) and its extensions to higher spins. Then in stationary limit Landau-Lifshitz equations of the corresponding models coincide with the nonlinear σ-model equations. As demonstrated in Refs. [8,9], the one-dimensional isotropic HM on a noncompact manifold of the constant negative curvature $S^{1,1}$ is gauge equivalent to a nonlinear Schrödinger equation (NLSE) of the repulsive type (as is well known, the attractive type NLSE corresponds to the O(3) HM defined on the sphere S^{2} [10]). At the same time the formulation of the Zakharov-Shabat-AKNS scheme on the superalgebra osp (2/1) [11,12] shows the superextension to be allowed only for the repulsive NLSE. The problem naturally arises to construct a σ-model associated with super-NLSE x) on the superalgebra osp (2/1) which is a supergeneralization of the O(2,1) HM [13]. In this paper we construct an integrable generalization of the continuous classical O(2,1) pseudospin Heisenberg model [9] to the case of the ospu (1,1/1) superalgebra. The gauge equivalence of the constructed model and the related NLSE is established. We indicate a method of generating classical solutions using the global supersymmetry ospu (1,1/1). The relationship between solutions to O(2,1) HM and superpartners of NLSE is obtained.

2. NLSE on the Superalgebra ospu (1,1/1)

The linear problem of the corresponding super-NLSE

$$\Phi_x = U\Phi, \qquad \Phi_t = V\Phi \tag{1}$$

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x) At this point and below the terms super-NLSE and super-HM mean that the corresponding Lax pairs are defined on superalgebra.

is given by the 3x3 operators

$$U = -i\begin{pmatrix} \lambda & -\bar{\varphi} & -\varkappa\bar{\psi} \\ \varphi & -\lambda & \psi \\ \psi & \varkappa\bar{\psi} & 0 \end{pmatrix}, \tag{2}$$

$$V = -2\lambda U + i\begin{pmatrix} |\varphi|^2-\rho+2\varkappa\bar{\psi}\psi & -i\bar{\varphi}_x & -2\varkappa i\bar{\psi}_x \\ -i\varphi_x & -|\varphi|^2+\rho-2\varkappa\bar{\psi}\psi & -2i\psi_x \\ -2i\psi_x & 2\varkappa i\bar{\psi}_x & 0 \end{pmatrix},$$

where $\varphi(x,t)$ and $\psi(x,t)$ are the complex boson and fermion fields respectively taking values in the Grassmann algebra, $\varkappa=\pm 1$. The matrices $U_c = iU$ and $V_c = -iV$ satisfy the pseudo-Hermiticity condition

$$\Gamma_\varkappa U_c^+ \Gamma_\varkappa = U_c, \quad \Gamma_\varkappa V_c^+ \Gamma_\varkappa = V_\varkappa, \tag{3}$$

where $\Gamma_\varkappa = \mathrm{diag}\ (1,-1,-\varkappa)$ and therefore are elements of the superalgebra $su(1,1/1)$. The conjugation conditions (3) show that in contrast to the superalgebra $su(2/1)$ /14/ in our case there exist two real superalgebras $su_\pm(1,1/1)$ corresponding to the values $\varkappa=\pm1$ and $\Gamma_\varkappa = \Gamma_\pm$. In fact our U-V pair belongs to some subalgebra of $su_\pm(1,1/1)$. In order to describe it we introduce a vector space $V(2/1)$ with two bosonic and one fermionic dimensions /15/. The orthosymplectic metric tensor $G = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ defines the scalar product

$$(x,y) = G_{\hat{\alpha}\hat{\beta}}\, x^{\hat{\alpha}} y^{\hat{\beta}} = -x^1y^2 + x^2y^1 + x^3y^3 \tag{4}$$

of two superspinors $x, y \in V(2/1)$. The transformations R in the superspace $V(2,1)$, $x' = Rx$, $y' = Ry$ conserving the scalar product (4), $(x', y') = (Rx, Ry) = (x, y)$ from the supergroup $OSP(2/1)$. The corresponding generators $R = e^{iA}$ form the superalgebra $osp\ (2/1)$ and satisfy the condition

