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SYSTEMS OF IMPRIMITIVITY FOR THE DIFFEOMORPHISM GROUP

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1. Introduction

We again accept notation introduced in [1]. $\mathrm{Diff}_c M$ denotes the group of compactly supported diffeomorphisms on a smooth manifold M. A couple (E,V) with E being a projection-valued measure on the σ-algebra of Borel sets B(M) and with V being a unitary representation of Diff, both in the same Hilbert space H, is a system of imprimitivity if $V(\varphi)E(S)V(\varphi)^{-1}=E(\varphi.S)$ holds for all $S \in B(M)$, $\varphi \in \mathrm{Diff}_c M$. According to the construction described in [1], to every unitary representation L of the group $J_\infty(n)=J_\infty$ consisting of jets of local diffeomorphisms at $0 \in \mathbb{R}^n$ (n=dim M) there is related a canonical system of imprimitivity (E^L,V^L) in the Hilbert space H^L of L-equivariant functions on the principal bundle $(P_\infty(M),\pi,M;J_\infty)$. In the case M is orientable and connected and if we restrict ourselves to the subgroup Diff_c^+M of diffeomorphisms preserving orientation and similarly to the component of the unity $J_\infty^+ \subset J_\infty$ then it holds
(E^{L_1},V^{L_1}), (E^{L_2},V^{L_2}) are equivalent if and only if L_1,L_2 are equivalent;
the $*$-algebras $C(E^L,V^L)$, $C(L)$ are isomorphic.

All canonical systems of imprimitivity satisfy the following additional condition - called the condition of locality in [1] : $\varphi|_S = id_S \Rightarrow V(\varphi)E(S)=E(S)$ for all $S \in B(M)$, $\varphi \in \mathrm{Diff}_c M$. In the present paper we call these systems of imprimitivity point supported. The aim of this article is to prove that in the case M is orientable and connected all point supported systems of imprimitivity are described, up to unitary equivalence, by the construction, i.e. to prove

Theorem 1. Let M be an orientable connected smooth manifold, (E,V) be a point supported system of imprimitivity for Diff_c^+M. Then there exists a unitary representation L of the group J_∞^+ such that (E,V) is unitarily equivalent to the canonical system of imprimitivity (E^L,V^L).

2. Preliminaries

We summarize some well-known facts. Most of them can be found with further details in [2]. Let us fix a σ-finite quasiinvariant measure μ in the unique invariant (with respect to $\mathrm{Diff}_c M$) measure class on $B(M)$. $\mathrm{Diff}_c^+ M$ acts transitively on M and so E must be homogeneous, i.e. up to unitary equivalence the Hilbert space H can be identified with $L^2(M,H_o,d\mu)$, $\dim H_o$=multiplicity of E, and

$$E(S)f = \chi_S . f, \quad S\in B(M), f\in H. \tag{1}$$

Every bounded operator A in H commuting with E is of the form $(Af)(x)=a(x)f(x)$, where a is an operator-valued Borel mapping, $\|A\| = \mathrm{ess}_x \sup |a(x)|$. A is unitary (self-adjoint) if and only if $a(x)$ is unitary (self-adjoint) almost everywhere. For $\varphi \in \mathrm{Diff}_c^+ M$ we put $[V_o(\varphi)f](x) = \sqrt{d\mu/d\varphi^*\mu}(\varphi^{-1}x)\, f(\varphi^{-1}x)$. Then $V_o(\varphi)$ is a unitary operator and $T(\varphi)=V(\varphi)V_o(\varphi)^{-1}$ commutes with E.
Hence V must be of the form

$$[V(\varphi)f](x) = \sqrt{d\mu/d\varphi^*\mu}(\varphi^{-1}x)\; h_\varphi(x)\; f(\varphi^{-1}x), \quad \varphi\in \mathrm{Diff}_c^+ M,\ f\in H, \tag{2}$$

where $h_\varphi : M \longrightarrow U(H_o)$ is a Borel mapping. Here and in the next $U(H_o)$ denotes the group of unitary operators in H_o. We shall write also $h(\varphi,x)$ instead of $h_\varphi(x)$. As V is a representation h satisfies the 1-cocycle identity

$$h(\varphi_1 \circ \varphi_2, x) = h(\varphi_1, x) h(\varphi_2, \varphi_1^{-1} x) \quad \text{almost everywhere} \tag{3}$$

for all φ_1, φ_2. Moreover, (E,V) is point supported if and only if

$$\varphi_1^{-1} = \varphi_2^{-1} \text{ on } S \Longrightarrow h(\varphi_1, x) = h(\varphi_2, x) \text{ almost everywhere on } S. \tag{4}$$

