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A NEW CLASS OF REALIZATIONS OF THE LIE ALGEBRA $u(q,n+1-q)$

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1. Introduction

1.1 The paper is continuation of the work done in Refs. 1-4. In Ref. 1 the method of constructing realizations (boson representation) for an arbitrary real semisimple algebra g was presented. This method gives the skew-Hermitean and Schurean realizations starting from a decompositions $g=n_+^b \oplus g_0^b \oplus n_-^b$, which is a simple generalisation of the triangle decomposition. In the papers [2-4] we have applied this method for the construction realizations of real algebras $gl(n+1,\mathbb{R})$, $sp(n,\mathbb{R})$ and $so(q,2n-q)$.

1.2 In the paper [5] (see also Ref. 6), extensive families of realizations for the real algebras $u(m,n)$ were constructed. The method is based on the recurrent formulae which give realizations of $u(m,n)$ in terms of $(2(m+n)-1)$ - boson pairs and generators $u(m-1,n-1)$. This realizations are not expressed as polynomials in canonical pairs q_i, p_i but the Weyl algebra W_{2N} is extended to the localisation (for details see Ref.5).

1.3 In the present paper, we apply the method of Ref. 1 to the case of real algebras $u(q,n+1-q)$, which are the real forms of the complex algebras $gl(n+1,\mathbb{C})$. We construct recurrent formulae which give realizations of $u(q,n+1-q)$ in terms of $(2n-1)$ - boson pairs and generators of the subalgebra $gl(1,\mathbb{C})^{\mathbb{R}} \oplus u(q-1,n-q)$. Here we use the explicit forms of the triangle decomposition of these algebras which we have constructed in previous paper [7]. The resulting realizations are Schurean and skew-Hermitean.

1.4 The article is organised as follows. Section 2 contains notations, definitions and some conventions. The results of the paper are found in Sec. 3. There are derived the new wide families of realizations. Section 4 contains a discussion of the results.

2. Basic notions

2.1 The standard basic of $gl(n+1,\mathbb{C})$ is given by the n^2 elements E_{ij} which are in their n x n - matrix representation, matrices with the matrix elements $(E_{ij})_{kl} = \delta_{ik}\delta_{jl}$. The commutation relations have the form

$$[E_{ij},E_{kl}] = \delta_{jk} E_{il} - \delta_{li} E_{kj}, \quad i,j,k,l=1,2,\dots,n+1. \tag{1}$$

2.2 In our article /7/ we have specified explicit form of the automorphisms which give the real forms of this algebra. Using these automorphisms we obtain for the algebras $u(q,n+1-q)$ the following bases:

$$\begin{aligned}
&X_{st}=(E_{st} - E_{q+t,q+s}),\ Y_{st}=i(E_{st} + E_{q+t,q+s}),\\
&X_{s,q+t}=(E_{s,q+t} - E_{t,q+s}),\ Y_{s,q+t}=i(E_{s,q+t} + E_{t,q+s}),\\
&X_{q+s,t}=(E_{q+s,t} - E_{q+t,s}),\ Y_{q+s,t}=i(E_{q+t,s} + E_{q+s,t}),\\
&X_{s,\alpha}=(E_{s,\alpha} + E_{\alpha,q+s}),\ Y_{s,\alpha}=i(E_{s,\alpha} - E_{\alpha,q+s}),\\
&X_{q+s,\alpha}=(E_{q+s,\alpha} + E_{\alpha,s}),\ Y_{q+s,\alpha}=i(E_{q+s,\alpha} - E_{\alpha,s}),\\
&X_{\alpha\beta}=(E_{\alpha\beta} - E_{\beta\alpha}),\ Y_{\alpha\beta} = i(E_{\alpha\beta} + E_{\beta\alpha}),
\end{aligned} \tag{2}$$

where $s,t=1,2,\dots,q$ and $\alpha,\beta = 2q+1,2q+2,\dots,n+1$. The commutation relation in these bases we bring in Appendix A.