$$A^{st} = -\, G A G^{-1}, \tag{5}$$

where A^{st} denotes the supertranspose of A. Let us call $ospu_\varkappa(1,1/1)$ a five-parameter supergroup with elements satisfying pseudounitarity conditions (3) and orthosymplecticity one (5). The generators of $OSPU_\varkappa(1,1/1)$ can be chosen in the form

$$E_k = \frac{1}{2}\left(\begin{array}{cc|c} \multicolumn{2}{c|}{\tau_k} & 0 \\ & & 0 \\ \hline 0 & 0 & 0 \end{array}\right), \quad S_1' = \frac{1}{2}\begin{pmatrix} 0 & 0 & -\varkappa \\ 0 & 0 & 1 \\ 1 & \varkappa & 0 \end{pmatrix}, \quad S_2' = \frac{i}{2}\begin{pmatrix} 0 & 0 & \varkappa \\ 0 & 0 & 1 \\ 1 & -\varkappa & 0 \end{pmatrix}, \tag{6}$$

$k = 1,2,3$

where E_k and S_1', S_2' are the bosonic and fermionic generators respectively and τ_k are the generators of the group $SU(1,1)$. They satisfy the commutation relations

$$\begin{aligned}
&[E_1, E_2] = -iE_3, && [E_2, E_3] = iE_1, && [E_1, E_3] = -iE_2, \\
&[E_1, S_1'] = \tfrac{\varkappa i}{2}S_1', && [E_1, S_2'] = -\tfrac{\varkappa i}{2}S_2', && \{S_1', S_1'\} = E_2 - \varkappa E_3, \\
&[E_2, S_1'] = \tfrac{\varkappa i}{2}S_2', && [E_2, S_2'] = \tfrac{\varkappa i}{2}S_1', && \{S_1', S_2'\} = -E_1, \\
&[E_3, S_1'] = \tfrac{i}{2}S_2', && [E_3, S_2'] = -\tfrac{i}{2}S_1', && \{S_2', S_2'\} = -E_2 - \varkappa E_3
\end{aligned} \tag{7}$$

or

$$[E_\alpha, E_\beta\} = E_\alpha E_\beta - (-1)^{P(\alpha)P(\beta)} E_\beta E_\alpha = i\, c_{\alpha\beta\gamma} E_\gamma, \tag{8}$$

where $p(\alpha)=0$ for bosonic generators and $p(\beta)=1$ for fermionic ones, $E_4 = S_1'$, $E_5 = S_2'$ and $c_{\alpha\beta\gamma}$ are the structure constants. Then $str\,(E_\alpha E_\beta) = \frac{1}{2} g_{\alpha\beta} = \frac{1}{2}\, diag(-1,-1,1,2\tau_1)$.

It is easy to verify that the U,V pair (2) in each point of space-time (x,t) is defined on an algebra of the noncompact supergroup $OUSP_\varkappa(1,1/1)$. The conditions of compatibility of system (I) $U_t - V_x + [U, V] = 0$ lead to the super-NLSE

$$i\varphi_t + \varphi_{xx} - 2\left(|\varphi|^2 - \rho + 2\varkappa\bar{\psi}\psi\right)\varphi + 4i\psi\psi_x = 0, \tag{9a}$$

$$i\psi_t + 2\psi_{xx} - \left(|\varphi|^2 - \rho\right)\psi + i\varkappa\left(2\varphi\bar{\psi}_x + \varphi_x\bar{\psi}\right) = 0. \tag{9b}$$

Let us note that we choose the U,V pair and equations of motion in such a way that the boundary conditions for the conventional NLSE of repulsive type $|\varphi|^2 \xrightarrow[x\to\pm\infty]{} \rho$ are a solution to the corresponding super-NLSE $\psi \xrightarrow[x\to\pm\infty]{} 0$. From Eq.(9a) we see that the sign of coupling constant $\varkappa$ characterizes interaction of the boson field $\varphi(x,t)$ with the fermion field $\psi(x,t)$ (the repulsion at $\varkappa=+1$ and the attraction for $\varkappa=-1$) and respectively two algebras of the linear problem (I) $ospu_\pm(1,1/1)$.

