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ON THE ORTHOGONAL DISSIPATIVE LAX - PHILLIPS SCATTERING THEORY

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1. Introduction

In 1973 P.D.Lax and R.S.Phillips [7] presented a further refinement of their scattering theory [5,6] allowing to handle dissipative hyperbolic systems. The generalization was based on the ideas developed in [5,6]. The main difference of [7] to these papers consists in replacing of the governing unitary group by a contraction semigroup. This replacement was forced by the philosophy that dissipative systems can be described by maximal dissipative operators, which generate contraction semigroups.

In [4] C.Foias started the characterization of the possible scattering matrices occurring in the dissipative Lax-Phillips scattering theory passing to a discrete Lax-Phillips framework. In this framework C.Foias gave a necessary and sufficient description in analytical terms of the possible scattering matrices. In [8] the investigations of C.Foias were continued and it was established that a strongly measurable contraction-valued function can be regarded as the scattering matrix of a dissipative Lax-Phillips scattering theory if and only if the adjoint function admits an analytically unitary synthesis.

In the following we restrict our attention to the so-called orthogonal dissipative Lax-Phillips scattering theory. Moreover, in accordance with [4] we prefer the discrete framework of this scattering theory. For the convience of the reader we recall the assumptions. A triplet $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ consisting of a contraction T defined on a separable Hilbert space $\mathcal{H}$, subspaces $\mathcal{D}_+$ and $\mathcal{D}_-$ of $\mathcal{H}$ forms an orthogonal dissipative Lax-Phillips scattering theory, if the following conditions are fulfilled.

(h1) $T\mathcal{D}_+ \subseteq \mathcal{D}_+$, $T^*\mathcal{D}_- \subseteq \mathcal{D}_-$,

(h2) $T \upharpoonright \mathcal{D}_+$ and $T^* \upharpoonright \mathcal{D}_-$ are isometries,

(h3) $\bigcap_{n \in \mathbb{Z}_+} T^n \mathcal{D}_+ = \{0\} = \bigcap_{n \in \mathbb{Z}_+} T^{*n} \mathcal{D}_-$,

(h4) $P^{\mathcal{H}}_{\mathcal{H} \ominus \mathcal{D}_+} T^n \longrightarrow 0$, $P^{\mathcal{H}}_{\mathcal{H} \ominus \mathcal{D}_-} T^{*n} \longrightarrow 0$ strongly for $n \longrightarrow +\infty$,

(h5) $\mathcal{D}_+ \perp \mathcal{D}_-$.

In the following we show that there is an one-to-one correspondence between the class $\mathcal{A}$ of all triplets $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ forming an orthogonal dissipative Lax-Phillips scattering theory and a class $\mathcal{P}$ of 6-tuples consisting of isometries and contractions which act on or between certain separable Hilbert spaces. It is possible to describe this class $\mathcal{P}$ in a simple manner and, roughly speaking, it can be said that modulo the Hilbert spaces which are involved in the definitions of the isometries and contractions these operators can be freely chosen. In such a way we obtain a parametrization of all orthogonal dissipative Lax-Phillips scattering theories which we denote by $\mathcal{P} \ni F \longrightarrow A_F \in \mathcal{A}$ and which is called the free parametrization of the orthogonal dissipative Lax-Phillips scattering theories. Moreover, introducing an equivalence relation into $\mathcal{A}$ generated by the unitary equivalence relation of operators on Hilbert spaces we are able to define a corresponding equivalence relation in the class of parameters $\mathcal{P}$.

Let $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ be an orthogonal dissipative Lax-Phillips scattering theory. Denoting by U on $\mathcal{K}$ the minimal unitary dilation of T and introducing the subspaces $\mathcal{H}_\pm$,

$$\mathcal{H}_\pm = \bigvee_{n \in \mathbb{Z}} U^n \mathcal{D}_\pm, \tag{1.1}$$

it is not hard to see that U is reduced by $\mathcal{H}_\pm$. Let $U_\pm = U \upharpoonright \mathcal{H}_\pm$. Because of (h3) the operators $U_\pm$ are bilateral shifts. The conditions (h1) - (h4) imply the existence of the wave operators $W_-(A)$,

$$W_-(A) = \operatorname*{s-lim}_{n \to +\infty} T^n P^{\mathcal{H}_-}_{\mathcal{D}_-} U_-^{-n}, \tag{1.2}$$

and $W_+(A)$,

$$W_+(A) = \operatorname*{s-lim}_{n \to +\infty} T^{*n} P^{\mathcal{H}_+}_{\mathcal{D}_+} U_+^n. \tag{1.3}$$

The scattering operator $S(A): \mathcal{H}_- \longrightarrow \mathcal{H}_+$ of the dissipative Lax-Phillips scattering theory A is defined by

$$S(A) = W_+(A)^* W_-(A). \tag{1.4}$$

By $\Phi_-: \mathcal{H}_- \longrightarrow L^2(\mathcal{G})$ and $\Phi_+: \mathcal{H}_+ \longrightarrow L^2(\mathcal{k})$ we denote the Fourier transformations of the bilateral shifts $U_\pm$ transforming U_- and U_+ into the multiplication operators induced by e^{it}, $t \in [0, 2\pi)$, on $L^2(\mathcal{G})$ and $L^2(\mathcal{k})$, respectively. We normalize these Fourier transformations by the conditions $\Phi_- \mathcal{D}_- = L^2(\mathcal{G}) \ominus H^2(\mathcal{G})$ and $\Phi_+ \mathcal{D}_+ = H^2(\mathcal{k})$. In the following we call these special representations of $U_\pm$ the spectral representations of $U_\pm$. In these spectral representations the scattering operator $\Phi_+ S(A)\Phi_-^{-1}$ is represented by a multiplication operator induced by an operator-valued function $\{\mathcal{G}, \mathcal{K}, S_A(t)\}$, which is usually called the scattering matrix of the dissipative Lax-Phillips scattering theory A.

Taking into account the parametrization $\mathcal{P} \ni F \longrightarrow A_F \in \mathcal{A}$ we obtain a parametrization of the scattering matrices $\mathcal{P} \ni F \longrightarrow \{\mathcal{G}, \mathcal{K}, S_F(t)\}$ via the map $\mathcal{P} \ni F \longrightarrow \{\mathcal{G}, \mathcal{K}, S_{A_F}(t)\}$.

The aim of the present note is to calculate all these parametrizations explicitly and, consequently, to obtain a detailed dependence on the parameters.

