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ON THE INVERSE PROBLEM OF A DISSIPATIVE SCATTERING THEORY

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1. Introduction

In a scattering theory the problem naturally arises of characterizing possible scattering operators or scattering matrices. Such a problem was posed and solved by V.M.Adamjan and D.Z.Arov [1] for the Lax-Phillips scattering theory with and without losses who found that every measurable contraction-valued function can be regarded as the scattering matrix of a Lax-Phillips scattering theory which is lossless in the case of a unitary-valued function and which is with losses in the opposite case.

This result was generalized by M.Wollenberg [2,14,15] who established that every operator which is unitary on the absolutely continuous subspace of the free evolution and elsewhere zero and which commutes with the free evolution can be viewed as a scattering operator of a complete scattering system of selfadjoint operators [2].

In [6] C.Foias characterizes the scattering matrices of the so-called non-conservative Lax-Phillips scattering theory [9].

A further dissipative scattering theory was considered in [10] which generalizes the scattering theory of selfadjoint operators [2] to maximal dissipative operators and which includes the non-conservative Lax-Phillips scattering theory. A first step to describe the possible scattering operators of this scattering theory has been made in [11]. It was shown that every contraction, which is zero outside the absolutely continuous subspace of the free evolution, which commutes with the free evolution and which satisfies the additional condition that the ranges of the contraction itself and its adjoint are dense in the absolutely continuous subspace of the free evolution, can be regarded as a scattering operator of a complete dissipative scattering system [10]. Moreover, in this special case the maximal dissipative operator describing the perturbed evolution can be chosen from the class C_{11}. Obviously, the result extends the conclusions of M.Wollenberg [2,14,15].

In this note we generalize the paper [11] in two directions. Firstly, we suppose that we have different free evolutions for the past and for the future. This generalization we need to compare our results with those of the dissipative Lax-Phillips scattering theory. Secondly, we allow arbitrary contractions intertwining the free evolutions and actually acting from the absolutely continuous subspace of the past free evolution into that of the future free evolution.

In section three we compare our results with those of [6,13]. We establish that in distinction to the dissipative or non-conservative Lax-Phillips scattering theory every measurable contraction-valued function can be viewed as a scattering matrix of a complete dissipative scattering system.

Naturally, the question arises to indicate the difference to V.M.Adamjan and D.Z.Arov [1]. In [1] the desired non-unitary matrix was obtained with the help of incompleteness of the scattering system full Hamiltonian of which is selfadjoint. We have a complete scattering system which forces the non-unitarity of the scattering matrix by the dissipativity of the full Hamiltonian.

In the last section we give an application of the obtained results to nuclear physics. Considering scattering processes in nuclear physics, which involve a number of particles, we are very often interested only in a part of the possible reactions. These interesting scattering reactions can be described by the compression of the full unitary scattering operator on a certain subspace of the state Hilbert space which reduces the free evolution.

Naturally, the following question can be posed: Is it possible to forget the complicated structure of the full system and to replace the evolution by a certain non-unitary evolution on the interesting subspace (taking into consideration in such a way the effects of the other reactions) such that the partial scattering operator is nearly uneffected by this replacement? This idea was firstly realized by H.Feshbach, C.E.Porter and V.F.Weisskopf [4]. In a greater generality this problem was solved by H.Feshbach [5,6] who introduced the so-called optical potential [3]. But in general it seems to us that the problem is unsolved. At the end of the paper we give a mathematically rigerous proof of the assertion that in every case such a replacement is possible. It is important to note that the solution is not unique in the class of dissipative operators. The disadvantage of the presented proof is that we cannot indicate the structure of the obtained full Hamiltonian how this is possible in the case of the optical potential. However, our dissipative operator exactly reproduces the partial scattering operator.

2. Inverse problem

Let $L_{\pm}$ be two selfadjoint operators on the separable Hilbert spaces $\mathcal{L}_{\pm}$. Further, let H be a maximal dissipative operator, $\mathrm{Im}(Hf,f) \leq 0$, $f \in \mathrm{dom}(H)$, on the separable Hilbert space $\mathfrak{h}$. Obviously, the operator H generates a one-parameter contraction semigroup T(t),

$$T(t) = e^{-itH}, \qquad (2.1)$$

$t \geq 0$. By $J_{\pm}$ we denote bounded linear operators acting from $\mathcal{L}_{\pm}$ into $\mathfrak{h}$, i.e. $J_{\pm}: \mathcal{L}_{\pm} \longmapsto \mathfrak{h}$, which we call the identification operators. Using these objects we introduce the wave operators W_+,

$$W_+ = \operatorname*{s-lim}_{t\to+\infty} e^{itH^*} J_+ e^{-itL_+} P^{ac}(L_+), \qquad (2.2)$$

and W_-,

$$W_- = \operatorname*{s-lim}_{t\to+\infty} e^{-itH} J_- e^{itL_-} P^{ac}(L_-), \qquad (2.3)$$

where $P^{ac}(.)$ denotes the projection onto the absolutely continuous subspace of a selfadjoint operator. We remark that if the operator H is selfadjoint, then the definitions of W_+ and W_- coincide with the usual ones of selfadjoint operators. We assume that the wave operator $W_\pm$ is complete [10,11].

We call the 5-tuple $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ a complete scattering system if the wave operators $W_\pm$ exist and are complete.

With every complete scattering system $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ we associate a scattering operator S defined by

$$S = W_+^* W_- \;. \qquad (2.4)$$

It is not hard to show that S is a contraction obeying the conditions

$$\operatorname{ima}(S) \subseteq \mathcal{L}_+^{ac}(L_+), \qquad (2.5)$$

$$\ker(S) \supseteq \mathcal{L}_- \ominus \mathcal{L}_-^{ac}(L_-), \qquad (2.6)$$

where $\mathcal{L}_\pm^{ac}(L_\pm)$ are the absolutely continuous subspaces of $L_\pm$, and

$$e^{-itL_+} S = S\, e^{-itL_-} , \qquad (2.7)$$

$t \in \mathbb{R}^1$. Our problem is the following: Suppose that the operators L_+,L_-,J_+ and J_- are given and that there is a contraction S satisfying the conditions (2.5) - (2.7). Does there exist a maximal dissipative operator H on $\mathfrak{h}$ such that $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ forms a complete scattering system whose scattering operator $S(\mathcal{A})$ coincides with S, i.e.

$$S(\mathcal{A}) = S? \qquad (2.8)$$

First of all we remark that the existence of such a solution depends on the identification operators J_+ and J_-. Only if the identification operators fulfil certain properties a solution is possible.