Eqs. (9) are Hamiltonian equations. The Poisson superbracket for two functionals A and B of the fields φ and ψ is defined conventionally

$$\{A,B\} = i\int dx \left\{ \left(\frac{\delta A}{\delta\varphi}\frac{\delta B}{\delta\bar\varphi} - \frac{\delta A}{\delta\bar\varphi}\frac{\delta B}{\delta\varphi} \right) - \frac{æ}{4}\left(A \frac{\overleftarrow{\delta}}{\delta\psi}\frac{\overrightarrow{\delta}}{\delta\bar\psi} B + B\frac{\overleftarrow{\delta}}{\delta\psi}\frac{\overrightarrow{\delta}}{\delta\bar\psi} A \right) \right\}. \tag{10}$$

Then canonically conjugate variables are φ, $\bar\varphi$ and ψ, $\bar\psi$

$$\{\varphi(x), \bar\varphi(y)\} = i\delta(x-y), \quad \{\psi(x), \bar\psi(y)\} = -\frac{æ i}{4}\delta(x-y). \tag{11}$$

Eqs. (10) have the Hamiltonian form

$$\varphi_t = \{H, \varphi\}, \quad \psi_t = \{H, \psi\}, \tag{12}$$

where the Hamiltonian function H is

$$H = \int dx \left\{ \bar\varphi_x\varphi_x + \left(|\varphi|^2 - \rho + 2æ\bar\psi\psi\right)^2 + 8æ\bar\psi_x\psi_x - 4i\left(\bar\varphi\psi\psi_x + \varphi\bar\psi\bar\psi_x\right) \right\}. \tag{13}$$

From Eq. (9) the continuity equation $\partial_\mu J_\mu = 0$, $\mu = 0,1$ follows, where $J_0 = |\varphi|^2 + 2æ\bar\psi\psi$, $J_1 = i(\bar\varphi\varphi_x - \bar\varphi_x\varphi) + 4æ(\bar\psi_x\psi - \bar\psi\psi_x)$.
The corresponding integral of motion (the "number of particles") has the form

$$I_1 = \int_{-\infty}^{\infty} dx \left(|\varphi|^2 + 2æ\bar\psi\psi \right) \tag{14}$$

and corresponds to the invariance of the super-NLSE(9) with respect to U(I) global symmetry $\varphi \to \varphi' = e^{2i\alpha}\varphi$, $\psi \to \psi' = e^{i\alpha}\psi$. Then the U,V pair (2) is transformed under global gauge transformations of the U(I) subgroup of OSPU (1,1/1) as $U' = R^{-1}UR$, $V' = R^{-1}VR$, where $R = \exp(iE_3\alpha)$. In the form of local gauge transformations the Galilei, Backlund and so on transformations can be realised, too.

3. OSPU (1,1/1) Continual Heisenberg Model

Let us construct a σ-model associated with the super NLSE (9). To do this we consider local gauge transformations of U,V pair (2):

$$\Phi = g\Phi', \quad U' = g^{-1}Ug - g^{-1}g_x, \quad V' = g^{-1}Vg - g^{-1}g_t, \tag{15}$$

where the solution of linear problem (I) in the point $\lambda = \lambda_0$: $g(x,t,\lambda_0) = \Phi(x,t,\lambda=\lambda_0)$ is chosen to be a gauge group element $g \in$ OSPU (1,1/1). For simplicity, we shall take $\lambda_0 = 0$ (see details in Ref. /9/). Then Φ' satisfies the linear system

$$\Phi'_x = U'\Phi', \qquad \Phi'_t = V\Phi', \tag{16}$$

where $U' = -i\lambda S'$, $V = -2i\lambda^2 S' + \lambda\left(2[S', S'_x] + 3S'^2 S'_x S'\right)$,

$$S' = g^{-1}E_3 g. \tag{17}$$

Since $g \in$ OSPU (1,1/1) and U(I)-local gauge transformations $g \to e^{iE_3\beta(x,t)}g$ keep the S matrix (2) unchanged, then $S' \in$ OSPU(1,1/1) / U(I). The matrix S can be parametrized as follows

$$S' = \sum_{i=1}^{5} S'_i E_i = \begin{pmatrix} S'_3 & iS'^- & -æ\bar c \\ iS'^+ & -S'_3 & c \\ c & æ\bar c & 0 \end{pmatrix}, \tag{18}$$

