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BRANCHING RULES FOR sp(2N) ALGEBRA REDUCTION ON THE CHAIN sp(2N-2)×sp(2)

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I. Introduction

Few years ago the so-called missing labels problem in the algebra reductions

$$sp(2N) \searrow sp(2N-2) \times A_N^N \qquad (1a)$$

and

$$sp(2N) \searrow sp(2N-2) \times sp(2) \qquad (1b)$$

was investigated[1] (here A_N^N is an operator from the Cartan subalgebra sp(2N)). Two different solutions on the set of the missing label operators were obtained, but only one solution **referred** here as (b) (see (1a, b)) contains the Casimir of the sp(2) subalgebra

$$C^2_{(N)} = 2(A_N^N)^2 + A_{-N}^N A_N^{-N} - A_N^{-N} A_{-N}^N , \qquad (2)$$

where A^α_β are the algebra generators.

The problem of the branching rules for the reduction (1a) has been solved many years ago in Zhelobenko paper[2]. Denoting shortly by $|(\Omega)\rangle$ the vectors belonging to the BUIR we have

$$|(\Omega)\rangle = \left| \begin{array}{cccccccccc} \Omega_1^N & & \Omega_2^N & & \Omega_3^N & \ldots & & \Omega_N^N & & \\ & \bar\Gamma_1^N & & \bar\Gamma_2^N & & \bar\Gamma_3^N \ldots & & \bar\Gamma_{N-1}^N & & \bar\Gamma_N^N \\ & \Omega_1^{N-1} & & \Omega_2^{N-1} & \Omega_3^{N-1} & \ldots & & \Omega_{N-1}^{N-1} & & \\ & \ldots & & \ldots & & \ldots & & \ldots & & \\ & & & & & & & \Omega_1^1 & & \\ & & & & & & & & \bar\Gamma_1^1 & \end{array} \right\rangle , \qquad (3a)$$

where the first row labels the UIR of the sp(2N) algebra, third row labels the UIR of the sp(2N-2) algebra, and so on. The eigenvalue for operator A_k^k is denoted by h_k and we have

$$h_k = \sum_{p=1}^{k} \Omega_p^k + \sum_{p=1}^{k-1} \Omega_p^{k-1} - 2\sum_{p=1}^{k} \Gamma_p^k . \qquad (3b)$$

All numbers entering to the pattern (3a) are positive integers and the branching rules for the reduction (1a) are contained in the systems of inequalities

$$\Omega_1^k \ge \Gamma_1^k \ge \Omega_2^k \ge \Gamma_2^k \ge \ldots \ge \Omega_k^k \ge \Gamma_k^k \ge 0 \qquad \text{for } k = N, N-1, \ldots, 1 \quad (3c)$$

$$\Gamma_1^k \ge \Omega_1^{k-1} \ge \Gamma_2^k \ge \Omega_2^{k-1} \ge \ldots \ge \Omega_{k-1}^{k-1} \ge \Gamma_k^k \qquad \text{for } k = N, N-1, \ldots, 2 . \quad (3d)$$

Hence we see that the same representation of the subalgebra sp(2k-2) x A_k^k may be found more than one times in the (Ω_1^k , Ω_2^k , ..., Ω_k^k) representation of the sp(2k) algebra and we use k-I missing label number $\Gamma_1^k, \Gamma_2^k, \ldots, \Gamma_{k-1}^k$ to distinguish them (here Γ_k^k is dependent on the h_k, see (3b)). If we take into the consideration the results of the paper[1] we see that the pattern (3a-d) is appropriate for labeling the states in the orthogonal base reduced on the chain (1a). In the next Section we will find branching rules and an appropriate pattern for the reduction (1b).

II. Branching Rules for Reduction sp(2N)$\searrow$sp(2N-2) x sp(2)

The following pattern may be used, instead of (3a), for labeling the vectors in BUIR of the sp(2N) algebra

$$|(\Omega)\rangle = \left|\begin{array}{ccccccc} \Omega_1^N & \Omega_2^N & \Omega_3^N & \ldots & \ldots & \Omega_N^N & \\ & \Gamma_1^N & \Gamma_2^N & \Gamma_3^N & \ldots & \Gamma_{N-1}^N & h_N \\ & \Omega_1^{N-1} & \Omega_2^{N-1} & \Omega_3^{N-1} & \ldots & \Omega_N^{N-1} & \\ & & \ldots & \ldots & \ldots & \ldots & \\ & & & & & \Omega_1^1 & \\ & & & & & & h_1 \end{array}\right\rangle , \qquad (4a)$$

where the meaning of the Ω's numbers is the same as in (3a). The eigenvalue of $C_{(k)}^2$ operators (2) are equal to $2\sigma_k(\sigma_k+2)$ and we have

$$\sigma_k = \sum_{p=1}^{k} \Omega_p^k + \sum_{p=1}^{k-1} \Omega_p^{k-1} - 2\sum_{p=1}^{k-1} \Gamma_p^k \qquad (4b)$$