<u>Definition 2.1:</u> We say the identification operators J_+ and J_- are admissible with respect to L_+ and L_- if there are two partial isometries $F_\pm$: $\mathcal{L}_\pm \longmapsto \mathfrak{h}$ obeying

(i) $F_\pm^* F_\pm = P^{ac}(L_\pm)$,

(ii) $F_+^* F_- = 0$

(iii) $\operatorname*{s-lim}_{t\to\pm\infty} (F_\pm - J_\pm) e^{-itL_\pm} P^{ac}(L_\pm) = 0$.

<u>Remark 2.2:</u> If $\mathcal{L}_+ = \mathcal{L}_- = \mathcal{L}$, $L_+ = L_- = L$ and $J_+ = J_- = J$, then it can be shown that the assumptions of Definition 2.1 are fulfilled if there is a partial isometry $F: \mathcal{L} \longmapsto \mathfrak{h}$ with $F^*F = P^{ac}(L)$ such that

$$\operatorname*{s-lim}_{t\to\pm\infty} (F - J) e^{-itL} P^{ac}(L) = 0 \qquad (2.9)$$

holds. Hence in this case Definitions 2.3 of [11] and Defenition 2.1 are equivalent.

The following theorem answers the proposed problem.

Theorem 2.3: Let $L_\pm$ be two selfadjoint operators on $\mathcal{L}_\pm$ and let J_+ and J_- be two identification operators which are admissible with respect to L_+ and L_-. If S is a contraction obeying (2.5) - - (2.7), then there is a maximal dissipative operator H on $\mathfrak{h}$ such that $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ is a complete scattering system whose scattering operator $S(\mathcal{A})$ coincides with S, i.e. $S(\mathcal{A}) = S$.

Remark 2.4: The assumption that J_+ and J_- are admissible with respect to L_+ and L_- is not only sufficient but also necessary. It can be proved that if $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ is a complete scattering system, then the identification operators J_+ and J_- are admissible with respect to L_+ and L_- [12].

Remark 2.5: If in addition S is a contraction obeying $\ker(S) = \mathcal{L}_- \ominus \mathcal{L}_-^{ac}(L_-)$ and $(\mathrm{ima}(S))^- = \mathcal{L}_+^{ac}(L_+)$, then the theorem can easily be obtained from Theorem 3.7 of [11] by reducing the problem to the form of [11] and taking into account Remark 2.2. Moreover, in this case H can be taken from the class C_{11}.

If S is even a partial isometry acting from $\mathcal{L}_-^{ac}(L_-)$ onto $\mathcal{L}_+^{ac}(L_+)$, then the solution follows already from the above mentioned papers of M.Wollenberg [2,14,15] and H can be chosen selfadjoint.

Remark 2.6: The solution of Theorem 2.3 is not unique. Having one solution it is not hard to construct a family of solutions.

Further we note that S = 0 fulfils the conditions (2.5) - - (2.7). Hence there is a complete scattering system $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ with $S(\mathcal{A}) = 0$.

In order to prove Theorem 2.3 we try to apply Theorem 3.7 of [11]. But this is only partially possible. The remaining part, which cannot reduce to Theorem 3.7 of [11], is handled by using a certain generalization of Lemma 3.1 of [11].

Lemma 2.7: Let L be an absolutely continuous selfadjoint operator

on the separable Hilbert space $\mathcal{L}$. Then there is a non-negative densely defined closed quadratic form $\gamma_0(.,.)$ on $\mathcal{L}$ with the following properties:

$$e^{-itL}\,\mathrm{dom}(\gamma_0) \subseteq \mathrm{dom}(\gamma_0),\ t \geq 0, \tag{2.10}$$

$$\|f\|^2 \leq \gamma_0(e^{-itL}f, e^{-itL}f) \leq \gamma_0(f,f),\ f \in \mathrm{dom}(\gamma_0),\ t \geq 0, \tag{2.11}$$

$$\lim_{t \to +\infty} \gamma_0(e^{-itL}f, e^{-itL}f) = \|f\|^2,\ f \in \mathrm{dom}(\gamma_0), \tag{2.12}$$

$$\mathfrak{D} = \{f \in \bigcap_{t \geq 0} e^{-itL}\,\mathrm{dom}(\gamma_0):\ \sup_{t \geq 0} \gamma_0(e^{itL}f, e^{itL}f) < < +\infty\} = \{0\}. \tag{2.13}$$

Proof: We prove Lemma 2.7 in several steps.

1. We introduce the Hilbert space $L^2(\mathbb{R}^1,\mathfrak{k})$, where $\mathfrak{k}$ is a separable infinite dimensional Hilbert space. Let B_0 be the generator of the shift group e^{-itB_0} on $L^2(\mathbb{R}^1,\mathfrak{k})$ defined by

$$(e^{-itB_0}f)(x) = f(x-t), \tag{2.14}$$

$f \in L^2(\mathbb{R}^1,\mathfrak{k})$, $t \in \mathbb{R}^1$. Since L is absolutely continuous we can regard L as a part of B_0. Hence there is an isometry F_0 from $\mathcal{L}$ into $L^2(\mathbb{R}^1,\mathfrak{k})$ such that

$$e^{-itB_0}F_0 = F_0 e^{-itL}, \tag{2.15}$$

$t \in \mathbb{R}^1$. Let $\{f_k\}_{k=1}^\infty$ be an orthonormal basis in $\mathcal{L}$. We choose an arbitrary sequence of complex numbers $\{a_k\}_{k=1}^\infty$ satisfying the properties $a_k \neq 0$, $k = 1,2,\dots$, and

$$\sum_{k=1}^{\infty} |a_k|^2 = 1. \tag{2.16}$$

With the help of this sequence we define the real function $g(.): \mathbb{R}^1 \longmapsto \mathbb{R}^1$,

$$g(x) = \sum_{k=1}^{\infty} |a_k|^2 \|(F_o f_k)(x)\|^2 \geqslant 0, \tag{2.17}$$

where $\|.\|$ is the norm of $\mathfrak{b}$. Obviously, we have

$$\int_{-\infty}^{+\infty} g(x)\, dx = 1. \tag{2.18}$$

We set

$$h(x) = \int_{-\infty}^{x} g(r)\, dr \geqslant 0 , \tag{2.19}$$

$x \in \mathbb{R}^1$. The function $h(.)$ satisfies the conditions

$$0 < h(x) \leqslant h(x') \leqslant 1, \tag{2.20}$$

$-\infty < x \leqslant x' \leqslant +\infty$, and

$$\lim_{x \to -\infty} h(x) = 0, \quad \lim_{x \to +\infty} h(x) = 1 . \tag{2.21}$$

Only the property $0 < h(x)$, $x > -\infty$, is not trivial. Assume that this property is not fulfilled. Then there is a real number x_o such that

$$\int_{-\infty}^{x_o} \|(F_o f_k)(x)\|^2\, dx = 0 \tag{2.22}$$

for every $k=1,2,\ldots$. Since $\{f_k\}_{k=1}^{\infty}$ is an orthonormal basis of $\mathcal{L}$ we get

$$\int_{-\infty}^{x_o} \|(F_o f)(x)\|^2\, dx = 0 \tag{2.23}$$

for every $f \in \mathcal{L}$. Taking into account (2.15) we obtain

$$\begin{aligned} 0 &= \int_{-\infty}^{x_o} \|(F_o e^{-itL} f)(x)\|^2\, dx = \int_{-\infty}^{x_o} \|(F_o f)(x-t)\|^2\, dt = \\ &= \int_{-\infty}^{x_o - t} \|(F_o f)(x)\|^2\, dx \end{aligned} \tag{2.24}$$

for every $f \in \mathcal{L}$ and $t \in \mathbb{R}^1$. But (2.24) yields $F_o f = 0$ for every $f \in \mathcal{L}$. Consequently, our assumption was false and we have $0 < h(x)$, $x > -\infty$.