where $c = \frac{1}{2}(S_4 + iS_5)$, $p(S_i) = 0$, $p(c) = 1$, $S^\pm = -\frac{i}{2}(S'_1 \pm iS'_2)$.
It follows from the definition of S (17) that

$$S'^3 = S' \tag{19a}$$

or in components

$$S'^2_3 - S'^+S'^- - 2æ\bar c c = 1. \tag{19b}$$

Condition (19) is a generalisation of the known condition $S'^2 = I$ of the theory of SU(1,1) HM /9/ to the superalgebra case and cannot be reduced to it because the matrix S is degenerate. The matrix $\begin{pmatrix} S_3 & iS^- \\ iS'^+ & -S_3 \end{pmatrix}$ is a boson block and coincides in the form with the pseudo-spin matrix for the Landau-Lifshitz SU(1,1) equation /9/. The consistency conditions for system (16) lead to the Landau-Lifshitz OSPU (1,1/1) / U(I) equation *

$$iS'_t = 2[S', S'_{xx}] + 3(S'^2 S'_x S')_x. \tag{20}$$

It follows from the definition of S' and g that

* Eqs. (16), (19) and (20) have been obtained by one of the authors (R.M.) earlier.

$$S'_x = g^{-1}[E_3, U]g. \tag{21}$$

Thus we have the following expression in terms of S for the particle number density (14):

$$|\varphi|^2 + 2\varkappa\bar{\Psi}\Psi = -\frac{1}{8}\,\mathrm{str}\left(7S_x'^2 - 6S'S_x'^2S'\right) = -\frac{1}{8}\,\mathrm{str}\left\{S_x'^2 + 3\left[(S'^2)_x\right]^2\right\}. \tag{22}$$

Since the integral of the particle number density is a conserved quantity, the right-hand side of Eq. (22) must be the energy density of OSPU (1,1/1) HM

$$H = \frac{1}{4}\int_{-\infty}^{\infty} dx\,\mathrm{str}\left\{S_x'^2 + 3\left[(S'^2)_x\right]^2\right\} \tag{23a}$$

or in components

$$H = \frac{1}{2}\int_{-\infty}^{\infty} dx\left\{S_{3x}'^2 - S_x'^+S_x'^- - 2\varkappa\bar{c}_x c_x - 3\bar{c}_x c_x\bar{c}c - 6\varkappa\left(\varkappa S_3'\bar{c} - iS'^-c\right)_x\left(\varkappa S_3'c + iS'^+\bar{c}\right)_x\right\}. \tag{23b}$$

Indeed, the Hamilton equations of motion $S_t'^\alpha = \{H, S'^\alpha\}$ with Hamilton function (23) and the Poisson's superbracket on the curved phase space associated with the superalgebra ospu (1,1/1)

$$\{A, B\} = \int dx \sum_{\alpha\beta\gamma=1}^{5} C_{\alpha\beta\gamma}\, A\frac{\overleftarrow{\delta}}{\delta S'_\alpha}\, S'_\gamma\, \frac{\overrightarrow{\delta}}{\delta S'_\beta} B, \tag{24}$$

where $C_{\alpha\beta\gamma}$ are the structure constants of OSPU (1,1/1) (see eqs. (7),(8)), coincide with Landau-Lifshitz equations (20). It is interesting that relation (22) is a nontrivial generalization of the well known relation between the NLSE particle number density and the energy density for the Landau-Lifshits SU(2) /10/ and SU (1,1) /9/ equations. Besides usual terms Hamiltonian (23) also contains four-fermion interaction terms and pair Bose-Fermi terms. If $S'^2 = I$, Hamiltonian (23) takes the conventional form /9/.