Furthermore, using Corollary 3.4 of [8] we get that $\{\mathcal{K}, \mathcal{G}, S_F(t)^*\}$ admits a Darlington synthesis. Moreover, taking into account Corollary 4.2 of [8] we obtain that the map $\mathcal{P} \ni F \longrightarrow \{\mathcal{K}, \mathcal{G}, S_F(t)\}$ establishes a parametrization of all analytical contraction-valued functions admitting a Darlington synthesis. But we remark that the parametrization may be not one-to-one. Nevertheless, on account of the explicit dependence of the scattering matrix on the operators of the 6-tuples of $\mathcal{P}$ we get a structure theorem of all those analytical contraction-valued functions admitting a Darlington synthesis in this way.

2. The free parametrization

To obtain the announced free parametrization of all triplets $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ obeying (h1) - (h5) we introduce the subspace $\mathcal{H}_0 = \mathcal{H} \ominus (\mathcal{D}_+ \oplus \mathcal{D}_-)$. With respect to the decomposition

$$\mathcal{H} = \mathcal{D}_+ \oplus \mathcal{H}_0 \oplus \mathcal{D}_- \qquad (2.1)$$

we obtain a block-matrix representation of T. Because of (h1) we get that this block-matrix representation has a triangular structure, i.e.

$$T = \begin{bmatrix} S_+ & X & Y \\ 0 & T_0 & Z \\ 0 & 0 & S_-^* \end{bmatrix}, \qquad (2.2)$$

where we have $S_+ = T \upharpoonright \mathcal{D}_+$ and $S_- = T^* \upharpoonright \mathcal{D}_-$. The operators S_+ and S_- are isometries and because of (h3) are unilateral shifts. The operator T_0 is given by

$$T_0 f = P^{\mathcal{H}}_{\mathcal{H}_0} Tf, \qquad (2.3)$$

$f \in \mathcal{H}_0$. Taking into account (h4) we get $T_0 \in C_{00}$.

It remains to determine the structure of X,Y,Z. To this end we remark that T is a contraction. Consequently, the operators $T\upharpoonright(\mathcal{D}_+ \oplus \mathcal{H}_0)$ and $T^*\upharpoonright(\mathcal{H}_0 \oplus \mathcal{D}_-)$ are contractions, too. Setting $P_+ = I_{\mathcal{D}_+} \ominus S_+S_+^*$ and $P_- = I_{\mathcal{D}_-} - S_-S_-^*$ we get that X and Z have the representations $X = \Gamma D_{T_0}$ and $Z = D_{T_0^*}\Lambda$, where $\Gamma : \mathcal{D}_{T_0} = (\mathrm{ima}(D_{T_0}))^- \longrightarrow P_+\mathcal{D}_+$ and $\Lambda : P_-\mathcal{D}_- \longrightarrow (\mathrm{ima}(D_{T_0^*}))^-$ are contractions.

Similarly, we find that Y has the representation $Y = \Omega P_-$, where $\Omega : P_-\mathcal{D}_- \longrightarrow P_+\mathcal{D}_+$ is a contraction, too. Hence we get

$$T = \begin{pmatrix} S_+ & \Gamma D_{T_o} & \Omega P_- \\ 0 & T_o & D_{T_o^*}\Lambda P_- \\ 0 & 0 & S_-^* \end{pmatrix}. \qquad (2.4)$$

Obviously, we have

$$\begin{aligned} &\| S_+f_+ + \Gamma D_{T_o} f_o + \Omega P_-f_-\|^2 + \\ &+ \| T_of_o + D_{T_o^*}\Lambda P_-f_-\|^2 + \|S_-^*f_-\|^2 \leq \\ &\leq \|f_+\|^2 + \|f_o\|^2 + \|f_-\|^2 \end{aligned} \qquad (2.5)$$

for every $f = \{f_+,f_o,f_-\} \in \mathcal{D}_+ \oplus \mathcal{H}_o \oplus \mathcal{D}_-$. Taking into account the structure (2.4) we find

$$\begin{aligned} &\| \Gamma h_o\|^2 + 2\mathrm{Re}(\Gamma h_o, \Omega h_-) + \|\Omega h_-\|^2 + \\ &+ 2\mathrm{Re}(T_oh_o, \Lambda h_-) + \|D_{T_o^*}\Lambda h_-\|^2 \leq \|h_o\|^2 + \|h_-\|^2 \end{aligned} \qquad (2.6)$$

for every $h_o \in \mathcal{D}_{T_o}$ and every $h_- \in P_-\mathcal{D}_-$. But from (2.6) we get

$$\|\Gamma h_o + \Omega h_-\|^2 \leq \|h_o - T_o^*\Lambda h_-\|^2 + \|D_\Lambda h_-\|^2, \qquad (2.7)$$

$h_o \in \mathcal{D}_{T_o}$, $h_- \in P_-\mathcal{D}_-$. Notice that $T_o^*\mathcal{D}_{T_o^*} \subseteq \mathcal{D}_{T_o}$ and choosing $h_o = T_o^*\Lambda h_-$, $h_- \in P_-\mathcal{D}_-$, we get

$$\|(\Gamma T_o^*\Lambda + \Omega)h_-\|^2 \leq \|D_\Lambda h_-\|^2, \qquad (2.8)$$

$h_- \in P_-\mathcal{D}_-$. But (2.8) yields the representation

$$\Gamma T_o^*\Lambda + \Omega = \Theta D_\Lambda, \qquad (2.9)$$

where $\Theta : \mathcal{D}_\Lambda \longrightarrow P_+\mathcal{D}_+$ is a contraction. In such a way we find

$$\begin{aligned} &\|\Gamma(h_o - T_o^*\Lambda h_-) + \Theta D_\Lambda h_-\|^2 \leq \\ &\leq \|h_o - T_o^*\Lambda h_-\|^2 + \|D_\Lambda h_-\|^2, \end{aligned} \qquad (2.10)$$

$h_o \in \mathcal{D}_{T_o}$, $h_- \in P_-\mathcal{D}_-$. Obviously, the linear map

$$\begin{pmatrix} g_o \\ g_- \end{pmatrix} = \begin{pmatrix} I_{\mathcal{D}_{T_o}} & -T_o^*\Lambda \\ 0 & I_{P_-\mathcal{D}_-} \end{pmatrix} \begin{pmatrix} h_o \\ h_- \end{pmatrix} \qquad (2.11)$$

defines an one-to-one correspondence on the Hilbert space $\mathcal{D}_{T_o} \oplus P_-\mathcal{D}_-$. Hence using this transformation we obtain

$$\|\Gamma g_o + \Theta D_\Lambda g_-\|^2 \leq \|g_o\|^2 + \|D_\Lambda g_-\|^2 \qquad (2.12)$$

for every $g_o \in \mathcal{D}_{T_o}$ and every $g_- \in P_-\mathcal{D}_-$. But this estimate yields the representation $\Theta = D_{\Gamma^*}\Pi$, where $\Pi : \mathcal{D}_\Lambda \longrightarrow \mathcal{D}_{\Gamma^*}$ is a contraction.