2. We introduce the function $\varsigma(.): \mathbb{R}^1 \longrightarrow \mathbb{R}^1_+$,

$$\varsigma(x) = (h(x))^{-1/4}, \tag{2.25}$$

$x \in \mathbb{R}^1$. Because of (2.20) this definition makes sense. From (2.20) and (2.21) we obtain

$$1 \leqslant \varsigma^2(x) \leqslant \varsigma^2(x') < +\infty, \tag{2.26}$$

$-\infty < x' \leqslant x \leqslant +\infty$,

$$\lim_{x \to -\infty} \varsigma^2(x) = +\infty, \tag{2.27}$$

$$\lim_{x \to +\infty} \varsigma^2(x) = 1. \tag{2.28}$$

The function $\varsigma(.)$ enables us to define the selfadjoint multiplication operator $M(\varsigma)$ on $L^2(\mathbb{R}^1, \mathfrak{h})$. We set

$$\mathrm{dom}(M(\varsigma)) = \{ f \in L^2(\mathbb{R}^1, \mathfrak{h}) : \varsigma(x) f(x) \in L^2(\mathbb{R}^1, \mathfrak{h}) \} \quad (2.29)$$

and

$$(M(\varsigma) f)(x) = \varsigma(x) f(x), \quad (2.30)$$

$f \in \mathrm{dom}(M(\varsigma))$.

Now we prove that the set $\mathcal{D}_\varsigma$,

$$\mathcal{D}_\varsigma = \{ f \in \mathcal{L} : F_o f \in \mathrm{dom}(M(\varsigma)) \}, \quad (2.31)$$

is dense in $\mathcal{L}$. To this end it is enough to establish that every f_k, k=1,2,..., belongs to $\mathcal{D}_\varsigma$. We find

$$\int_{-\infty}^{+\infty} |\varsigma(x)|^2 \, \| (F_o f_k)(x) \|^2 \, dx \leq$$

$$\leq |a_k|^{-2} \int_{-\infty}^{+\infty} (h(x))^{-1/2} g(x) \, dx = 2|a_k|^{-2}, \quad (2.32)$$

which implies $f_k \in \mathcal{D}_\varsigma$, k=1,2,... .

3. We define the quadratic form $\tau_o(.,.)$ setting

$$\tau_o(f,g) = (M(\varsigma) F_o f, M(\varsigma) F_o g), \quad (2.33)$$

$f,g \in \mathrm{dom}(\tau_o) = \mathcal{D}_\varsigma$. The quadratic form $\tau_o(.,.)$ is non-negative, densely defined and closed.

Taking into account (2.15) and (2.26) we get

$$\| f \|^2 \leq \int_{-\infty}^{+\infty} |\varsigma(x)|^2 \| (F_o e^{-itL} f)(x) \|^2 \, dx \quad (2.34)$$

$$= \int_{-\infty}^{+\infty} |\varsigma(x)|^2 \| (F_o f)(x-t) \|^2 \, dx$$

$$= \int_{-\infty}^{+\infty} |\varsigma(x+t)|^2 \| (F_o f)(x) \|^2 \, dx$$

$$\leq \int_{-\infty}^{+\infty} |\varsigma(x)|^2 \| (F_o f)(x) \|^2 \, dx,$$

$f \in \mathrm{dom}(\tau_o)$, $t \geq 0$. But (2.34) yields (2.10) and (2.11). Because of the Lebesgue dominated convergence theorem, (2.26) and (2.28) we obtain (2.12) from (2.34).

It remains to show (2.13). To this end we assume the existence of a nontrivial element $f \in \mathcal{D}$, i.e. $f \neq 0$. For every $t \geq 0$ we get

$$e^{itL} f \in \mathrm{dom}(\tau_o). \quad (2.35)$$

Consequently, we find

$$\tau_o(e^{itL} f, e^{itL} f) = \int_{-\infty}^{+\infty} |\varsigma(x)|^2 \| (F_o e^{itL} f)(x) \|^2 \, dx$$

$$= \int_{-\infty}^{+\infty} |\varsigma(x-t)|^2 \| (F_o f)(x) \|^2 \, dx \quad (2.36)$$

$$\geq \int_{-\infty}^{t/2} |\varsigma(x-t)|^2 \| (F_o f)(x) \|^2 \, dx,$$

$t \geq 0$. Because of (2.26) we get

$$\tau_o(e^{itL} f, e^{itL} f) \geq |\varsigma(-t/2)|^2 \int_{-\infty}^{t/2} \| (F_o f)(x) \|^2 \, dx, \quad (2.37)$$

$t \geq 0$. Taking into account (2.27)

$$\sup_{t \geq 0} \tau_o(e^{itL} f, e^{itL} f) = +\infty. \quad (2.38)$$

But this is in contradiction with $f \in \mathcal{D}$.

The quadratic form $\gamma_0(.,.)$ is closed. Hence there is a self-adjoint operator $Z \geq I_{\mathcal{L}}$ such that the representation

$$\gamma_0(f,g) = (Z^{1/2}f, Z^{1/2}g), \tag{2.39}$$

$f,g \in dom(\gamma_0) = dom(Z^{1/2})$, holds.