We also remind (c.f. [2,3]) that according to Mackey's theory, for a locally compact group G satisfying the second axiom of countability and acting on a homogeneous space $M=G/K$ equivalence classes of unitary representations of the isotropy group K are in one-to-one correspondence with equivalence classes of systems of imprimitivity. Particularly, if K is trivial then every system of imprimitivity is a sum of at most countably many copies of the unique irreducible system of imprimitivity.

<u>Notation</u>. For $S\in B(M)$ we denote the subgroup of diffeomorphisms with supports contained in S by $D(S) \subset \mathrm{Diff}_c^+ M$. If S is open then $D(S) \equiv \mathrm{Diff}_c^+ S$, if S is closed then $D(S)$ is a closed subgroup in $\mathrm{Diff}_c^+ M$, if S is compact then $D(S)$ is metrizable. Details conserning the topology

of the diffeomorphism group can be found in [4]. $B(S)$ denotes the family of Borel subsets of S and $H(S)=E(S).H$ denotes the subspace in H corresponding to S.

Given a system of imprimitivity (E,V) we can consider the restriction to the σ-algebra $B(S)$ and to the subgroup $D(S)$ and both E and V having been restricted act in the invariant subspace $H(S)$.

3. <u>Continuity of the 1-cocycle</u>

We need to smooth out the 1-cocycle h. The following simple lemma turns out to be useful since it enables to apply Mackey's "Imprimitivity Theorem".

<u>Lemma 2</u>. Let $B\subset \mathbb{R}^n$ be an open ball. Then there exists an injective continuous homomorphism $\Phi: \mathbb{R}^n \longrightarrow \mathrm{Diff}_c \mathbb{R}^n$: $t \longmapsto \varphi_t$ such that

i) $\Phi(\mathbb{R}^n) \subset \mathrm{Diff}_c \mathbb{R}^n$ is a closed subgroup and $\Phi: \mathbb{R}^n \longrightarrow \Phi(\mathbb{R}^n)$ is an isomorphism of topological groups,

ii) $\mathrm{supp}\ \varphi_t \subset \bar{B}$ for all $t\in \mathbb{R}^n$,

iii) $\Phi(\mathbb{R}^n)$ acts on B transitively and freely,

iv) $\mathbb{R}^n \longrightarrow B$: $t \longmapsto \varphi_t(0\cdot)$ is a diffeomorphism.

<u>Proof</u>. We can restrict ourselves to the case of the unite ball centred in the origin. Let us for $t\in\mathbb{R}^n$ denote by τ_t the translation $\tau_t(x)=x+t$. Let ρ be a positive smooth function on $\langle 0,+\infty)$ with properties: $\rho=1$ identically on some neighbourhood of 0 and $\rho(x) = (x \ln^2 x)^{-1}$ for x large enough. For $x\geq 0$ we put $f(x) = K \int_0^x \rho(s)\, ds$, $1/K = \int_0^\infty \rho(s)\, ds$. Then f maps $\langle 0,+\infty)$ diffeomorphically onto $\langle 0,1)$ and $f(x)=Kx$ in a neighbourhood of 0, $f(x)=1-(K/\ln x)$ for x large enough. We define a diffeomorphism $F: \mathbb{R}^n \longrightarrow B$ by $F(x)=(f(|x|)/|x|)x$ and put $\varphi_t(x)=F\circ\tau_t\circ F^{-1}(x)$ for $x\in B$, $\varphi_t(x)=x$ for $x\notin B$.
Now, all assertions (i - iv) can be verified directly using the explicit expressions for φ_t.