P r o p o s i t i o n 2.1. The triplet $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ fulfils the assumptions (h1) - (h5) if and only if there are two unilateral shifts $S_\pm$ defined on $\mathcal{D}_\pm$, a contraction T_o of class C_{oo} on $\mathcal{H}_o = \mathcal{H} \ominus (\mathcal{D}_+ \oplus \mathcal{D}_-)$ and three contractions $\Gamma: \mathcal{D}_{T_o} \longrightarrow P_+\mathcal{D}_+$, $\Lambda: P_-\mathcal{D}_- \longrightarrow \mathcal{D}_{T_o^*}$ and $\Pi: \mathcal{D}_\Lambda \longrightarrow \mathcal{D}_{\Gamma^*}$ such that with respect to the decomposition (2.1) the operator T admits the block-matrix representation

$$T = \begin{bmatrix} S_+ & \Gamma D_{T_o} & D_{\Gamma^*}\Pi D_\Lambda - \Gamma T_o^* \Lambda P_- \\ 0 & T_o & D_{T_o^*}\Lambda P_- \\ 0 & 0 & S_-^* \end{bmatrix}. \tag{2.13}$$

For a given triplet $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ obeying (h1) - (h5) the operators S_+, S_-, T_o, Γ, Λ and Π are uniquely determined.

P r o o f: It was shown that for every triplet $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ obeying (h1) - (h5) there exist operators S_+, S_-, T_o, Γ, Λ and Π such that the desired block-matrix representation (2.13) of T is valid. Conversely, if with respect to a decomposition (2.1) the operator T admits the block-matrix representation (2.13), then a direct computation shows that T is a contraction and the triplet $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ fulfils the assumptions (h1) - (h3). To prove $P^{\mathcal{H}}_{\mathcal{H}\ominus\mathcal{D}_+}T^n \longrightarrow 0$ strongly for $n \longrightarrow +\infty$ we establish

$$\begin{bmatrix} T_o & D_{T_o^*}\Lambda P_- \\ 0 & S_-^* \end{bmatrix}^n \longrightarrow 0 \tag{2.14}$$

strongly for $n \longrightarrow +\infty$. Because of $T_o \in C_{oo}$ this is obvious for every $\{f_o, 0\}$, $f_o \in \mathcal{H}_o$. We set $f_- = \sum_{n=0}^{N} S_-^n f^{(n)}$, where

$f^{(n)} \in P_-\mathcal{D}_-$, $n = 0,1,2,\ldots$. A simple calculation proves

$$\begin{bmatrix} T_o & D_{T_o^*}\Lambda P_- \\ 0 & S_-^* \end{bmatrix}^n \begin{bmatrix} 0 \\ f_- \end{bmatrix} \longrightarrow 0 \tag{2.15}$$

strongly for $n \longrightarrow +\infty$ and for $N = 0,1,2,\ldots$. Hence the relation (2.14) holds for every $\{f_o, \sum_{n=0}^{N} S_-^n f^{(n)}\}$, $N = 0,1,2,\ldots$, which implies the validity of (2.14) for every element of $\mathcal{H}_o \oplus \mathcal{D}_-$. Similarly, we prove $P^{\mathcal{H}}_{\mathcal{H}\ominus\mathcal{D}_-}T^{*n} \longrightarrow 0$ strongly for $n \longrightarrow +\infty$. The condition (h5) is obvious.

It remains to show the uniqueness. Obviously, the operators S_+, S_-, T_o, X, Y and Z are uniquely determined. But the uniqueness of X and Z implies the uniqueness of Γ and Λ. Using this fact it is not hard to see that on account of the uniqueness of Y, T_o, Γ and Λ the contraction Π is uniquely determined, too. ∎

In the following we denote by $\mathcal{P}$ the class of all those 6-tuples $F = \{S_+, S_-, T_o; \Gamma, \Lambda, \Pi\}$ of contractions such that S_+ and S_- are unilateral shifts, T_o is a contraction of class C_{oo} and $\Gamma: \mathcal{D}_{T_o} \longrightarrow P_+\mathcal{D}_+$, $\Lambda: P_-\mathcal{D}_- \longrightarrow \mathcal{D}_{T_o^*}$ and $\Pi: \mathcal{D}_\Lambda \longrightarrow \mathcal{D}_{\Gamma^*}$ are contractions. Because of the converse part of Proposition 2.1 a triplet $A_F = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ obeying (h1) - (h5) corresponds to every $F \in \mathcal{P}$. Now using again Proposition 2.1 we see that the correspondence $F \longrightarrow A_F$ establishes an one-to-one correspondence between $\mathcal{P}$ and the class of all possible orthogonal dissipative Lax-Phillips scattering theories $\mathcal{A}$. In the following we call this representation the free parametrization of the orthogonal dissipative Lax-Phillips scattering theories. The parametrization is called free because the operators S_+, S_-, T_o can be freely chosen in their classes of unilateral shifts and C_{oo}-contractions, respectively, and the contractions Γ, Λ, Π can be freely chosen up to the definition and range spaces, too.

We arrive at the case investigated by D.Z.Arov [1,2,3] demanding instead of $T_o \in C_{oo}$ the condition $T_o \in C_o \subset C_{oo}$. For the definition of the class C_o the reader is referred to [9].

Further we remark that we obtain an orthogonal conservative Lax-Phillips scattering theory [5,6] if T is a unitary operator. In terms of the parameter F this means that the corresponding scattering theory is a conservative one if and only if the operator Γ is an isometry, the operator Λ is a co-isometry and Π is an isometry from D_Λ onto D_{Γ^*}.