<u>Corollary 2.8:</u> The operator Z obeys the properties

$$(ima(Z^{-1/2}))^- = \mathcal{L}, \tag{2.40}$$

$$e^{-itL} Z^{-1} e^{itL} \leq Z^{-1}, \; t \geq 0, \tag{2.41}$$

$$\text{s-lim}_{t\to+\infty} (I - Z^{-1/2})e^{-itL} = 0, \tag{2.42}$$

$$\text{s-lim}_{t\to-\infty} Z^{-1/2} e^{-itL} = 0 \tag{2.43}$$

<u>Proof:</u> The property (2.40) follows from $ima(Z^{-1/2}) = dom(Z^{1/2}) = dom(\gamma_0)$. From (2.11) we obtain (2.41). The estimate

$$((I - Z^{-1})e^{-itL} f, e^{-itL} f) \leq \leq (Z^{1/2} e^{-itL} f, Z^{1/2} e^{-itL} f) - (f,f), \tag{2.44}$$

$f \in dom(Z^{1/2})$, $t \geq 0$, and (2.12) imply (2.42). Taking into account (2.41) and Theorem 3.3 of T.Kato [8] we find the existence of $\text{s-lim}_{t\to+\infty} e^{-itL} Z^{-1} e^{itL} = X_+$. Obviously, the operator X_+ commutes with e^{-itL}, i.e.

$$e^{-itL} X_+ = X_+ e^{-itL}, \tag{2.45}$$

$t \in \mathbb{R}^1$, and fulfils the relation

$$X_+ \leq Z^{-1}. \tag{2.46}$$

Because of Corollary 7-2 [7] there is a contraction Y such that

$$X_+^{1/2} = Z^{-1/2} Y. \tag{2.47}$$

Hence we find

$$ima(X_+^{1/2}) \subseteq ima(Z^{-1/2}) = dom(Z^{1/2}) = dom(\gamma_0). \tag{2.48}$$

In addition we get

$$\| Z^{1/2} X_+^{1/2} f \| \leq \| f \|, \tag{2.49}$$

$f \in \mathcal{L}$. Since (2.45) we have

$$e^{-itL} X_+^{1/2} = X_+^{1/2} e^{-itL}, \tag{2.50}$$

$t \in \mathbb{R}^1$. Hence we obtain

$$e^{-itL} ima(X_+^{1/2}) = ima(X_+^{1/2}) \subseteq dom(\gamma_0), \tag{2.51}$$

$t \in \mathbb{R}^1$. Because of (2.49) we find

$$\gamma_0(e^{itL} X_+^{1/2} f, e^{itL} X_+^{1/2} f) = \| Z^{1/2} X_+^{1/2} e^{itL} f \|^2 \leq \leq \| f \|^2, \tag{2.52}$$

$f \in \mathcal{L}$, $t \geq 0$. But (2.51) and (2.52) yield $ima(X_+) \subseteq \mathcal{D}$. Taking into account (2.13) we obtain $X_+ = 0$. ■

We notice that Lemma 2.7 and Corollary 2.8 generalize Lemma 3.3 and Corollary 3.6 of [11] to the case $S^{-1} = 0$.

With the help of Lemma 2.7 and Corollary 2.8 we prove the following

Proposition 2.9: Let L be a selfadjoint operator on the separable Hilbert space $\mathcal{L}$ and let $J:\mathcal{L} \longmapsto \mathfrak{H}$ be an identification operator. If there is an isometry $F_+:\mathcal{L} \longrightarrow \mathfrak{H}$ obeying $F_+^*F_+ = P^{ac}(L)$, $ima(F_+) = \mathfrak{H}$ and

$$\underset{t\to+\infty}{s\text{-}lim}\ (F_+ - J)e^{-itL}P^{ac}(L) = 0, \tag{2.53}$$

then there is a maximal dissipative operator H on $\mathfrak{H}$ of class C_{10} such that the wave operator $W_+(H,L;J)$ exists and is complete.

Proof: Without loss of generality we assume that L is absolutely continuous. Let Z be defined by (2.39). We introduce the operator $W:\mathcal{L} \longrightarrow \mathfrak{H}$,

$$W = F_+Z^{-1/2}. \tag{2.54}$$

The operator W is a contraction fulfilling $\ker(W) = \{0\}$. Because of (2.40) we have

$$(ima(W))^- = \mathfrak{H}. \tag{2.55}$$

On account of (2.41) the relation

$$T(t)^*Wf = We^{itL}f, \tag{2.56}$$

$f \in \mathcal{L}$, $t \geq 0$, defines a contraction semigroup $T(t)$, $t \geq 0$, on $\mathfrak{H}$.

Let the maximal dissipative operator H be defined by $T(t) = e^{-itH}$, $t \geq 0$. Taking into consideration (2.43) we find

$$\underset{t\to+\infty}{s\text{-}lim}\ T(t)^* = 0. \tag{2.57}$$

Hence $H \in C_{.0}$. The definition (2.56) and the property (2.55) imply

$$\lim_{t\to+\infty} \|W^*T(t)f\| = \|W^*f\| > 0, \tag{2.58}$$

$0 \neq f \in \mathfrak{H}$. But (2.58) yields $H \in C_{10}$.

Now we establish the existence of the wave operator $W_+(H,L;J)$. From (2.42) we derive

$$\underset{t\to+\infty}{s\text{-}lim}\ (W - F_+)e^{-itL} = 0. \tag{2.59}$$

Hence the wave operator $W_+(H,L;F_+)$ exists and we have $W = W_+(H,L;F_+)$. Because of Theorem 3.5 of [10] the wave operator $W_+(H,L;F_+)$ is complete, if the condition

$$\underset{t\to+\infty}{s\text{-}lim}\ (W^* - F_+)e^{-itH} = 0 \tag{2.60}$$

is satisfied. The representation

$$W^* = \underset{t\to+\infty}{w\text{-}lim}\ e^{itL}F_+^*e^{-itH} \tag{2.61}$$

follows from the existence of $W_+(H,L;F_+)$. Our aim is now to replace the weak limit by the strong limit. This can be done if the condition

$$\lim_{t\to+\infty} \|F_+^*e^{-itH}f\|^2 = \|W^*f\|^2, \tag{2.62}$$

$f \in \mathfrak{H}$, is fulfilled. From (2.56) we obtain

$$e^{-itL}\, \mathrm{ima}(W^*) \subseteq \mathrm{ima}(W^*), \tag{2.63}$$

$t \geq 0$, and

$$e^{-itH}\,(W^*)^{-1} f = (W^*)^{-1}\, e^{-itL} f, \tag{2.64}$$

$f \in \mathrm{ima}(W^*)$, $t \geq 0$. Using (2.12) we find

$$\begin{aligned} &\lim_{t\to+\infty} \| F_+^*\, e^{-itH}\, (W^*)^{-1} f \|^2 = \\ &= \lim_{t\to+\infty} \| Z^{1/2}\, e^{-itL} f \|^2 = \| f \|^2, \end{aligned} \tag{2.65}$$

$f \in \mathrm{ima}(W^*) = \mathrm{dom}(Z^{1/2})$. But (2.65) proves (2.62).

Taking into account (2.53) the existence and completeness of $W_+(H,L;F_+)$ imply the existence and completeness of $W_+(H,L;J)$.■

Obviously, a similar result holds for the opposite time direction.

Corollary 2.10: If there is an isometry $F_-: \mathcal{L} \longrightarrow \mathfrak{H}$ obeying $F_-^* F_- = P^{ac}(L)$, $\mathrm{ima}(F_-) = \mathfrak{H}$ and

$$\operatorname*{s-lim}_{t\to+\infty} (F_- - J) e^{-itL} P^{ac}(L) = 0, \tag{2.66}$$

then there is a maximal dissipative operator H on $\mathfrak{H}$ of class C_{01} such that the wave operator $W_-(H,L;J)$ exists and is complete.