4. Global Supersymmetry of OSPU (1,1/1) HM

Let us consider global gauge transformations from OSPU (1,1/1) generated by the fermion generators E_4, E_5

$$R = \exp i(\theta_1 E_4 + \theta_2 E_5) = \exp i(\theta q_2 - \bar{\theta} q_1), \tag{25}$$

where $\theta = \frac{1}{2}(\theta_1 + i\theta_2)$, $\bar{\theta} = (\theta)^*$ are the Grassmann parameters $q_1 = -\varkappa(E_4 + iE_5)$ and $q_2 = E_4 - iE_5$ are the generators of the superalgebra osp (2/1). Since in this case $S' \to S'' = R^{-1}S'R$ and $S'_x \to S_x'' = R^{-1}S'_x R$ Hamilton function (23) and the form of the equations of motion (20) are invariant and the U,V pair (16) are transformed under the similarity transformation $U' = R^{-1}UR$, $V' = R^{-1}VR$. We have

$$\delta S' = [S', R], \quad R = I + i(\theta q_2 - \bar{\theta} q_1) \tag{26}$$

in the infinitesimal form, or

$$\delta S_3' = \bar{\theta}c - \bar{c}\theta, \quad \delta S'^+ = 2c\theta, \quad \delta c = -iS_3'\theta + \varkappa S'^+\bar{\theta} \tag{27}$$

in the component form. Owing to the nilpotent property of the Grassmann parameters θ, $\bar{\theta}$ the infinite series (25) is broken up, and one can obtain a matrix of finite supersymmetry transformations $R = I + i(\theta q_2 - \bar{\theta} q_1) + \frac{\bar{\theta}\theta}{2}[q_1, q_2]$ which generates transformation of the S fields

$$S_3'' = S_3'(1 + \bar{\theta}\theta) + i(\bar{\theta}c - \bar{c}\theta), \quad S'^{+\prime} = S'^+(1 + \bar{\theta}\theta) + 2c\theta,$$
$$c' = c\left(1 - \frac{3}{2}\bar{\theta}\theta\right) - iS_3'\theta + \varkappa S'^+\bar{\theta}. \tag{28}$$

Transformations of global supersymmetry can be realised in the Hamiltonian form. In fact, the components of the "supermagnetisation" vector

$$M_\alpha = \int_{-\infty}^{\infty} S'_\alpha(x,t)\,dx \tag{29}$$

on Poisson's superbrackets (24) satisfy the algebra OSPU (1,1/1)

$$\{M_\alpha, M_\beta\} = \sum_{\gamma=1}^{5} C_{\alpha\beta\gamma} M_\gamma \tag{30}$$

and "commute" with Hamilton function (23) $\{H, M_\alpha\} = 0$. They generate rotations of the vector $\vec{S}' = \{S'_\alpha\}$ around the corresponding axes:

$$\delta S'_\alpha = \sum_\beta \{M_\beta, S'_\alpha\}\,\theta_\beta, \tag{31}$$

where θ_β is the rotation parameter in "superspace". Among transformations (31) there are SU(1,1) rotations of boson components generated by M_1, M_2, M_3 and super-rotations (27) generated by M_4, M_5 and mixing boson and fermion components. It is important, however, that the choice of boundary conditions for $\vec{S}(x,t)$ reduces the number of functionals M_α allowed.

5. Classical Solutions and Supersymmetry

Global supersymmetry transformations (28) allow one to obtain classical solutions of the OSPU (1,1/1) HM using solutions of the usual SU(1,1) HM . In fact, super-HM(20) has the following solutions: $S'_3 = \tilde{S}_3$, $S'^{+} = \tilde{S}^{+}$, $c = \hat{c} = 0$, where $\tilde{S}$ is the solution of the SU(1,1) HM . Using (28), we can obtain a new solution of equation (20) which depends on two Grassmann parameters θ, $\bar{\theta}$:

$$
\begin{aligned}
S'_3 &= \tilde{S}_3 \left(1 + \varkappa \bar{\theta}\theta\right), \\
S'^{+} &= \tilde{S}^{+} \left(1 + \varkappa \bar{\theta}\theta\right), \\
c &= -i \tilde{S}'_3 \theta + \varkappa \tilde{S}^{+} \bar{\theta}.
\end{aligned}
\tag{32}
$$

The further rotations allow one to generate new solutions of Eq.(20) from Eq.(32). Let us consider, for example, a superanalogue of the pseudospin wave (a classical analogue of the Bogolubov condensate) [18]

$$
\begin{aligned}
\tilde{S}'_3 &= \frac{\sqrt{k^2 + 4\rho}}{k}, \\
\tilde{S}^{+} &= \frac{2\sqrt{\rho}}{k} e^{-i\beta(x,t)}, \\
\beta(x,t) &= k\left(x - \sqrt{k^2 + 4\rho}\, t\right).
\end{aligned}
\tag{33}
$$