We say the dissipative Lax-Phillips scattering theories $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ and $A' = \{T', \mathcal{D}'_+, \mathcal{D}'_-\}$ are equivalent, if there is an isometry R from $\mathcal{H}$ onto $\mathcal{H}'$ such that we have $R\mathcal{D}_\pm = \mathcal{D}'_\pm$ and

$$T' = R\,T\,R^{-1}. \tag{2.16}$$

Obviously, we define an equivalence relation in this way. Further, we say two 6-tuples $F = \{S_+, S_-, T_o; \Gamma, \Lambda, \Pi\}$ and $F' = \{S'_+, S'_-, T'_o; \Gamma', \Lambda', \Pi'\}$ of $\mathcal{P}$ are equivalent if there is a triplet of isometries $\{R_+, R_o, R_-\}$ such that the isometries R_+, R_o and R_- acting from $\mathcal{D}_+$, $\mathcal{H}_o$ and $\mathcal{D}_-$ onto

$\mathcal{D}'_+$, $\mathcal{H}'_o$ and $\mathcal{D}'_-$, respectively, fulfil

$$S'_\pm = R_\pm\, S_\pm\, R_\pm^{-1}, \tag{2.17}$$

$$T'_o = R_o\, T_o\, R_o^{-1}, \tag{2.18}$$

$$\Gamma' = R_+\, \Gamma\, R_o^{-1}, \tag{2.19}$$

$$\Lambda' = R_o\, \Lambda\, R_-^{-1}, \tag{2.20}$$

$$\Pi' = R_+\, \Pi\, R_-^{-1}. \tag{2.21}$$

Obviously, in this way we find an equivalence relation in $\mathcal{P}$.

P r o p o s i t i o n 2.2. The orthogonal dissipative Lax-Phillips scattering theories A_F and $A_{F'}$, $F, F' \in \mathcal{P}$, belong to the same equivalence class if and only if F and F' belong to same equivalence class.

We left the proof to the reader.

3. A special matrix representation

Next we transform (2.13) to a form in which the shift operators S_+ and S_- are realized by a canonical shift representation. To this end we introduce the subspaces $\mathcal{K} = \mathcal{D}_+ \ominus S_+\mathcal{D}_+$ and $\mathcal{G} = \mathcal{D}_- \ominus S_-\mathcal{D}_-$ and define the Hilbert space $\mathcal{H}'$ by

$$\mathcal{H}' = \bigoplus_{j=-\infty}^{-1} \mathcal{K}_j \oplus \mathcal{H}_o \oplus \bigoplus_{j=1}^{+\infty} \mathcal{G}_j\,, \tag{3.1}$$

where $\mathcal{K} = \mathcal{K}_{-1} = \mathcal{K}_{-2} = \dots$ and $\mathcal{G} = \mathcal{G}_1 = \mathcal{G}_2 = \dots$.
In this Hilbert space every element $f \in \mathcal{H}'$ is given by a sequence $\{\dots, k_{-2}, k_{-1}, \boxed{f_0}, g_1, g_2, \dots\}$, where $k_j \in \mathcal{K}$, $j = -1,-2,\dots$ and $g_j \in \mathcal{G}$, $j = 1,2,\dots$. We define the operator T' as follows.

$$T'\{\dots, k_{-2}, k_{-1}, \boxed{f_0}, g_1, g_2, \dots\} =$$

$$= \{\dots, k_{-2}, k_{-1}, \Gamma D_{T_0} f_0 + D_{\Gamma^*}\Gamma D_\Lambda g_1 - \Gamma T_0^* \Lambda g_1, \quad (3.2)$$

$$\boxed{T_0 f_0 + D_{T_0^*} \Lambda g_1}, g_2, g_3, \dots\}.$$

Setting $\mathcal{D}'_+ = \bigoplus_{j=-\infty}^{-1} \mathcal{K}_j$ and $\mathcal{D}'_- = \bigoplus_{j=1}^{+\infty} \mathcal{G}_j$ it is not hard to see that $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ and $A' = \{T', \mathcal{D}'_+, \mathcal{D}'_-\}$ are equivalent orthogonal dissipative Lax-Phillips scattering theories. On account of this fact we trop the symbol ' in the following. We call the representation (3.2) the special matrix representation of $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$.

The special matrix representation remembers at the matrix construction of a unitary dilation of a contraction described by B.Sz.-Nagy and C.Foias in [9,chapter I]. For further considerations we recall this construction. Let T_0 be a contraction on $\mathcal{H}_0$. We set

$$\mathcal{K}_0 = \bigoplus_{j=-\infty}^{+\infty} \mathcal{H}_j, \qquad (3.3)$$

where $\mathcal{H}_0 = \mathcal{H}_{\pm 1} = \mathcal{H}_{\pm 2} = \dots$. In accordance with [9] we define the unitary dilation U_0 of T_0 by

$$U_0\{\dots, f_{-2}, f_{-1}, \boxed{f_0}, f_1, f_2, \dots\} = \qquad (3.4)$$

$$= \{\dots, f_{-2}, f_{-1}, D_{T_0} f_0 - T_0^* f_1, \boxed{T_0 f_0 + D_{T_0^*} f_1}, f_2, \dots\}.$$

In the following we mostly work in this representation. Especially, we calculate the wave operators in this representation. In order to do this we need some further operators which we will introduce now. We assume $\dim(\mathcal{H}_0) \geq \dim(\mathcal{G})$ and $\dim(\mathcal{H}_0) \geq \dim(\mathcal{K})$. This assumption is obviously fulfilled if $\mathcal{H}_0$ is an infinite dimensional Hilbert space. Because of this assumption there are isometries $V_+ : \mathcal{G} \longrightarrow \mathcal{H}_0$ and $V_- : \mathcal{K} \longrightarrow \mathcal{H}_0$. We define the operators $\Gamma_n : \mathcal{K}_0 \longrightarrow \mathcal{H}$ by

$$\Gamma_n\{\dots, f_{-n}, f_{-(n-1)}, \dots, f_{-1}, \boxed{f_0}, f_1, f_2, \dots\} = \qquad (3.5)$$

$$= \{\dots, V_-^* f_{-(n+1)}, \Gamma Q_- f_{-n}, \Gamma Q_- f_{-(n-1)}, \dots, \Gamma Q_- f_{-1},$$

$$\boxed{f_0}, V_+^* f_1, V_+^* f_2, \dots\},$$

where $Q_- = P^{\mathcal{H}_0}_{\mathcal{D}_{T_0}}$, and the operators $\Lambda_n : \mathcal{H} \longrightarrow \mathcal{K}_0$ by

$$\Lambda_n\{\dots, k_{-2}, k_{-1}, \boxed{f_0}, g_1, \dots, g_{(n-1)}, g_n, g_{(n+1)}, \dots\} \qquad (3.6)$$

$$= \{\dots, V_- k_{-2}, V_- k_{-1}, \boxed{f_0}, \Lambda g_1, \dots, \Lambda g_{(n-1)}, \Lambda g_n, V_+ g_{(n+1)}, \dots$$

$n = 1,2,\dots$. Further we introduce the operators $L_n : \mathcal{H} \longrightarrow \mathcal{H}$,

$$L_n\{\dots, k_{-2}, k_{-1}, \boxed{f_0}, g_1, \dots, g_n, g_{n+1}, \dots\} =$$

$$= \{\ldots, 0, D_{\Gamma^*}\Pi D_\Lambda g_1, D_{\Gamma^*}\Pi D_\Lambda g_2, \ldots, D_{\Gamma^*}\Pi D_\Lambda g_n, \boxed{0}, 0, 0, \ldots\}, \tag{3.7}$$

$n = 1, 2, \ldots$.