We left the proof to the reader. We notice that Proposition 2.9 and Corollary 2.10 imply the existence of maximal dissipative operators belonging to the class C_{10} and C_{01} such that their residual parts are unitarily equivalent to a given absolutely continuous selfadjoint operator.

We are now in a position to prove our theorem.

Proof of Theorem 2.3: We introduce the subspaces $\mathcal{R}_+ = (\mathrm{ima}(S))^-$, $\mathcal{R}_- = (\mathrm{ima}(S^*))^-$ and $\mathcal{N}_\pm = \mathcal{L}_\pm \ominus \mathcal{R}_\pm$. The subspaces $\mathcal{R}_\pm$ and $\mathcal{N}_\pm$ reduce the selfadjoint operators $L_\pm$. We set $R_\pm = L_\pm \restriction \mathrm{dom}(L_\pm) \cap \mathcal{R}_\pm$ and $N_\pm = L_\pm \restriction \mathrm{dom}(L_\pm) \cap \mathcal{N}_\pm$. The selfadjoint operators $R_\pm$ are absolutely continuous, while the operators $N_\pm$ have in general singularly continuous parts.

The operator S can be regarded as a contraction acting from $\mathcal{R}_-$ into $\mathcal{R}_+$. Doing so we denote the contraction S by S', i.e. $Sf = S' P^{\mathcal{L}_-}_{\mathcal{R}_-} f$, $f \in \mathcal{L}_-$. Obviously, we have $(\mathrm{ima}(S'))^- = \mathcal{R}_+$ and $(\mathrm{ima}(S'^*))^- = \mathcal{R}_-$.

By F_+ and F_- we denote the partial isometries of Definition 2.1. We define $G_\pm = F_\pm \restriction \mathcal{N}_\pm$ and $F'_\pm = F_\pm \restriction \mathcal{R}_\pm$ as well as $\mathfrak{H}_\pm = G_\pm \mathcal{N}_\pm$ and $\mathfrak{H}' = F'_+ \mathcal{R}_+ \oplus F'_- \mathcal{R}_-$. On account of (ii) of Definition 2.1 all these subspaces are orthogonal to each other. We introduce the subspace $\mathfrak{H}_s$,

$$\mathfrak{H}_s = \mathfrak{H} \ominus \{\mathfrak{H}_- \oplus \mathfrak{H}' \oplus \mathfrak{H}_+\}. \tag{2.67}$$

Further we need the notation $Y_- = P^{\mathfrak{H}}_{\mathfrak{H}_-} J_- \restriction \mathcal{N}_-$, $Z_\pm = P^{\mathfrak{H}}_{\mathfrak{H}'} J_\pm \restriction \mathcal{R}_\pm$ and $Y_+ = P^{\mathfrak{H}}_{\mathfrak{H}_+} J_+ \restriction \mathcal{N}_+$. Obviously, we have

$$\operatorname*{s-lim}_{t\to-\infty} (G_- - Y_-) e^{-itN_-} P^{ac}(N_-) = 0, \tag{2.68}$$

$$\operatorname*{s-lim}_{t\to\pm} (F'_\pm - Z_\pm) e^{-itR_\pm} = 0, \tag{2.69}$$

$$\operatorname*{s-lim}_{t\to+\infty} (G_+ - Y_+) e^{-itN_+} P^{ac}(N_+) = 0. \tag{2.70}$$

Because of $F_+^* F_- = 0$ we obtain $F'^*_+ F'_- = 0$. Consequently, the identification operators Z_+ and Z_- are admissible with respect to R_+ and R_-.

Now we apply Theorem 3.7 of [11] and Remark 2.5 to S', R_+, R_-, Z_+ and Z_-. We obtain a maximal dissipative operator H' on $\mathfrak{H}'$ of class C_{11} such that $\mathcal{A}' = \{H';R_+,R_-;Z_+,Z_-\}$ forms a complete scattering system whose scattering operator $S(\mathcal{A}')$ coincides with S', i.e. $S(\mathcal{A}') = S'$.

Further, Proposition 2.9 and Corollary 2.10 imply the existence of two maximal dissipative operators $H_+ \in C_{10}$ and $H_- \in C_{01}$ defined on $\mathfrak{H}_+$ and $\mathfrak{H}_-$, respectively, such that the wave operators $W_+(H_+,N_+;Y_+)$ and $W_-(H_-,N_-;Y_-)$ exist and are complete.

By H_s we denote an arbitrary singularly continuous selfadjoint operator on $\mathfrak{H}_s$. In accordance with the orthogonal decomposition $\mathfrak{H} = \mathfrak{H}_- \oplus (\mathfrak{H}' \oplus \mathfrak{H}_s) \oplus \mathfrak{H}_+$ we set

$$H = H_- \oplus (H' \oplus H_s) \oplus H_+ . \tag{2.71}$$

It remains to show that $\mathcal{A} = \{H;L_+,L_-;J_+,J_-\}$ is a complete scattering system with $S(\mathcal{A}) = S$. To this end it is enough to show that the identification operators $\tilde{J}_\pm: \mathcal{L}_\pm \longrightarrow \mathfrak{H}$, $\tilde{J}_- f = Y_- P^{\mathcal{L}_-}_{\mathcal{N}_-} f + Z_- P^{\mathcal{L}_-}_{\mathcal{R}_-} f$, $f \in \mathcal{L}_-$, $\tilde{J}_+ f = Y_+ P^{\mathcal{L}_+}_{\mathcal{N}_+} f + Z_+ P^{\mathcal{L}_+}_{\mathcal{R}_+} f$, $f \in \mathcal{L}_+$, and $J_\pm$ are equivalent, i.e.

$$\underset{t\to\pm\infty}{\text{s-lim}} (\tilde{J}_\pm - J_\pm)e^{-itL_\pm} P^{ac}(L_\pm) = 0. \tag{2.72}$$

Because of (2.68) - (2.70) we obtain

$$\underset{t\to\pm\infty}{\text{s-lim}} (F_\pm - \tilde{J}_\pm)e^{-itL_\pm} P^{ac}(L_\pm) = 0. \tag{2.73}$$