2.3 The Weyl algebra W_{2N} is the associative algebra over $\mathbb{C}$ with identity generated by 2N elements q_i and p_i, $i=1,2,\dots,N$, which satisfy the relations

$$p_i\, q_j - q_j\, p_i = \delta_{ij}\, 1, \quad i,j = 1,2,\dots,N. \tag{3}$$

According to the Poincaré - Birkhoff theorem, a basis in W_{2N} is given by the ordered monomials

$$q_1^{k_1}\dots q_N^{k_N}\, p_1^{l_1}\dots p_N^{l_N} \tag{4}$$

2.4 Let g, g_o are real Lie algebras. By $\tilde{g}$, $\tilde{g}_o$ we denote their complexifications, furthermore, $U(\tilde{g})$, $U(\tilde{g}_o)$ are the enveloping algebras of these complexifications.

Definition: A realization of a Lie algebra g in $W_{2N}\otimes U(\tilde{g}_o)$ is a homomorphism

$$g \longrightarrow W_{2N}\otimes U(\tilde{g}_o). \tag{5}$$

We shall assume this homomorphism in all cases to be already uniquely extended to a homomorphism of the enveloping algebra $U(\tilde{g})$ of g into $W_{2N}\otimes U(\tilde{g}_o)$.

2.5 Definition: The realization τ is called Shur-realization if every central element C of the enveloping algebra $U(\tilde{g})$ is realised by $1\otimes C_o$ where C_o is central element of the enveloping algebra $U(\tilde{g}_o)$.

2.6 For possible applications to representation theory we introduce in $W_{2N}\otimes U(\tilde{g}_o)$ the involution "+" through the following relations

$$\begin{aligned} q_i^+ &= -q_i\,,\\ p_i^+ &= p_i\,, \end{aligned} \tag{6a}$$

and

$$Y^+ = -Y \qquad \text{for } Y\in g_o. \tag{6b}$$

Definition: Let g be a real Lie algebra and let "+" be the involution on $W_{2N}\otimes U(\tilde{g}_o)$ defined above. A realization τ of g on $W_{2N}\otimes U(\tilde{g}_o)$ is called skew-Hermitean, if

$$(\tau(X))^+ = -\tau(X) \tag{7}$$

holds for all $X\in g$.

2.7 For $b=E_{11}+E_{q+1,q+1}$ we define a decomposition of algebra $u(q,2n-q)$ in this way:

$$\begin{aligned}
g &= n_+^b \oplus g_o^b \oplus n_-^b\\
n_+^b &= \mathbb{R}\{X\in g; [b,X] = \alpha_X X \quad \text{where} \quad \alpha_X > 0\}\\
g_o^b &= \mathbb{R}\{X\in g; [b,X] = 0\}\\
n_-^b &= \mathbb{R}\{X\in g; [b,X] = -\alpha_X X \quad \text{where} \quad \alpha_X>0\}.
\end{aligned} \tag{8}$$

This decompositions we use as a starting point for our construction (see also Ref. 4 Sec. 4).

3. Construction of realizations

3.1 Using the commutation relations in algebra $u(q,n+1-q)$ (see Appendix A) we can bring the decomposition (8) into the form

$$\begin{aligned} n_+^b &= R\{X_{1i},\ Y_{1i},\ X_{1,q+i},\ Y_{1,q+i},\ X_{1\alpha},\ Y_{1\alpha},\ Y_{1,q+1}\}, \\ g_o^b &= R\{X_{ij},\ Y_{ij},\ X_{i,q+j},\ Y_{i,q+j},\ X_{q+i,j},\ Y_{q+i,j}, \\ &\qquad X_{i\alpha},\ Y_{i\alpha},\ X_{q+i,\alpha},\ Y_{q+i,\alpha},\ X_{\alpha\beta},\ Y_{\alpha\beta},\ X_{11},\ Y_{11}\}, \qquad (9) \\ n_-^b &= R\{X_{i1},\ Y_{i1},\ X_{q+i,1},\ Y_{q+i,1},\ X_{q+1,\alpha},\ Y_{q+1,\alpha},\ Y_{q+1,1}\}, \end{aligned}$$

where $i,j=2,3,\ldots,q$ and $\alpha,\beta = 2q+1,2q+2,\ldots,n+1$.