L e m m a 3.1. Let $A_F = \{T, \mathcal{D}_+, \mathcal{D}_-\}$, $F \in \mathcal{P}$, be an orthogonal dissipative Lax-Phillips scattering theory. Using the special matrix representation of A_F the operator T^n can be represented by

$$T^n = \Gamma_n U_0^n \Lambda_n + L_n, \tag{3.8}$$

$n = 1, 2, \ldots$.

The proof is straightforward. Therefore we omit it .

4. Wave operators

In the special matrix representation we can identify the subspaces $\mathcal{H}_+$ and $\mathcal{H}_-$ with the subspaces $\bigoplus_{j=-\infty}^{+\infty} \mathcal{k}_j$ and $\bigoplus_{j=-\infty}^{+\infty} \mathcal{G}_j$, respectively. We recall that the subspaces $\mathcal{D}_+$ and $\mathcal{D}_-$ are given by $\bigoplus_{j=-\infty}^{-1} \mathcal{k}_j$ and $\bigoplus_{j=1}^{+\infty} \mathcal{G}_j$, respectively. The operator U_- acts now as follows.

$$U_-\{\ldots, g_{-2}, g_{-1}, \boxed{g_0}, g_1, g_2, \ldots\} = \{\ldots, g_{-2}, g_{-1}, g_0, \boxed{g_1}, g_2, \ldots\}. \tag{4.1}$$

Similarly, the operator U_+ is given by

$$U_+\{\ldots, k_{-2}, k_{-1}, \boxed{k_0}, k_1, k_2, \ldots\} = \{\ldots, k_{-2}, k_{-1}, k_0, \boxed{k_1}, k_2, \ldots\}. \tag{4.2}$$

To calculate the wave operators $W_+(A)$ it is necessary to introduce some new operators. We set $L_-: \mathcal{H}_- \longrightarrow \mathcal{H}$,

$$L_-\{\ldots, g_{-2}, g_{-1}, \boxed{g_0}, g_1, g_2, \ldots\} = \{\ldots, D_{\Gamma^*}\Pi D_\Lambda g_{-2}, D_{\Gamma^*}\Pi D_\Lambda g_{-1}, D_{\Gamma^*}\Pi D_\Lambda g_0, \boxed{0}, 0, 0, \ldots\}, \tag{4.3}$$

$L_+: \mathcal{H}_+ \longrightarrow \mathcal{H}$,

$$L_+\{\ldots, k_{-2}, k_{-1}, \boxed{k_0}, k_1, k_2, \ldots\} = \{\ldots, 0, 0, \boxed{0}, D_\Lambda \Pi^* D_{\Gamma^*} k_0, D_\Lambda \Pi^* D_{\Gamma^*} k_1, \ldots\}, \tag{4.4}$$

$\Lambda_-: \mathcal{H}_- \longrightarrow \mathcal{K}_0$,

$$\Lambda_-\{\ldots, g_{-2}, g_{-1}, \boxed{g_0}, g_1, g_2, \ldots\} = \{\ldots, \Lambda g_{-2}, \Lambda g_{-1}, \boxed{\Lambda g_0}, V_+ g_1, V_+ g_2, \ldots\}, \tag{4.5}$$

$\Lambda_+: \mathcal{K}_0 \longrightarrow \mathcal{H}$,

$$\Lambda_+\{\ldots, f_{-2}, f_{-1}, \boxed{f_0}, f_1, f_2, \ldots\} = \{\ldots, V_-^* f_{-2}, V_-^* f_{-1}, \boxed{f_0}, \Lambda^* Q_+ f_1, \Lambda^* Q_+ f_2, \ldots\}, \tag{4.6}$$

$Q_+ = P^{\mathcal{H}_0}_{\mathcal{D}_{T_0^*}}$, $\Gamma_-: \mathcal{K}_0 \longrightarrow \mathcal{H}$,

$$\begin{aligned}&\Gamma_-\{\dots,f_{-2},f_{-1},\boxed{f_0},f_1,f_2,\dots\} = \\ &= \{\dots,\Gamma Q_- f_{-2},\Gamma Q_- f_{-1},\boxed{f_0},V_+^* f_1,V_+^* f_2,\dots\}\end{aligned} \tag{4.7}$$

and $\Gamma_+: \mathcal{H}_+ \longrightarrow \mathcal{K}_0$,

$$\begin{aligned}&\Gamma_+\{\dots,k_{-2},k_{-1},\boxed{k_0},k_1,k_2,\dots\} = \\ &= \{\dots,V_- k_{-2},V_- k_{-1},\boxed{\Gamma^* k_0},\Gamma^* k_1,\Gamma^* k_2,\dots\}.\end{aligned} \tag{4.8}$$

The incoming and outgoing subspaces of the unitary dilation U_0 are defined by

$$\mathcal{K}_- = \bigoplus_{j=1}^{+\infty} \mathcal{H}_j \tag{4.9}$$

and

$$\mathcal{K}_+ = \bigoplus_{j=-\infty}^{-1} \mathcal{H}_j . \tag{4.10}$$

Further we define the shift operator S_0 on $\mathcal{K}_0$ by

$$\begin{aligned}&S_0\{\dots,f_{-2},f_{-1},\boxed{f_0},f_1,f_2,\dots\} = \\ &= \{\dots,f_{-2},f_{-1},f_0,\boxed{f_1},f_2,\dots\}.\end{aligned} \tag{4.11}$$

T h e o r e m 4.1. Let $A_F = \{T,\mathcal{D}_+,\mathcal{D}_-\}, F \in \mathcal{P}$, be an orthogonal dissipative Lax-Phillips scattering theory. In the special matrix representation of A_F the wave operators $W_\pm(A_F)$ are given by

$$W_-(A_F) = \Gamma_-(\operatorname*{s-lim}_{n\to+\infty} U_0^n P^{\mathcal{K}_0}_{\mathcal{K}_-} S_0^{-n})\Lambda_- + L_- \tag{4.12}$$

and

$$W_+(A_F) = \Lambda_+(\operatorname*{s-lim}_{n\to+\infty} U_0^{-n} P^{\mathcal{K}_0}_{\mathcal{K}_+} S_0^{n})\Gamma_+ + L_+ . \tag{4.13}$$