But condition (iii) of Definition 2.1 and (2.73) imply (2.72). ∎

3. Scattering matrix

It is well-known that every selfadjoint operator admits a representation as a multiplication operator on some direct integral of Hilbert spaces [2]. Usually, such a representation is called the spectral representation of a selfadjoint operator. In the following we consider the spectral representations of the absolutely continuous parts $L^{ac}_\pm$ of the selfadjoint operators $L_\pm$. We denote these spectral representations by $L^2(\mathbb{R}^1, \mu_\pm; \mathfrak{h}^\pm_\lambda, \mathfrak{f}_\pm)$, where $\mu_\pm$ are some measures on $\mathbb{R}^1$, $\{\mathfrak{h}^\pm_\lambda\}_{\lambda\in\mathbb{R}^1}$ are families of separable Hilbert spaces and $\mathfrak{f}_\pm$ are separable admissible (with respect to $\mu_\pm$) subsystems of $\underset{\lambda\in\mathbb{R}^1}{X} \mathfrak{h}^\pm_\lambda$ [2]. The selfadjoint operators $L^{ac}_\pm$ are now unitarily equivalent to multiplication operators $L^\pm_\lambda$ on $L^2(\mathbb{R}^1, \mu_\pm; \mathfrak{h}^\pm_\lambda, \mathfrak{f}_\pm)$. The operators $L^{ac}_\pm$ are absolutely continuous. Hence the measures $\mu_\pm$ are absolutely continuous with respect to the Lebesgue measure $|.|$ on $\mathbb{R}^1$. Consequently, there are measurable subsets $\Lambda_\pm$ of $\mathbb{R}^1$ such that the measures $\mu_\pm$ can be chosen as restrictions of the Lebesgue measure on $\mathbb{R}^1$, i.e. $\mu_\pm(\Delta) = |\Delta \cap \Lambda_\pm|$ for every measurable set $\Delta \subseteq \mathbb{R}^1$. In the following we specify the measures $\mu_\pm$ to this form.

Every contraction S obeying (2.5) - (2.7) can be now represented as the multiplication operator with a measurable operator-valued family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ of bounded operators acting from $\mathfrak{h}^-_\lambda$ into $\mathfrak{h}^+_\lambda$ [2]. The family is uniquely defined (up to a set of Lebesgue measure zero) on the subset $\Lambda = \Lambda_+ \cap \Lambda_-$. Further, we believe that $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ is zero outside Λ, i.e. $S(\lambda) = 0$ for $\lambda \in \mathbb{R}^1\setminus\Lambda$. This well corresponds to the fact that in case $|\Lambda| = 0$ the operators $L^{ac}_\pm$ have no unitarily equivalent parts. Therefore, $S = 0$ is the unique operator which fulfils (2.7). Since S is a contraction the family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ consists of contractions acting from $\mathfrak{h}^-_\lambda$ into $\mathfrak{h}^+_\lambda$ for a.e. $\lambda\in\mathbb{R}^1$.

With the above-described conventions a uniquely defined (up to a set of Lebesgue measure zero) family of contractions $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ acting from $\mathfrak{h}_\lambda^-$ into $\mathfrak{h}_\lambda^+$ corresponds to every contraction S obeying (2.5) - (2.7) and every given spectral representation of $L_\pm^{ac}$. Let $\mathcal{S} = \{H;L_+,L_-;J_+,J_-\}$ be a complete scattering system. Then we call this uniquely defined family of contractions, which corresponds to $S(\mathcal{S})$, the scattering matrix of the scattering system $\mathcal{S}$.

Theorem 2.3 implies now the following:

<u>Theorem 3.1:</u> A family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ of contractions acting from the separable Hilbert space $\mathfrak{h}_\lambda^-$ into the separable Hilbert space $\mathfrak{h}_\lambda^+$ can be regarded as the scattering matrix of a complete scattering system if and only if there are separable admissible (with respect to the Lebesgue measure $|\cdot|$ on $\mathbb{R}^1$) subsystems $\mathcal{F}_\pm \subseteq \times_{\lambda\in\mathbb{R}^1}\mathfrak{h}_\lambda^\pm$ such that for every $f\in\mathcal{F}_-$ $\{S(\lambda)f(\lambda)\}_{\lambda\in\mathbb{R}^1}$ is strongly measurable with respect to $\mathcal{F}_+$ [2].

<u>Proof:</u> To complete the necessity of this theorem it remains to find two separable admissible (with respect to the Lebesgue measure $|\cdot|$ on $\mathbb{R}^1$) subsystems $\tilde{\mathcal{F}}_\pm$ of $\times_{\lambda\in\mathbb{R}^1}\mathfrak{h}_\lambda^\pm$ such that $\tilde{\mathcal{F}}_\pm\restriction\Lambda_\pm = \mathcal{F}_\pm$. But this can easily be done. Now for every $f\in\mathcal{F}_-$ $\{S(\lambda)f(\lambda)\}_{\lambda\in\mathbb{R}^1}$ is strongly measurable with repsect to $\mathcal{F}_+$. Using $S(\lambda)\restriction(\mathbb{R}^1\setminus\Lambda) = 0$ we obtain that for every $f\in\tilde{\mathcal{F}}_-$ $\{S(\lambda)f(\lambda)\}_{\lambda\in\mathbb{R}^1}$ is strongly measurable with respect to $\tilde{\mathcal{F}}_+$.

To prove the converse we consider the multiplication operators $L_\lambda^\pm = L_\pm$ on $\mathcal{L}_\pm = L^2(\mathbb{R}^1,|\cdot|;\mathfrak{h}_\lambda^\pm,\tilde{\mathcal{F}}_\pm)$. The selfadjoint operators $L_\pm$ are absolutely continuous. On account of the measurability assumption the family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ induced a multiplication operator S acting from $\mathcal{L}_-$ into $\mathcal{L}_+$ which is a contraction. Further, let $\mathfrak{h}$ be an auxiliary separable Hilbert space. Because $\mathcal{L}_\pm$ are separable [2] there are isometries $F_\pm$ acting from $\mathcal{L}_\pm$ into $\mathfrak{h}$

such that $F_+^*F_- = 0$. Applying Theorem 2.3 to S,L_+,L_-,F_+ and F_- we find a maximal dissipative operator H on $\mathfrak{h}$ such that $\mathcal{S} = \{H;L_+,L_-;F_+,F_-\}$ forms a complete scattering system with $S(\mathcal{S}) = S$. Now it is not hard to see that the scattering matrix $\{S_{\mathcal{S}}(\lambda)\}_{\lambda\in\mathbb{R}^1}$ of $\mathcal{S}$ coincides with $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$. ∎

If $\mathfrak{h}_\lambda^- = \mathcal{K}_-$ and $\mathfrak{h}_\lambda^+ = \mathcal{K}_+$, $\lambda\in\mathbb{R}^1$, then the measurability condition of Theorem 3.1 reduces to the assumption that for every $f\in\mathcal{K}_-$ $\{S(\lambda)f\}_{\lambda\in\mathbb{R}^1}$ is strongly measurable. In this case we call $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ measurable.

<u>Corollary 3.2:</u> Every measurable family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ of contractions acting from $\mathcal{K}_-$ into $\mathcal{K}_+$ can be regarded as the scattering matrix of a complete scattering system.