We parametrize a ball $B\subset\mathbb{R}^n$ using the diffeomorphism $\mathbb{R}^n \longrightarrow B$: $t \longmapsto \varphi_t(0)$ given by Lemma 2. Consequently, the group $D(B)$ is identified with $\mathrm{Diff}_c \mathbb{R}^n$, $D(\bar{B})$ with a closed subgroup $D\subset \mathrm{Diff}^+ \mathbb{R}^n$, the diffeomorphism φ_t with the translation $\tau_t \in D$. The 1-cocycle h in this parametrization will be distinguished by the dash: h'. So, h'_φ is defined almost everywhere on $\mathbb{R}^n$ for all $\varphi\subset D$ and the identity (3) is fulfilled for all $\varphi_1, \varphi_2 \in D$. We also identify $B(B)$ with $B(\mathbb{R}^n)$ and $H(B)$ with $L^2(\mathbb{R}^n, H_o, d^n x)$.

We consider the restricted system of imprimitivity for the group D (acting on $\mathbb{R}^n$) which is written in the form (1), (2) (in the Hilbert space $L^2(\mathbb{R}^n, H_o, d^n x)$. If we restrict the system of imprimitivity once more to the subgroup of translations we can use "Imprimitivity

Theorem". It follows that there exists a unitary transformation preserving the projection-valued measure such that the transformed 1-cocycle h' fulfils

$$h'(\tau_t, x) = 1 \quad \text{identically on } \mathbb{R}^n \text{ for all } t \in \mathbb{R}^n. \tag{5}$$

In the next we suppose (5) to be valid.

According to (3), for each $\varphi \in D$ and all $z \in \mathbb{R}^n$, $h'(\tau_z \circ \varphi, x) = h'(\varphi, x-z)$ holds for almost all $x \in \mathbb{R}^n$. Let us denote by D_0 the isotropy group of the origin. Each $\psi \in D$ has a unique decomposition $\psi = \tau_z \circ \varphi$ with $z \in \mathbb{R}^n$, $\varphi \in D_0$. We can redefine h'_ψ on a measure zero set and assume that

$$h'(\tau_z \circ \varphi, x) = h'(\varphi, x-z) \quad \text{for all } x, z \in \mathbb{R}^n \tag{6}$$

and for each $\varphi \in D_0$. Clearly, the condition (6) is then valid even for each $\varphi \in D$.

We say that a Borel set has full measure if its complement has zero measure.

Lemma 3. For each $\varphi \in D$ we can redefine h'_φ on a measure zero set in such a way that h'_φ is continuous on $\mathbb{R}^n$.

Proof. Let us denote $T(\varphi) = V(\varphi) V_0(\varphi)^{-1}$, i.e. $[T(\varphi)f](x) = h'(\varphi, x) f(x)$. Since $z \mapsto T(\tau_z \circ \varphi)$ is a continuous mapping the sequence of non-negative numbers $\varepsilon_n = \sup_{|z|<1/n} \| T(\tau_z \circ \varphi) - T(\varphi) \|$ converges to zero for $n \to \infty$. From the Fubini Theorem and from the assumption (6) it follows that the sets C_n consisting of points $(x,y) \in \mathbb{R}^n \times \mathbb{R}^n$ such that $|x-y| < 1/n$ and $|h'(\varphi, x) - h'(\varphi, y)| > \varepsilon_n$ have measure zero for all $n \in \mathbb{N}$. Let A be the complement of the union $\bigcup C_n$. Then A has full measure and it holds $\forall \varepsilon > 0$, $\exists \delta > 0$, $\forall (x,y) \in A$, $|x-y| < \delta \Rightarrow |h'(\varphi, x) - h'(\varphi, y)| < \varepsilon$. The following lemma completes the proof.

Lemma 4. Let h be a Borel mapping from $\mathbb{R}^n$ into a complete metric space (X, ρ). Let $A \subset \mathbb{R}^n \times \mathbb{R}^n$ be a Borel subset of full measure with the property: $\forall \varepsilon > 0$, $\exists \delta > 0$, $\forall (x,y) \in A$, $|x-y| < \delta \Rightarrow \rho(h(x), h(y)) < \varepsilon$. Then there exists a uniformly continuous mapping $\bar{h}: \mathbb{R}^n \to X$ which coincides with h on a set of full measure.