The branching rules for the reduction (1b) are contained in the system of inequalities (k = N,N-1,...,2)

$$\Omega_1^k \ge \Gamma_1^k \ge \Omega_2^k \ge \Gamma_2^k \ge \ldots \ge \Gamma_{k-1}^k \ge \Omega_k^k \qquad (4c)$$

$$\Gamma_1^k \ge \Omega_1^{k-1} \ge \Gamma_2^k \ge \Omega_2^{k-1} \ge \ldots \ge \Gamma_{k-1}^k \ge \Omega_{k-1}^{k-1} \qquad (4d)$$

$$\sum_{p=a}^{k-1} (\Omega_{p+1}^k + \Omega_p^{k-1}) \ge \Gamma_a^k + 2\sum_{p=a}^{k-1} \Gamma_p^k \qquad \text{for } a = 1,2,\ldots,k-1 . \quad (4e)$$

(4f)

Proof

Let us consider two sets $\Delta^k(A_1, A_2, \ldots, A_k; M)$ and $\bar{\Delta}^k(A_1, A_2, \ldots, A_k; M)$. The first one contains points $X = (x_1, x_2, \ldots, x_k)$, where the x_i are integer or halfinteger numbers bounded by the system inequalities (5) and M is given by (6)

$$A_i \ge x_i \ge -A_i \qquad \text{for } i = 1,2,\ldots,k \qquad (5)$$

$$M = \sum_{i=1}^{k} x_i . \qquad (6)$$

The second one contains points $Y = (y_1, y_2, \ldots, y_k)$ where coordinates y_i are integer or halfinteger numbers satysfying relations

$$A_{p+1} + y_{p-1} \ge y_p \ge |A_{p+1} - y_{p-1}| \qquad \text{for } p=1,2,\ldots,k-1 \qquad (7a,b,c)$$

$$y_{k-1} \ge y_k \ge -y_{k-1} . \qquad (7d)$$

Here $y_0 = A_1$ and $y_k = M$

The dimensions of the sets Δ^k and $\bar{\Delta}^k$ are the same. The above result follows from the relation

$$D[\bar{\Delta}^k(A_1, A_2, \ldots, A_k; M] = \sum D[\bar{\Delta}^{k-1}(A_1, A_2, \ldots, A_{k-1}; M-m)] \times D[\Delta^1(A_k, m)], \qquad (8)$$

where by $D[R]$ we denote the dimension of the set R. The proof of (8) is rather easy by induction for k.

It is obvious from (3b-d) and (10) that the representation of the algebra sp(2k-2) x A_k^k : $(\Omega_1^{k-1}, \Omega_2^{k-1}, \ldots, \Omega_{k-1}^{k-1}) \times h_k$ enters $D[\Delta^k(A_1, A_2, \ldots, A_k; 1/2\, h_k)]$ times into the representation $(\Omega_1^k, \Omega_2^k, \ldots, \Omega_k^k)$, where

$$A_i = \frac{1}{2}(B_i - C_i) \qquad \text{for } i=1,\ldots,k \tag{9a}$$

$$B_k = \Omega_1^k \qquad C_1 = 0 \tag{9b,c}$$

$$B_p = \mathrm{Min}(\Omega_{k+1-p}^k, \Omega_{k-p}^{k-1}) \qquad \text{for } p=1,\ldots,k-1 \tag{9d}$$

$$C_p = \mathrm{Max}(\Omega_{k+2-p}^k, \Omega_{k+1-p}^{k-1}) \qquad \text{for } p=2,3,\ldots,k \tag{9e}$$

$$X_p = -\bar{\Gamma}_{k+1-p}^k + \frac{1}{2}(B_p + C_p). \tag{10}$$

Now if we assume a simple relation between the missing label numbers occuring in the pattern (4a) and coordinates

$$y_p = \frac{1}{2}\{B_1 + \sum (B_p + C_p - 2\Gamma_{k-p+1}^k)\} \tag{11}$$

for a = 1,2,...,k-1 we immediately obtained from (7a,b,c) the system of inequalities (4c-e) and because **σ is equal to** $2y_k$ we get also (4f) from (7d) what was to show.

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Правила ветвления для приведения алгебры sp(2N) на цепочке sp(2N-2) x sp(2)

Найдены правила ветвления для приведения sp(2N) ↘ sp(2N-2)× ×sp(2) и предлагается новая схема для обозначения векторов, принадлежащих базису унитарного неприводимого представления /БУНТ/.

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Branching Rules for sp(2N) Algebra Reduction on the Chain sp(2N-2) x sp(2)

We find branching rules for reduction sp(2N) ↘ sp(2N-2)× ×sp(2), and we propose a new pattern for labeling vectors belonging to the base for unitary irreducible representation (BUIR).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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