The proof is obvious. The last corollary allows a comparison with the scattering matrices of the well-known dissipative or non-conservative Lax-Phillips scattering theory [9].

First of all we remark that the dissipative Lax-Phillips scattering theory can be embedded in ours and is complete in our sense. The dissipative Lax-Phillips scattering theory is characterized by a 5-tuple of operators $\mathcal{S} = \{H;L_+,L_-;J_+,J_-\}$ and two subspaces $\mathcal{D}_+$ and $\mathcal{D}_-$ of $\mathfrak{h}$ as well as of $\mathcal{L}_+$ and $\mathcal{L}_-$, respectively, such that

$$e^{-itL_+}\mathcal{D}_+ \subseteq \mathcal{D}_+,\ e^{-itH}\mathcal{D}_+ \subseteq \mathcal{D}_+,\ t\geq 0, \tag{3.1}$$

$$e^{itL_-}\mathcal{D}_- \subseteq \mathcal{D}_-,\ e^{itH^*}\mathcal{D}_- \subseteq \mathcal{D}_-,\ t\geq 0, \tag{3.2}$$

$$e^{-itL_+}\restriction\mathcal{D}_+ = e^{-itH}\restriction\mathcal{D}_+,\ t\geq 0 \tag{3.3}$$

$$e^{itL_-}\restriction\mathcal{D}_- = e^{itH^*}\restriction\mathcal{D}_-,\ t\geq 0, \tag{3.4}$$

$$\bigvee_{t\in\mathbb{R}^1} e^{-itL_\pm}\mathcal{D}_\pm = \mathcal{L}_\pm, \tag{3.5}$$

$$\bigcap_{t\in\mathbb{R}^1} e^{-itL_+}\mathcal{D}_+ = \{0\} = \bigcap_{t\in\mathbb{R}^1} e^{-itL_-}\mathcal{D}_-, \tag{3.6}$$

$$\left.\begin{array}{l} \text{s-}\lim_{t\to+\infty} P^{\mathfrak{H}}_{\mathfrak{H}\ominus\mathcal{D}_+} e^{-itH} = 0 \\ \text{s-}\lim_{t\to+\infty} P^{\mathfrak{H}}_{\mathfrak{H}\ominus\mathcal{D}_-} e^{itH^*} = 0. \end{array}\right. \tag{3.7}$$

Defining the identification operators $J_\pm: \mathcal{L}_\pm \longrightarrow \mathfrak{H}$ by

$$J_\pm f = P^{\mathcal{L}_\pm}_{\mathcal{D}_\pm} f, \tag{3.8}$$

$f \in \mathcal{L}_\pm$, we can introduce the operators $W_\pm(H,L_\pm;J_\pm)$ which exist under the assumptions (3.1) - (3.7). Condition (3.7) enables us to show that these wave operators are complete in the sense of [10]. Consequently, $\mathcal{J} = \{H,L_+,L_-,J_+,J_-\}$ forms a complete scattering system. The scattering operator $S(\mathcal{J})$ is defined by $S(\mathcal{J}) = W_+(H,L_+;J_+)^* W_-(H,L_-;J_-)$. Because of (3.2), (3.5) and (3.6) the selfadjoint operators $L_\pm$ are unitarily equivalent to multiplication operators $L^\pm_\lambda$ on some Hilbert spaces $L^2(\mathbb{R}^1, |.|; \mathcal{N}_\pm)$, where $L^2(\mathbb{R}^1, |.|; \mathcal{N}_\pm)$ are usual Hilbert spaces of square integrable vector-valued functions (with respect to the Lebesgue measure $|.|$ on $\mathbb{R}^1$). Obviously, $L^2(\mathbb{R}^1, |.|; \mathcal{N}_\pm)$ together with $L^\pm_\lambda$ are spectral representations of $L_\pm$. By $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ we denote the uniquely determined scattering matrix in these spectral representations. Naturally, the question arises to characterize all those measurable families $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ of contractions acting from $\mathcal{N}_-$ into $\mathcal{N}_+$ which can be regarded as the scattering matrix of a dissipative Lax-Phillips scattering theory. This problem was solved by

C.Foias in [6]. A new solution of this problem was given in [13]. In both papers the so-called discrete case was considered. Transforming the results of [13] to our situation we obtain that a measurable contraction-valued function $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$, $S(\lambda): \mathcal{N}_- \longrightarrow \mathcal{N}_+$, can be regarded as the scattering matrix of a dissipative Lax-Phillips scattering theory if and only if there are separable Hilbert spaces $\mathfrak{A}_\pm$ and measurable contraction-valued families $\{\mathcal{N}_-, \mathfrak{A}_+; A_*(\lambda)\}_{\lambda\in\mathbb{R}^1}$, $\{\mathfrak{A}_-, \mathcal{N}_+; A(\lambda)\}_{\lambda\in\mathbb{R}^1}$ and $\{\mathfrak{A}_-, \mathfrak{A}_+; \theta(\lambda)\}_{\lambda\in\mathbb{R}^1}$, which can analytically be extended in the upper half plane, such that

$$S'(\lambda) = \begin{pmatrix} \theta(\lambda) & A_*(\lambda) \\ A(\lambda) & S(\lambda) \end{pmatrix} : \begin{array}{c} \mathfrak{A}_- \\ \oplus \\ \mathcal{N}_- \end{array} \longrightarrow \begin{array}{c} \mathfrak{A}_+ \\ \oplus \\ \mathcal{N}_+ \end{array} \tag{3.9}$$

forms a measurable unitary-valued function. It can be shown that this condition cannot be fulfilled by every measurable contraction-valued family $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$. The restrictions on the set of scattering matrices result from the special assumptions which we have made in (3.1) - (3.4) and (3.7).

These special assumptions are omitted in our dissipative scattering theory and replaced by the completeness condition. Taking into account Corollary 3.2 we see that this replacement is accompanied by an enlargement of the set of possible scattering matrices. Moreover, it turns out that the condition to obtain the desired full Hamiltonian in the class of maximal dissipative operators is not a real restriction. In terms of a block-matrix representation this can be expressed as follows.

<u>Corollary 3.3:</u> Let $\{S(\lambda)\}_{\lambda\in\mathbb{R}^1}$ be a measurable family of contractions acting from $\mathcal{N}_-$ into $\mathcal{N}_+$. Then there are measurable fami-

lies of contractions $\{\mathcal{N}_-, \mathcal{O}_+; A_*(\lambda)\}_{\lambda\in\mathbb{R}^1}$, $\{\mathcal{O}_-, \mathcal{N}_+; A(\lambda)\}_{\lambda\in\mathbb{R}^1}$ and $\{\mathcal{O}_-, \mathcal{O}_+; \theta(\lambda)\}_{\lambda\in\mathbb{R}^1}$ such that $\{\mathcal{O}_-, \mathcal{O}_+; \theta(\lambda)\}_{\lambda\in\mathbb{R}^1}$ can be analytically extended in the upper half plane and $S'(\lambda)$ defined by (3.9) forms a measurable family of unitary operators.