P r o o f. Taking into account Lemma 3.1 we get

$$W_-(A_F) = \operatorname*{s-lim}_{n\to+\infty} (\Gamma_n U_0^n \Lambda_n + L_n) P^{\mathcal{H}_-}_{\mathcal{D}_-} U_-^{-n} . \tag{4.14}$$

Using (4.1) and (3.7) we obtain

$$\begin{aligned}&\lim_{n\to+\infty} L_n P^{\mathcal{H}_-}_{\mathcal{D}_-} U_-^{-n}\{\dots,g_{-2},g_{-1},\boxed{g_0},g_1,g_2,\dots\} = \\ &\lim_{n\to+\infty} L_n\{\dots,0,0,\boxed{0},g_{-(n-1)},\dots,g_{-1},g_0,g_1,g_2,\dots\} = \\ &\lim_{n\to+\infty}\{\dots D_{\Gamma^*}\Pi D_\Lambda g_{-(n-1)},\dots,D_{\Gamma^*}\Pi D_\Lambda g_0,\boxed{0},0,0,\dots\} = \\ &L_-\{\dots,g_{-2},g_{-1},\boxed{g_0},g_1,g_2,\dots\}.\end{aligned} \tag{4.15}$$

Further we find

$$\begin{aligned}&\lim_{n\to+\infty} \Gamma_n U_0^n \Lambda_n P^{\mathcal{H}_-}_{\mathcal{D}_-} U_-^{-n}\{\dots,g_{-2},g_{-1},\boxed{g_0},g_1,g_2,\dots\} = \\ &\lim_{n\to+\infty} \Gamma_n U_0^n \Lambda_n\{\dots,0,0,\boxed{0},g_{-(n-1)},\dots,g_{-1},g_0,g_1,g_2,\dots\} =\end{aligned}$$

$$\lim_{n\to+\infty} \Gamma_n U_o^n \{\dots,0,0,\boxed{0}, \Lambda g_{-(n-1)},\dots,$$

$$\Lambda g_{-1}, \Lambda g_o, V_+ g_1, V_+ g_2, \dots\} = \tag{4.16}$$

$$\lim_{n\to+\infty} \Gamma_n U_o^n P^{\mathcal{K}_o}_{\mathcal{K}_-} S_o^{-n} \Lambda_-\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} =$$

$$\Gamma_- \lim_{n\to+\infty} U_o^n P^{\mathcal{K}_o}_{\mathcal{K}_-} S_o^{-n} \Lambda_-\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\}.$$

Both formulas imply (4.12). Similarly, we prove (4.13). ∎

5. Scattering operator

Our next aim is to calculate the scattering operator $S(A)$ of $A = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ in the special matrix representation. We introduce the wave operators $\overset{o}{W}_\pm$,

$$\overset{o}{W}_\pm = \text{s-lim}_{n\to\pm\infty} U_o^{-n} P^{\mathcal{K}_o}_{\mathcal{K}_\pm} S_o^n, \tag{5.1}$$

which exist. The corresponding scattering operator is denoted by S_o,

$$S_o = \overset{o}{W}{}_+^* \overset{o}{W}_-. \tag{5.2}$$

Beside these notations we need the following operators.
$\hat{\Lambda}: \mathcal{H}_- \to \mathcal{K}_o$,

$$\hat{\Lambda}\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} =$$
$$= \{\dots,\Lambda g_{-2},\Lambda g_{-1},\boxed{\Lambda g_o},\Lambda g_1,\Lambda g_2,\dots\}, \tag{5.3}$$

$\hat{\Gamma}: \mathcal{K}_o \to \mathcal{H}_+$,

$$\hat{\Gamma}\{\dots,f_{-2},f_{-1},\boxed{f_o},f_1,f_2,\dots\} =$$
$$= \{\dots,\Gamma Q_- f_{-2},\Gamma Q_- f_{-1},\boxed{\Gamma Q_- f_o},\Gamma Q_- f_1,\Gamma Q_- f_2,\dots\} \tag{5.4}$$

and $\hat{L}: \mathcal{H}_- \to \mathcal{H}_+$,

$$\hat{L}\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} =$$
$$= \{\dots,D_{\Gamma^*}\Pi D_\Lambda g_{-2},D_{\Gamma^*}\Pi D_\Lambda g_{-1},D_{\Gamma^*}\Pi D_\Lambda g_o,$$
$$\boxed{D_{\Gamma^*}\Pi D_\Lambda g_1},D_{\Gamma^*}\Pi D_\Lambda g_2,D_{\Gamma^*}\Pi D_\Lambda g_3,\dots\}. \tag{5.5}$$

T h e o r e m 5.1. Let $A_F = \{T, \mathcal{D}_+, \mathcal{D}_-\}$, $F \in \mathcal{P}$, be an orthogonal dissipative Lax-Phillips scattering theory. In the special matrix representation of A_F the scattering operator $S(A_F)$ equals

$$S(A_F) = \hat{\Gamma} S_o \hat{\Lambda} + \hat{L}. \tag{5.6}$$

P r o o f. To prove this theorem we use (4.12) and (4.13). We find

$$S(A_F) = \Gamma_+^* \overset{o}{W}{}_+^* \Lambda_+^* \Gamma_- \overset{o}{W}_- \Lambda_- + L_+^* \Gamma_- \overset{o}{W}_- \Lambda_- +$$
$$+ \Gamma_+^* \overset{o}{W}{}_+^* \Lambda_+^* L_- + L_+^* L_-. \tag{5.7}$$

To calculate the first summand we introduce the operator

$\Gamma'_-: \mathcal{H}_o \to \mathcal{H}_o$

$$\Gamma'_-\{\dots,f_{-2},f_{-1},\boxed{f_o},f_1,f_2,\dots\} = \{\dots,V_-\Gamma Q_-f_{-2},V_-\Gamma Q_-f_{-1},\boxed{f_o},f_1,f_2,\dots\}. \qquad (5.8)$$

We find

$$\Lambda_+^* \Gamma_- U_o^n P^{\mathcal{H}_o}_{\mathcal{H}_-} S_o^{-n}\Lambda_-\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} = \Gamma'_- U_o^n P^{\mathcal{H}_o}_{\mathcal{H}_-} S_o^{-n}\hat{\Lambda}\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\}, \qquad (5.9)$$

$n = 1,2,\dots$. Similarly we prove

$$\Gamma_+^* S_o^{-n} P^{\mathcal{H}_o}_{\mathcal{H}_+} U_o^n \Gamma'_-\{\dots,f_{-2},f_{-1},\boxed{f_o},f_1,f_2,\dots\} = \hat{\Gamma} S_o^{-n} P^{\mathcal{H}_o}_{\mathcal{H}_+} U_o^n\{\dots,f_{-2},f_{-1},\boxed{f_o},f_1,f_2,\dots\}. \qquad (5.10)$$