Proof. Replacing A by $A \cup A^{-1} \cup \Delta$ where A^{-1} is the inverse relation and Δ is the diagonal we can suppose A to be symmetric and reflexive. For $x \in \mathbb{R}^n$ let us denote by A_x the corresponding section, i.e. $y \in A_x \subset \mathbb{R}^n$ if $(x,y) \in A$. It holds $x \in A_x$ and if $x \in A_y$ then $y \in A_x$. It will be sufficient to show that there exist sets $W \subset Z \subset \mathbb{R}^n$ such that W is countable and dense in $\mathbb{R}^n$, Z has full measure and

$$\forall \varepsilon > 0, \ \exists \delta > 0, \ \forall x \in W, \ \forall y \in Z, \ |x-y| < \delta \Rightarrow \rho(h(x), h(y)) < \varepsilon. \tag{7}$$

Clearly, h is uniformly continuous on W. There exists a unique uniformly continuous mapping $\bar{h}$ on $\mathbb{R}^n$ which coincides with h on W. Moreover, for each $x \in Z$ there exists a sequence (w_n) in W, $w_n \to x$ and $\bar{h}(x) = \lim h(w_n) = h(x)$ due to (7).

Now we are going to construct W and Z. According to the Fubini Theorem, there exists a set $C \subset \mathbb{R}^n$ of full measure such that A_x has full measure for all $x \in C$. Let (x_n) be a sequence of points in $\mathbb{R}^n$ which as a subset is dense in $\mathbb{R}^n$. We define by induction a sequence (w_n) of points in C requiring $|w_n - x_n| < 1/n$ and $w_n \in A_{w_1} \cap \ldots \cap A_{w_{n-1}}$. Let $W = \{ w_n ; n \in \mathbb{N} \}$ and $Z = \bigcap A_{w_n}$. Then W is countable and dense in $\mathbb{R}^n$, $W \subset Z$, Z has full measure and $Z \subset A_w$ for all $w \in W$, so $W \times Z \subset A$. Hence W, Z have the desired property. This completes the proof.

A reformulation of Lemma 3 leads to

Proposition 5. Let (E,V) be a system of imprimitivity for the group $\mathrm{Diff}_c \mathbb{R}^n$ in the Hilbert space $L^2(\mathbb{R}^n, H_0 d^n x)$ written in the form (1), (2). Then for every open ball $B \subset \mathbb{R}^n$ there exists a unitary transformation in H(B) preserving the projection-valued measure such that having performed it (and possibly having redefined h_φ on a measure zero set) h_φ is continuous on B for all $\varphi \in D(\bar{B})$.

4. Representation of the jet group

From now on we investigate point supported systems of imprimitivity, i.e. the condition (4) holds. Let (E,V) be a system of imprimitivity for $\mathrm{Diff}_c \mathbb{R}^n$ in $L^2(\mathbb{R}^n, H_o, d^n x)$ and let $B \subset \mathbb{R}^n$ be an open ball. We keep notation from the previous section. B is again parametrized by $t \mapsto \varphi_t(0)$. We can assume (for the restricted system of imprimitivity) that h_φ is continuous on $\mathbb{R}^n$ for all $\varphi \in D$.

The mapping $L_0 : D_0 \to U(H_o)$, $L_0(\varphi) = h'(\varphi, 0)$ defines a unitary representation of the isotropy group D_0. The condition (4) implies that $L_o(\varphi)$ depends only on the germ of φ at 0. According to the next lemma we obtain a representation of the group of germs of local diffeomorphisms at the origin.

Lemma 6 (Palais, c.f. [5]). Let $\varphi : \mathbb{R}^n \to \mathbb{R}^n$ be a local diffeomorphism preserving the orientation, defined on a neighbourhood of a point x. Then there exists $\psi \in \mathrm{Diff}_c \mathbb{R}^n$ such that the germs of φ and ψ at the point x coincide.

