объединенный
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исследований
дубна

E5-87-867

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ON TWO-DIMENSIONAL TOPOLOGICAL SOLITONS IN ANISOTROPIC MAGNETS

Submitted to "Письма в ЖЭТФ"

1987

Soon after the soliton solutions [1] for the two-dimensional isotropic Heisenberg magnet had been found numerous attempts were undertaken to generalize these solutions to the anisotropic versions of the Heisenberg model (see [2] and references cited therein). Difficulties of analytical obtaining of mentioned solutions stimulated their numerical search. By means of a computer the localized distributions having soliton-like shape have been obtained and their main parameters have been calculated [2]. But it is easy to demonstrate (see below), using scaling transformations [3], that the existence of static soliton solutions is forbidden within the framework of the discussed two-dimensional anisotropic models.

Consider two-dimensional anisotropic Heisenberg model having rather general form of the static Hamiltonian density:

$$\mathcal{H} = \alpha^2 (\partial_i s^a)^2 + \beta^2 w_{an}(s^a(x_i)) + \gamma^2 F(\partial_i s^a) = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3, \tag{1}$$

here the anisotropy energy density $w_{an} \geq 0$, a non-negative addendum F is of the fourth order in differentiation with respect to x_i, $\partial_1 = \partial_x$, $\partial_2 = \partial_y$, $s^a s^a = 1$, $a = 1,2,3$, α, β and γ are constants.

Consider continuous localized distributions $s^a(x_i)$ with the topological charge $Q_t \geq 1$, here Q_t is the degree of map from R^2 (with the condition $\vec{s}(\infty) = \vec{s}_0$ taken into account) to the sphere S^2 [4,2]. The following inequalities are valid for such configurations

$$H_K = \int \mathcal{H}_K d^2x > 0, \quad K = 1,2,3, \qquad H = H_1 + H_2 + H_3 > 0. \tag{2}$$

Under scaling transformations, $\vec{s}(\vec{x}) \to \vec{s}(a\vec{x})$, $H_1(a=1)$ is transformed to $H_1(1)$, $H_2 \to a^{-2} H_2(1)$, $H_3(1) \to a^2 H_3(1)$. Such transformations determine at $|a - 1| \ll 1$ small variations of initial distributions $s^a(x_i)$. It is nessesary for the soliton existence, that

$\frac{dH}{da}(1)=0$, i.e.

$$H_3 - H_2 = 0. \qquad (3)$$

One can see that Eq.3 has no solutions at $F=0$, i.e. the functional H cannot reach even a local minimum, and, hence, static solitons cannot exist in the model (1) with $F=0$. When $F\neq 0$, one can deduce from Eq.3, that the existence of soliton solutions to the equation $\frac{\delta H}{\delta s^a}=0$ is not excluded a priori, and if they exist the relationship $H_2=H_3$ is valid for them.

As an example consider an easy-axis type anisotropy, $w_{an}=\sin^2\theta$, here θ is the angle between the vector s^a and the z axis. Specify the addendum F as well , choosing it in the Skyrme [5] form

$$F_S = [(\partial_i s^a \partial_i s^a)^2 - (\partial_i s^a \partial_j s^a)^2]. \qquad (4)$$

Such the F option is characterized by an absence of the soliton solutions of the form

$$s^1=\sin\theta,\quad s^2=0,\quad s^3=\cos\theta,\quad \theta=\theta(x,y), \qquad (5)$$

representing the $Q_t=0$ sector, since for the configurations defined by (5) we have : $F_S=0$, $H_3=0$ and therefore $\frac{dH}{da}(1)\neq 0$.

Now consider the distributions $s^a(x_i)$ having $Q_t\geq 1$.

Introduce the dimensionless variables X, Y, R:

$$x=\frac{\alpha}{\beta}X,\quad y=\frac{\alpha}{\beta}Y,\quad R=\sqrt{X^2+Y^2}. \qquad (6)$$

Using the well-known ansatz,

$$s^1=\sin\theta\frac{X}{R},\quad s^2=\sin\theta\frac{Y}{R},\quad s^3=\cos\theta,\quad \theta=\theta(R), \qquad (7)$$

we find

$$\beta^{-2}\mathcal{H}(R) = \left(\frac{d\theta}{dR}\right)^2+\frac{1}{R^2}\sin^2\theta+\sin^2\theta+\frac{2p}{R^2}\sin^2\theta\left(\frac{d\theta}{dR}\right)^2, \qquad (8)$$

where $p=\beta^2\gamma^2\alpha^{-4}$ is the dimensionless parameter.