There is a corresponding superpseudospin wave with the boson component

$$
S'(x,t) = \tilde{S}(x,t)\left(1 + \varkappa \bar{\theta}\theta\right)
\tag{34}
$$

and the fermion component

$$
C(x,t) = \frac{i}{k}\left(-\sqrt{k^2 + 4\rho}\,\theta + 2\varkappa\sqrt{\rho}\, e^{-i\beta}\bar{\theta}\right).
\tag{35}
$$

The related fermion component density is constant $\bar{C}(x,t)\cdot C(x,t) = \bar{\theta}\theta$. As shown in Ref. [9], solutions of the SU(1,1) HM can be obtained using Jost solutions for NLSE. Now we shall show that Jost solution for the linear problem of the usual NLSE allows one to obtain solutions

for its superpartner as well [16]. Let us consider an expansion of the field variables of super-NSE (I0) in the basis of the Grassmann two-dimensional algebra

$$
\begin{aligned}
\varphi(x,t) &= \varphi_1(x,t)\,\theta + \varphi_2(x,t)\,\bar{\theta}\theta, \\
\psi(x,t) &= \psi_1(x,t)\,\theta + \psi_2(x,t)\,\bar{\theta},
\end{aligned}
\tag{36}
$$

where φ_i, ψ_i are the usual C-number functions. Substituting (36) into (9) we obtain

$$
i\varphi_{1t} + \varphi_{1xx} - 2\left(|\varphi_1|^2 - \rho\right)\varphi_1 = 0,
\tag{37}
$$

$$
\begin{aligned}
i\varphi_{2t} + \varphi_{2xx} - 4|\varphi_1|^2\varphi_2 + 2\rho\varphi_2 - 2\varphi_1^2\bar{\varphi}_2 - 4\varkappa\left(|\psi_1|^2 - |\psi_2|^2\right) + \\
+ 4i\left(\psi_2\psi_{1x} - \psi_1\psi_{2x}\right) = 0,
\end{aligned}
\tag{38}
$$

$$
i\psi_{jt} + 2\psi_{jxx} - \left(|\varphi_1|^2 - \rho\right)\psi_j - \varkappa\left[2\varphi_1(\tau_1)_{jk}\bar{\psi}_{kx} + \varphi_{1x}(\tau_1)_{jk}\bar{\psi}_k\right] = 0.
\tag{39}
$$

It is seen that (37) being the usual NLSE of the repulsive type for $\varphi_1(x,t)$ can be integrated via the inverse problem method [17]. A linear problem for NLSE (37) has the form

$$
\begin{aligned}
\Phi_x &= \left[-i\lambda\sigma_3 + i\begin{pmatrix} 0 & \bar{\varphi}_1 \\ -\varphi_1 & 0 \end{pmatrix}\right]\Phi, \\
\Phi_t &= \left[-2\lambda U + i\begin{pmatrix} |\varphi_1|^2 - \rho & -i\bar{\varphi}_{1x} \\ -i\varphi_{1x} & -|\varphi_1|^2 + \rho \end{pmatrix}\right]\Phi,
\end{aligned}
\tag{40}
$$

where

$$
U = -i\lambda\sigma_3 + i\begin{pmatrix} 0 & \bar{\varphi}_1 \\ -\varphi_1 & 0 \end{pmatrix}.
$$

Eliminating the spectral parameter λ from system (40), we obtain equations for the components $\Phi^t = (\bar{\Phi}_1, \Phi_2)$:

$$
i\Phi_{jt} + 2\Phi_{jxx} - \left(|\varphi_1|^2 - \rho\right)\Phi_j - \left[2\varphi_1(\tau_1)_{jk}\bar{\Phi}_{kx} + \varphi_{1x}(\tau_1)_{jk}\bar{\Phi}_k\right] = 0.
\tag{41}
$$