From (5.9) and (5.10) we obtain

$$\Gamma_+^* \mathring{W}_+^* \Lambda_+^* \Gamma_- \mathring{W}_- \Lambda_- = \hat{\Gamma} S_o \hat{\Lambda}. \qquad (5.11)$$

Next we calculate the second summand of (5.7). From (4.4) we conclude that the operator $L_+^*: \mathcal{H} \to \mathcal{H}_+$ acts as follows.

$$L_+^*\{\dots,k_{-2},k_{-1},\boxed{f_o},g_1,g_2,\dots\} = \{\dots,0,0,\boxed{D_{\Gamma^*}\Pi D_\Lambda g_1}, D_{\Gamma^*}\Pi D_\Lambda g_2,\dots\}. \qquad (5.12)$$

Hence we find

$$L_+^*\Gamma_- U_o^n P^{\mathcal{H}_o}_{\mathcal{H}_-} S_o^{-n}\Lambda_-\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} = \{\dots,0,0,\boxed{D_{\Gamma^*}\Pi D_\Lambda g_1},D_{\Gamma^*}\Pi D_\Lambda g_2,\dots\}, \qquad (5.13)$$

$n = 1,2,\dots$. Similarly we prove

$$\Gamma_+^* S_o^{-n} P^{\mathcal{H}_o}_{\mathcal{H}_+} U_o^n \Lambda_+^* L_-\{\dots,g_{-2},g_{-1},\boxed{g_o},g_1,g_2,\dots\} = \{\dots,D_{\Gamma^*}\Pi D_\Lambda g_{-2}, D_{\Gamma^*}\Pi D_\Lambda g_{-1}, D_{\Gamma^*}\Pi D_\Lambda g_o, \boxed{0},0,0,\dots\}, \qquad (5.14)$$

$n = 1,2,\dots$. Summing up (5.13) and (5.14) we get

$$L_+^*\Gamma_-\mathring{W}_-\Lambda_- + \Gamma_+^*\mathring{W}_+^*\Lambda_+^* L_- = \hat{L}. \qquad (5.15)$$

Taking into account (4.3) and (4.4) we obtain $L_+^*L_- = 0$. Hence (5.7), (5.11) and (5.15) prove the representation (5.6). ∎

R e m a r k 5.2. In the previous and in this section we have assumed $\dim(\mathcal{H}_o) \geqslant \dim(\mathcal{K})$ and $\dim(\mathcal{H}_o) \geqslant \dim(\mathcal{G})$. Looking at the formula (5.6) and the definitions of the operators $\hat{\Lambda}$, $\hat{\Gamma}$ and $\hat{L}$ we see that this formula makes sense independent on the assumptions $\dim(\mathcal{H}_o) \geqslant \dim(\mathcal{K})$ and $\dim(\mathcal{H}_o) \geqslant \dim(\mathcal{G})$. In such a way it seems naturally to expect that formula (5.6) holds in every case. This can be really proved. We left the proof to the reader.

6. Scattering matrix

To calculate the scattering matrix of $A_F = \{T, \mathcal{D}_+, \mathcal{D}_-\}$ we introduce the Fourier transformations $\Phi_-: \mathcal{H}_- \to L^2(\mathfrak{g})$,

$$\Phi_-\{\dots, g_{-2}, g_{-1}, \boxed{g_0}, g_1, g_2, \dots\} = \sum_{n=-\infty}^{+\infty} g_{-n} e^{int}, \tag{6.1}$$

and $\Phi_+: \mathcal{H}_+ \to L^2(\mathfrak{k})$,

$$\Phi_+\{\dots, k_{-2}, k_{-1}, \boxed{k_0}, k_1, k_2, \dots\} = \sum_{n=-\infty}^{+\infty} k_{-(n+1)} e^{int}. \tag{6.2}$$

Obviously, we have $\Phi_-\mathcal{D}_- = L^2(\mathfrak{g}) \ominus H^2(\mathfrak{g})$ and $\Phi_+\mathcal{D}_+ = H^2(\mathfrak{k})$. Moreover, we find

$$\Phi_- U_- = M_g \Phi_- \tag{6.3}$$

and

$$\Phi_+ U_+ = M_k \Phi_+, \tag{6.4}$$

where M_g and M_k denote the multiplication operators induced by e^{it} on $L^2(\mathfrak{g})$ and $L^2(\mathfrak{k})$, respectively.

T h e o r e m 6.1. Let $A_F = \{T, \mathcal{D}_+, \mathcal{D}_-\}$, $F \in \mathcal{P}$, be an orthogonal dissipative Lax-Phillips scattering theory. Then the operator $\Phi_+ S(A_F)\Phi_-^{-1}$ acts from $L^2(\mathfrak{g})$ into $L^2(\mathfrak{k})$ as

a multiplication operator $M(S_{A_F})$ induced by the operator-valued function $\{\mathfrak{g}, \mathfrak{k}, S_{A_F}(t)\}$,

$$S_{A_F}(t) = D_{\Pi^*}\Pi D_\Lambda + \Gamma\Theta_{T_0}(e^{it})^*\Lambda, \tag{6.5}$$

$t \in [0, 2\pi)$, where $\{\mathcal{D}_{T_0}, \mathcal{D}_{T_0^*}, \Theta_{T_0}(\lambda)\}$ is the characteristic function of the contraction T_0.

P r o o f. We introduce the Fourier transformations $\overset{\circ}{\Phi}_-: \mathcal{K}_- \to L^2(\mathcal{H}_0)$,

$$\overset{\circ}{\Phi}_-\{\dots, f_{-2}, f_{-1}, \boxed{f_0}, f_1, f_2, \dots\} = \sum_{n=-\infty}^{+\infty} f_{-n} e^{int}, \tag{6.6}$$

and $\overset{\circ}{\Phi}_+: \mathcal{K}_+ \to L^2(\mathcal{H}_0)$,

$$\overset{\circ}{\Phi}_+\{\dots, f_{-2}, f_{-1}, \boxed{f_0}, f_1, f_2, \dots\} = \sum_{n=-\infty}^{+\infty} f_{-(n+1)} e^{int}. \tag{6.7}$$