Calculating the variation $\frac{\delta H}{\delta\theta}\delta\theta$, when $\mathcal{H}(R)$ is given by (8),

we obtain the equation for the $\theta(R)$ function

$$\frac{d^2\theta}{dR^2}+\frac{1}{R}\frac{d\theta}{dR}-\left(\frac{1}{R^2}+1\right)\sin\theta\cos\theta + 2p\left[\frac{\sin^2\theta}{R^2}\frac{d^2\theta}{dR^2}+\frac{\sin\theta\cos\theta}{R^2}\left(\frac{d\theta}{dR}\right)^2-\frac{\sin^2\theta}{R^3}\frac{d\theta}{dR}\right]=0. \qquad (9)$$

To find the soliton solutions of Eq.(9) with $Q_t=1$ one should solve this equation with the following boundary conditions : $\theta(0)=\pi$, $\theta(\infty)=0$ (we assume that $s^a(\infty)=\delta_{3a}$). From the Eq.(9) we find that $\theta(R)=\pi-CR+O(R^3)$ when $R\to 0$. Using this expansion, we solve (9) numerically, to get boundary condition $\theta(\infty)=0$ satisfied we vary "the shooting parameter" C.

The numerical studies have been made in the wide region of the p parameter: from $p_{min}=10^{-6}$ to $p_{max}=500$. These investigations demonstrate, that at least the local minimum of the functional $h[\theta(R)]$, $h=\alpha^{-2}\int\mathcal{H}d^2x$, is reached on the obtained solutions $\theta_s(R;p)$. We shall prove in a separate paper, that these solutions give the absolute minima of the energy $h(p)$ in the sector $Q_t=1$.

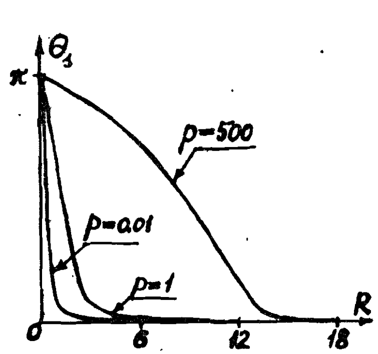

Fig.1

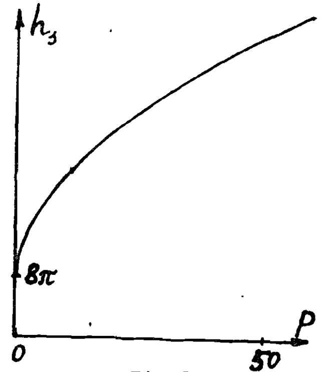

Fig.2

The radial functions $\theta_s(R)$ are displayed in Fig.1. The soliton half-width R_s monotonously increases with the increase of the parameter p ; when $p\to 0$ $R_s\to 0$ ($R_s\approx 4\cdot 10^{-2}$ at $p=10^{-6}$, $R_s\approx 0.4$ at $p=10^{-2}$, so when $p\to 0$ the half-width R_s is proportional to p^{ν}, here $\nu=\frac{1}{4}$). The soliton energy $h_s(p)$ is the monotonously increasing function of the parameter ρ too(see Fig.2), the curve $h_3(\rho)$ is described at $\rho \ll 1$ by the formula

$$h_3(\rho) = 8\pi + B\rho^{\varkappa}, \quad B \approx 37, \quad \varkappa \approx 0.4.$$

The soliton solutions $\theta_3(R)$ found numerically in this paper may be investigated analytically at small values of ρ by means of the perturbation theory in the parameter ρ, and the localized solutions of the zero approximation equation are the Belavin-Polyakov solitons in the isotropic Heisenberg ferromagnet [1].

In conclusion we note, that in the course of the sector $Q_t=2$ investigation the soliton-type distributions have been found at all ρ having the energy less than the double energy of the $Q_t=1$ soliton; this circumstance proves the existence of the bound state of two solitons with $Q_t = 1$ within the frameworks of the model under consideration.

The authors are grateful to Profs. E.P.Zhidkov, V.G.Makhankov and the participants of the LCTA seminar on the nonlinear mathematical physics for their interest in this investigation.

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Received by Publishing Department
on December 14, 1987.

Боголюбская А.А., Боголюбский И.Л. E5-87-867
О двумерных топологических солитонах в анизотропных магнетиках

Показано, что в двумерных моделях магнетиков гейзенберговского типа с различными видами анизотропии не может быть статических солитонных решений, описывающих локализованные распределения трехкомпонентного поля $s^a(x)$, $a = 1, 2, 3$, $s^a s^a = 1$. Включение в гамильтониан модели скирмовского слагаемого делает возможным существование устойчивых топологических солитонов. С помощью ЭВМ исследуются статические солитоны с топологическим зарядом $Q_t = 1$ в двумерном легкоосном магнетике.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Bogolubskaya A.A., Bogolubsky I.L. E5-87-867
On Two-Dimensional Topological Solitons in Anisotropic Magnets

It is shown that in two-dimensional Heisenberg-type magnets with various anisotropy forms static solitons, describing localized distributions of the three-component field $s^a(x)$, $a = 1,2,3$, $s^a s^a = 1$, cannot exist. Inclusion of the Skyrme addendum into the Hamiltonian of the model makes the existence of the stable topological solitons possible. By means of the computer we investigate the static solitons having the topological charge $Q_t = 1$ in the easy-axis two-dimensional magnet.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987