More is true. The following lemma is proved exactly in the same way as Theorem 1.2 in [6].

Lemma 7. $L_0(\varphi)$ depends only on the jet $j_\infty(\varphi)_0$.

Proof. If $j_\infty(\varphi_1^{-1})_0 = j_\infty(\varphi_2^{-1})_0$ then there exist open sets O_1, O_2 and $\psi \in D_0$ such that 0 belongs to the intersection of the closures $\bar{O}_1 \cap \bar{O}_2$ and ψ^{-1} coincides with φ_i^{-1} on O_i, i=1,2. Since h' is continuous and the condition (4) holds we find

$$L_0(\varphi_1) = \lim_{\substack{x\to 0\\ x\in O_1}} h'(\varphi_1,x) = \lim_{x\to 0} h'(\psi,x) = \lim_{\substack{x\to 0\\ x\in O_2}} h'(\varphi_2,x) = L_0(\varphi_2).$$

This proves the lemma.

According to the Borel "Extension Lemma" (c.f. [6]), the group of infinite invertible jets in $\mathbb{R}^n$ is a projective limit of the groups of finite jets, $J_\infty(n) = \varprojlim J_k(n)$. For every $j \in \varprojlim J_k(n)$ there exists a local diffeomorphism φ at the origin such that $j = j_\infty(\varphi)_0$. We have obtained a unitary representation L of the group $J_\infty^+ = J_\infty^+(n)$ defined by $L\colon j_\infty(\varphi)_0 \longmapsto L_0(\varphi)$.

From (3) it follows $h'(\varphi,x) = h'(\tau_{-u}\circ\varphi\circ\tau_{\varphi^{-1}u}, x-u)$ for all $x,u \in \mathbb{R}^n$. Putting u=x we have

$$h'(\varphi,x) = L(j_\infty(\tau_{-x}\circ\varphi\circ\tau_{\varphi^{-1}x})_0). \tag{8}$$

Hence the restricted system of imprimitivity is equivalent to the canonical system of imprimitivity (E^L, V^L). The principal fibre bundle $P_\infty^+(\mathbb{R}^n) \longrightarrow \mathbb{R}^n$ with the structure group J_∞^+ is trivializable. The mapping $x \longmapsto \sigma(x) = j_\infty(\tau_x)_0^x$ defines a smooth section σ. Then the desired unitary mapping from the Hilbert space H^L of L-equivariant functions on $P_\infty^+(\mathbb{R}^n)$ onto $L^2(\mathbb{R}^n, H_0, d^n x)$ is defined by $\tilde{f} \longmapsto f$, $\tilde{f}(\sigma(x)) = f(x)$.

We add a remark to the general case. Let (E,V) be a (not necessarily point supported) system of imprimitivity for $\mathrm{Diff}_c M$ written in the form (1),(2). Let $\pi\colon P \longrightarrow M$ be a principal bundle with a structure group J and let L be a unitary representation of J in H_0. L need not be related to the 1-cocycle h. Then we can transform (E,V) unitarily to a system of imprimitivity $(\tilde{E},\tilde{V})$ in the Hilbert space H^L of L-equivariant functions on P.

$$\tilde{E}(S)\,\tilde{f} = \chi_{\pi^{-1}(S)}\,\tilde{f}, \tag{9}$$

$$[\tilde{V}(\varphi)\tilde{f}](p) = \pi^*\sqrt{d\mu/d\varphi^*\mu}\,(\varphi^{-1}p)\; k_\varphi(p)\; \tilde{f}(\varphi^{-1}p), \tag{10}$$

where $\tilde{f} \in H^L$ and k_φ is a Borel 1-cocycle on P with values in $U(H_0)$ and

$$k(\varphi,pj) = L(j^{-1})\; k(\varphi,p)\; L(j), \quad j \in J, \tag{11}$$

holds on almost all fibres $P_x = \pi^{-1}(x)$, $x \in M$. We can construct a unitary transformation using again a Borel section $\sigma\colon M \longrightarrow P$. Moreover, every unitary transformation $\tilde{U}$ in H^L commuting with $\tilde{E}$ is of the form $(\tilde{U}\tilde{f})(p) = \tilde{u}(p)\tilde{f}(p)$, where $\tilde{u}\colon P \longrightarrow U(H_0)$ is a Borel mapping fulfilling

$$\tilde{u}(pj) = L(j^{-1})\; \tilde{u}(p)\; L(j), \; j \in J, \tag{12}$$

on almost all fibres.