By a straightforward calculation we find that $\overset{\circ}{\Phi}_+ S_0 \overset{\circ}{\Phi}_-^{-1}$ equals the multiplication operator induced by the operator-valued function $S_0(t) = \operatorname{s-lim}_{r \uparrow 1} \widetilde{\Theta}_{T_0}(r \cdot e^{it})^*: \mathcal{H}_0 \to \mathcal{H}_0$, $t \in [0, 2\pi)$,

$$\widetilde{\Theta}_{T_0}(\lambda) = -T_0 + \lambda D_{T_0^*}(I - \lambda T_0^*)^{-1} D_{T_0}, \tag{6.8}$$

$|\lambda| < 1$. The restriction $\widetilde{\Theta}_{T_0}(\lambda) \upharpoonright \mathcal{D}_{T_0}$, $|\lambda| < 1$, defines an operator-valued function $\theta_{T_o}(\lambda)$, $|\lambda| < 1$, acting from $\mathcal{D}_{T_o}$ into $\mathcal{D}_{T_o^*}$. This function $\{\mathcal{D}_{T_o}, \mathcal{D}_{T_o^*}, \theta_{T_o}(\lambda)\}$ is usually called the characteristic function of T_o.

Taking into account Theorem 5.1 and the previous considerations we calculate the operator $\Phi_+ S(A_F)\Phi_-^{-1}$. We find

$$\Phi_+ S(A_F)\Phi_-^{-1} = \qquad (6.9)$$
$$= \Phi_+ \hat{L}\Phi_-^{-1} + \Phi_+ \hat{\Gamma}\, \overset{o}{\Phi}{}_+^{-1}\, \overset{o}{\Phi}_+\, S_o\, \overset{o}{\Phi}{}_-^{-1}\, \overset{o}{\Phi}_-\, \hat{\Lambda}\Phi_-^{-1}.$$

It is easy to see that $\Phi_+\hat{L}\Phi_-^{-1}: L^2(\mathcal{G}) \rightarrow L^2(\mathcal{k})$ is a multiplication operator induced by the constant operator-valued function $D_{\Gamma^*}\Pi D_\Lambda : \mathcal{G} \rightarrow \mathcal{k}$. Similarly, we obtain that $\Phi_+ \hat{\Gamma}\overset{o}{\Phi}{}_+^{-1}: L^2(\mathcal{H}_o) \rightarrow L^2(\mathcal{k})$ and $\overset{o}{\Phi}_- \hat{\Lambda}\Phi_-^{-1}: L^2(\mathcal{G}) \rightarrow$ $\rightarrow L^2(\mathcal{H}_o)$ are multiplication operators induced by the constant operator-valued functions $\Gamma P^{\mathcal{H}_o}_{\mathcal{D}_{T_o}} : \mathcal{H}_o \rightarrow \mathcal{k}$ and $\Lambda: \mathcal{G} \rightarrow \mathcal{D}_{T_o^*} \subseteq \mathcal{H}_o$, respectively. Taking into account the previous considerations we obtain (6.5). ■

Obviously, the map $\mathcal{P} \ni F = \{S_+, S_-, T_o, \Gamma, \Lambda, \Pi\} \longrightarrow$ $\rightarrow \{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$, where $\{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$ is given by

$$S_F(t) = D_{\Gamma^*}\Pi D_\Lambda + \Gamma\,\theta_{T_o}(e^{it})^*\Lambda, \qquad (6.10)$$

$t \in [0,2\pi)$, defines a parametrization of the occurring scattering matrices of the orthogonal dissipative Lax-Phillips scattering theory. Moreover, participating the point of view that the main object of the Lax-Phillips scattering theory is the scattering matrix we can regard the map $\mathcal{P} \ni F \rightarrow \{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$ as the Lax-Phillips scattering theory itself forgetting the definitions of the wave and scattering operators.

We remark that $\{P_+\mathcal{D}_+, P_-\mathcal{D}_-, S_F(t)^*\}$ is an analytical contraction-valued function. This fact is a consequence of the assumption $\mathcal{D}_+ \perp \mathcal{D}_-$.

Further we note that if F and F' belong to the same equivalence class, then the scattering matrices $\{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$ and $\{P'_-\mathcal{D}'_-, P'_+\mathcal{D}'_+, S_{F'}(t)\}$ are equivalent, too. This means there exists a pair of isometries $\{V_+, V_-\}$ such that V_+ and V_- acting from $P_+\mathcal{D}_+$ and $P_-\mathcal{D}_-$ onto $P'_+\mathcal{D}'_+$ and $P'_-\mathcal{D}'_-$, respectively, and fulfilling

$$V_+^{-1} S_{F'}(t) V_- = S_F(t) \qquad (6.11)$$

for a.e. $t \in [0,2\pi)$. Unclear is the converse of this conclusion. This means, if F and F' belong to $\mathcal{P}$ and $\{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$ and $\{P'_-\mathcal{D}'_-, P'_+\mathcal{D}'_+, S_{F'}(t)\}$ are equivalent what can be said with respect to F and F'.

Moreover, we remark that in accordance with [8] an analytical contraction-valued function can be regarded as the adjoint scattering matrix of an orthogonal dissipative Lax-Phillips scattering theory if and only if this function admits a Darlington synthesis. It can be directly shown that $\{P_+\mathcal{D}_+, P_-\mathcal{D}_-, S_F(t)\}$ admits a Darlington synthesis. In such a way $\{P_-\mathcal{D}_-, P_+\mathcal{D}_+, S_F(t)\}$ fulfils the assumptions of C.Foias [4].

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Найдхардт Х. E5-87-416
Об ортогональной диссипативной теории рассеяния Лакса-Филлипса

Работа посвящена так называемой ортогональной диссипативной теории рассеяния Лакса-Филлипса. Получена параметризация всех возможных ортогональных диссипативных теорий рассеяния Лакса-Филлипса в терминах упорядоченного шестикратного семейства, состоящего из ортогональных сдвигов и сжимающих операторов, которые можно, вообще говоря, свободно выбирать. В этой параметризации волновые операторы, оператор рассеяния, а также матрица рассеяния явно вычисляются. Более того, дано описание всех аналитических сжимающих операторозначных функций, допускающих синтез по Дерлингтону.

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Neidhardt H. E5-87-416
On the Orthogonal Dissipative Lax-Phillips Scattering Theory

The paper is devoted to the so-called orthogonal dissipative Lax-Phillips scattering theory. It is obtained a parametrization of all possible orthogonal dissipative Lax-Phillips scattering theories in terms of ordered 6-tuples consisting of unilateral shifts and contractions which can be roughly speaking freely chosen. In this parametrization the wave and scattering operators as well as the scattering matrix are explicitly calculated. Moreover, a description of all analytical contraction-valued functions admitting a Darlington synthesis is found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.
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