Corollary 3.3 can be proved using Corollary 3.2 and the considerations of [13]. We left the proof to the reader. Corollary 3.3 clearly shows in which places our scattering theory and the non-conservative Lax-Phillips scattering theory agree and in which they are different.

4. Application to nuclear physics

In nuclear physics one is rarely able to solve the corresponding many-body problems exactly. In order to get some information about the scattering process, we must therefore select a suitable part of the whole state Hilbert space from which the needed quantities can be extracted, while the influence of the rest is taken into account in some phenomenological way. Consequently, it is assumed that the Hilbert space $\mathfrak{H}$ can be decomposed into two subspaces,

$$\mathfrak{H} = \mathfrak{H}_0 \oplus \mathfrak{H}_1. \tag{4.1}$$

Further, it is assumed that the free Hamiltonian K_0 is reduced by $\mathfrak{H}_0$ and $\mathfrak{H}_1$. We set $L = K_0 \restriction (\mathrm{dom}(K_0) \cap \mathfrak{H}_0)$ and $H_1 = K_0 \restriction (\mathrm{dom}(K_0) \cap \mathfrak{H}_1)$. For simplicity we assume that L is absolutely continuous. Let K be the full Hamiltonian. We assume that the wave operators $W_\pm = \text{s-lim}_{t\to\pm\infty} e^{itK} e^{-itK_0} P^{ac}(K_0)$ exist and are complete, i.e. $\mathcal{S}_0 = \{K; \mathfrak{K}_0, K_0; I_{\mathfrak{H}}, I_{\mathfrak{H}}\}$ forms a complete scattering system. Denoting by P_0 the projection from $\mathfrak{H}$ onto $\mathfrak{H}_0$ we are interested in the part $S_{00} = P_0 S \restriction \mathfrak{H}_0$ of the unitary (on the absolutely continuous subspace of K_0) scattering operator $S = W_+^* W_-$.

The problem now is to find a maximal dissipative operator H on $\mathfrak{H}_0$ such that the wave operators $W_\pm(H, L; I_{\mathfrak{H}_0})$ exist and are complete and, furthermore, such that the scattering operator $S_A = W_+(H, L; I_{\mathfrak{H}_0})^* W_-(H, L; I_{\mathfrak{H}_0})$ is a good approximation of the partial scattering operator S_{00}, i.e. $S_{00} \approx S_A$.

There are several possibilities to solve this problem. One solution was proposed by H.Feshbach [5]. Assuming that the full Hamiltonian K is given by $K = K_0 + gV$, $g \in \mathbb{R}^1$, representing V by

$$V = \begin{pmatrix} V_{00} & V_{01} \\ V_{10} & V_{11} \end{pmatrix} : \begin{matrix} \mathfrak{H}_0 \\ \oplus \\ \mathfrak{H}_1 \end{matrix} \longmapsto \begin{matrix} \mathfrak{H}_0 \\ \oplus \\ \mathfrak{H}_1 \end{matrix}, \tag{4.2}$$

where for simplicity we set $V_{11} = 0$, and introducing the generalized optical potential $V_F(E)$,

$$V_F(E) = gV_{00} - g^2 \lim_{\varepsilon\to+0} V_{01}(H_1 - E - i\varepsilon)^{-1} V_{10}, \tag{4.3}$$

$E \in \mathbb{R}^1$, H.Feshbach uses the operator $H_F(E) = L + V_F(E)$. It can be shown that the corresponding scattering operator $S_F = W_+(H_F(E), L; I_{\mathfrak{H}_0})^* W_-(H_F(E), L; I_{\mathfrak{H}_0})$ gives a good approximation of S_{00} for a suitable class of perturbations V [3]. The parameter E can be used to improve the approximation.

Another method, which is widely applied in nuclear physics, takes into account the influence of the reaction channels $\{\mathfrak{H}_0 \to \mathfrak{H}_1\}$ and $\{\mathfrak{H}_1 \to \mathfrak{H}_0\}$ by some local potentials with negative imaginary parts which are usually called local optical potentials (Saxon-Woods potentials).

But it seems to us that the problem, whether in every case a maximal dissipative operator H on $\mathfrak{h}_o$ can be found such that the wave operators $W_\pm(H,L;I_{\mathfrak{h}_o})$ exist and are complete as well as $S_{oo} \approx S_A$ or even $S_{oo} = S_A$, is unsolved. Theorem 2.3 enables us to answer this question.

Proposition 4.1: Let K_o be a selfadjoint operator on the separable Hilbert space $\mathfrak{h}$, which is reduced by the subspace $\mathfrak{h}_o \subset \mathfrak{h}$. Assume that $L = K_o \restriction (\mathrm{dom}(K_o) \cap \mathfrak{h}_o)$ is absolutely continuous. If $\mathcal{S}_o = \{K;K_o,K_o;I_{\mathfrak{h}},I_{\mathfrak{h}}\}$ is a complete scattering system, then there is a maximal dissipative operator H on $\mathfrak{h}_o$ such that $\mathcal{S} = \{H;L,L;I_{\mathfrak{h}_o},I_{\mathfrak{h}_o}\}$ forms a complete scattering system whose scattering operator $S(\mathcal{S})$ coincides with the partial scattering operator S_{oo} defined by $S_{oo} = P_o S(\mathcal{S}_o) \restriction \mathfrak{h}_o$, i.e. $S_{oo} = S(\mathcal{S})$.

Proof: We try to apply Theorem 2.3 to $\mathcal{L}_+ = \mathcal{L}_- = \mathfrak{h}_o$, $J_+ = J_- = I_{\mathfrak{h}_o}$ and $L_+ = L_- = L$. By obvious changes of the notation the contraction S_{oo} fulfils the assumptions (2.5) - (2.7). Taking into account Remark 2.2 we can apply Theorem 2.3 and obtain the desired maximal dissipative operator H on $\mathfrak{h}_o$. ■

On account of Remark 2.6 the solution of Proposition 4.1 is not unique.

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Об обратной задаче диссипативной теории рассеяния

В операторно-теоретическом подходе решается обратная задача диссипативной теории рассеяния. Результат сопоставляется с теорией рассеяния Лакса–Филлипса. В качестве применения дается математически строгое обоснование оптической аппроксимации, применяемой в ядерной физике.

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On the Inverse Problem of a Dissipative Scattering Theory

In an operator-theoretical framework the abstract inverse scattering problem of a dissipative scattering theory is solved. The result is related to the non-conservative version of the Lax-Phillips scattering theory. As an application, a mathematically rigorous foundation of the optical approximation used in nuclear physics is given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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