5. Proof of Theorem 1

The case $M=\mathbb{R}^n$. Let (E,V) be a point supported system of imprimitivity for the group $\mathrm{Diff}_c\mathbb{R}^n$. According to the previous section, we can relate to every open ball $B \subset \mathbb{R}^n$ a unitary representation of the group J_∞^+. From [1] we know that L is unique up to unitary equivalence. If $B \subset B'$ are two nested balls then $D(B) \subset D(B')$ and for the same reason the corresponding representations L,L$'$ are equivalent. It follows that there is a unique equivalence class of unitary representations of J_∞^+ which is related to (E,V). Let us fix a representation L in this class. We can write (E,V) in the form (9),(10) with $P = P_\infty^+(\mathbb{R}^n)$, $J = J_\infty^+$.

Again from the previous section it follows that for every open ball B there exists a unitary transformation in H(B) preserving the projection-valued measure such that having performed it $k_\varphi = 1$ identically on $\pi^{-1}(B)$ for all $\varphi \in D(B)$. This unitary representation is determined by a Borel mapping $u\colon \pi^{-1}(B) \longrightarrow U(H_0)$ which fulfils (12) on almost all fibres $P_x \subset \pi^{-1}(B)$. The results of [1] imply that any other u' with this property has the form $u'(p) = wu(p)$, where $w \in C(L) \cap U(H_0)$. Hence if $B \subset B'$ are nested balls then u can be extended from B to B$'$ in a unique way (up to a measure zero set). In this way we can define u even on $\mathbb{R}^n$. This proves the assertion in this case.

The case M is orientable and connected. We use the following well-known fact (in the same manner as it was used in [7]):

Every connected manifold M admits a cell decomposition with a unique cell in the top dimension. Therefore there is a smooth imbedding of $\mathbb{R}^n$ into M whose image has complement of measure zero.

Let (E,V) be a point supported system of imprimitivity for Diff_c^+M in a Hilbert space H. Let $N \subset M$ be an open subset diffeomorphic to $\mathbb{R}^n$ with the complement $Z = M \setminus N$ of measure zero. We consider the restricted system of imprimitivity for the group $D(N) \cong \mathrm{Diff}_c N$ in the space $H(N)$. Clearly, $H(N)=H$. According to the case $M=\mathbb{R}^n$, there exists a representation L of J_∞^+ and a unitary transformation $H \longrightarrow H^L$ such that the transformed and restricted system of imprimitivity coincides with the canonical one, i.e. (E,V) is expressed in the form (9),(10) and $k_\varphi=1$ identically on P_x for almost all $x \in N$ and for all $\varphi \in \mathrm{Diff}_c N$.

Now, let $\varphi \in \mathrm{Diff}_c^+ M$ and $x \in N \setminus \varphi(Z) = M \setminus (Z \cup \varphi(Z))$. Then according to Lemma 6, there exists $\psi \in D(N)$ such that the germs of φ^{-1}, ψ^{-1} at x coincides. As we suppose (4) to be valid a neighbourhood O of the point x exists such that $k_\varphi = k_\psi$ on P_y for almost all $y \in O$. The set $Z \cup \varphi(Z)$ has zero measure and so $k_\varphi=1$ on P_x for almost all $x \in M$. This completes the proof.

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Системы импримитивности для группы диффеоморфизмов

Доказывается, что конструкция, которая была предложена в предшествующей статье[1], описывает с точностью до унитарной эквивалентности все системы импримитивности для группы диффеоморфизмов, предполагая выполненным дополнительное условие локальности.

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Systems of Imprimitivity for the Diffeomorphism Group

It is proved that the construction investigated in the previous article[1] describes, up to unitary equivalence, all systems of imprimitivity for the group of diffeomorphisms provided the additional condition of locality is fulfilled.

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