Comparing systems (39) and (41) we see that at $\varkappa = 1$ they coincide, i.e. the corresponding variables can be identified $\psi_j(x,t) = \Phi_j(x,t)$. Thus if one knows the soliton solutions of U(0,1) NLSE (37) and the corresponding Jost solutions for the linear problem (40), one can construct solutions for the supersymmetrical partners (39). Choosing a definite relation for the components which is in agreement with (39) only under certain conditions, one can reduce Eq.(38) to the equation for the function ψ_{1x}. As was mentioned above, Jost solutions for U(0,1) NLSE allow generating solutions for SU(1,1) HM [9]. On the other hand, the relation also follows of U(0,1) NLSE superpartners with SU(1,1) HM from what was mentioned above. In fact the condition $\psi_j = \Phi_j$ implies that the pseudospin matrix $S' = \Phi^{-1}\sigma_3\Phi$, where $\Phi = \begin{pmatrix} \bar\psi_1 & \bar\psi_2 \\ \psi_2 & \psi_1 \end{pmatrix} \in SU(1,1)$ can be parameterized by ψ_j superpartners of U(0,1) of NLSE in the form

$$S_3 = \frac{|\psi_1|^2 + |\psi_2|^2}{|\psi_1|^2 - |\psi_2|^2}, \quad S^+ = \frac{2\bar\psi_1 \psi_2}{|\psi_1|^2 - |\psi_2|^2}. \tag{42}$$

Thus, the solutions for ψ_j -superpartners of NLSE generate solutions for SU(1,1) HM. It means, in particular, that pseudospin wave (33), a classical analogue of the Bogolubov condensate [18], is naturally expressed through the superpartners of NLSE. In conclusion we note that (42) leads to the pseudostereographic projection of HM:

$$S'_3 = \frac{1+|\zeta|^2}{1-|\zeta|^2}, \quad S'^+ = \frac{2\zeta}{1-|\zeta|^2},$$

where $\zeta = \psi_2(x,t)/\psi_1(x,t)$.

Here the functions $\zeta(x,t)$ will be a solution for the modified NLSE [9]:

$$i\zeta_t + \zeta_{xx} + \frac{2\bar\zeta(\zeta_x)^2}{1-|\zeta|^2} = 0.$$

More detailed results will be presented separately.

In conclusion let us note that the integrable super generalization of the continuous classical $O(3)$ Heisenberg ferromagnet model is uosp(2/1) HM [13]. The equation of motion and the Lax pair of uosp (2/1) HM has the same form as (20) and (16), respectively, where

$$S = \begin{pmatrix} S_3 & S^- & \varkappa\tilde c \\ S^+ & -S_3 & c \\ c & -\varkappa\tilde c & 0 \end{pmatrix}$$

and obeys the condition $S^3 = S$ on $S_3^2 + S^+S^- + 2\varkappa\tilde c c = 1$. Here tilde denotes an adjoint operation of the second kind [19]. The matrix $i\cdot S$ belongs to uosp (2/1) at $\varkappa = +1$. The uosp (2/1) HM

is gauge equivalent to uosp (2/1) NLSE, the latter being a super generalization of the conventional NLSE of the attractive type.[13]

More detailed results concerning uosp (2/1) HM and uosp (2/1) NLSE will be published by one of the authors (R.M.) separately.

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Маханьков В.Г., Мырзакулов Р., Пашаев О.К. E5-88-4
Калибровочная эквивалентность, суперсимметрия и классические решения ospu(1, 1/1) - МГ и НУШ

Построено интегрируемое обобщение непрерывной классической o(2,1) псевдоспиновой модели Гейзенберга на случай супералгебры ospu(1, 1/1). Калибровочная эквивалентность построенной модели и соответствующего НУШ установлена. Указан метод генерирования классических решений с использованием глобальной суперсимметрии ospu(1, 1/1). Получена связь между решениями для суперпартнеров НУШ и o(2,1) - МГ.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Makhankov V.G., Myrzakulov R., Pashaev O.K. E5-88-4
Gauge Equivalence, Supersymmetry and Classical Solutions of ospu(1, 1/1) Heisenberg Model and Nonlinear Schrödinger Equation

An integrable generalization of the continuous classical O(2,1) pseudospin Heisenberg model for the case of the ospu (1,1/1) superalgebra is constructed. The gauge equivalence of the constructed model and the corresponding NLSE is established. We indicate a method of generating classical solutions using the global ospu (1, 1/1) supersymmetry. The relationship between solutions of O(2,1) HM and superpartners of NLSE is obtained.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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