Evidently, the set $\{X_{1i},Y_{1i},X_{1,q+i},Y_{1,q+i},X_{1\alpha},Y_{1\alpha},Y_{1,q+1};\ i=2,3,\ldots,q;\ \alpha,\beta = 2q+1,\ 2q+2,\ldots,n+1\}$ is a basis in $\tilde{n}_+^b$. We write the element of this basis as the matrix

$$\left|\begin{matrix} X_{12},\ X_{13},\ \ldots,\ X_{1q} \\ X_{1,q+2},X_{1,q+3},\ldots,X_{1,n+1} \\ Y_{12},\ Y_{13},\ \ldots,\ Y_{1q} \\ Y_{1,q+1},Y_{1,q+2},\ldots,Y_{1,n+1} \end{matrix}\right| . \qquad (10)$$

We introduce an ordering in the above basis in which its elements are ordered lexicographically. The monomials of $U(\tilde{n}_+^b)$ can be then written as the matrices

$$\left|\begin{matrix} n_2^X,\ \ldots,\ n_q^X \\ n_{q+2}^X,\ldots,n_{n+1}^X \\ n_2^Y,\ \ldots,\ n_q^Y \\ n_{q+1}^Y,\ldots,n_{n+1}^Y \end{matrix}\right| = \left(X_{12}^{n_2}\ldots X_{1q}^{n_q}\right)\left(X_{1,q+2}^{n_{q+2}}\ \ldots\ X_{1,n+1}^{n_{n+1}}\right) \times \left(Y_{12}^{n_2}\ldots Y_{1q}^{n_q}\right)\left(Y_{1,q+1}^{n_{q+1}}\ \ldots\ Y_{1,n+1}^{n_{n+1}}\right), \qquad (11)$$

where of course n_k^X, n_k^Y belongs to N_o, the set of all non-negative integers.

3.2 Now we are able to apply the general construction described in Ref.1. Let σ be an auxiliary representation of the algebra $g_o^b \oplus n_-^b$ on a vector space V such that

$$\sigma(n_-^b) = 0$$
$$\sigma|g_o^b \text{ is faithful.} \qquad (12)$$

We denote by W the carrier space of the induced representation $\rho = \mathrm{ind}(g,\sigma)$. If $\{v_1,\ldots,v_d\}$ is a basis in the space V, then the vectors

$$\left|\begin{matrix} n_2^X,\ \ldots,\ n_q^X \\ n_{q+2}^X,\ldots,n_{n+1}^X \\ n_2^Y,\ \ldots,\ n_q^Y \\ n_{q+1}^Y,\ldots,n_{n+1}^Y \end{matrix}\right| \otimes v_i \qquad (13)$$

form a basis in W.

3.3 We define the creation and annihilation operators $\bar{a}_k^Y$, a_k^Y on the space W in the following way:

$$\bar{a}_k^Y \left|\begin{matrix} n_2^X, & \ldots, & n_q^X \\ n_{q+2}^X, & \ldots, & n_{n+1}^X \\ n_2^Y,\ldots,n_k^Y,\ldots,n_q^Y \\ n_{q+1}^Y, & \ldots, & n_{n+1}^Y \end{matrix}\right| \otimes v_i = \left|\begin{matrix} n_2^X, & \ldots, & n_q^X \\ n_{q+2}^X, & \ldots, & n_{n+1}^X \\ n_2^Y,\ldots,n_k^Y+1,\ldots,n_q^Y \\ n_{q+1}^Y, & \ldots, & n_{n+1}^Y \end{matrix}\right| \otimes v_i$$

$$a_k^Y \left|\begin{matrix} n_2^X, & \ldots, & n_q^X \\ n_{q+2}^X, & \ldots, & n_{n+1}^X \\ n_2^Y,\ldots,n_k^Y,\ldots,n_q^Y \\ n_{q+1}^Y, & \ldots, & n_{n+1}^Y \end{matrix}\right| \otimes v_i = n_k^Y \left|\begin{matrix} n_2^X, & \ldots, & n_q^X \\ n_{q+2}^X, & \ldots, & n_{n+1}^X \\ n_2^Y,\ldots,n_k^Y-1,\ldots,n_q^Y \\ n_{q+1}^Y, & \ldots, & n_{n+1}^Y \end{matrix}\right| \otimes v_i \qquad (14a)$$

and similarly we define the operators $\bar{a}_k^X$, a_k^X. Furthermore we define the operators $\tilde{X}$ for X $gl(1,C)^R \oplus u(q-1,n+1-q)$ by the relation

$$\tilde{X} = 1 \otimes \sigma(X). \qquad (14b)$$

3.4 According to theorem 3.6 of Ref.1, the induced representation $\rho = \mathrm{ind}(g,\sigma)$ can be rewritten using the above defined operators (14a-b) and further the sought skew-Hermitean realizations are obtained easily by replacing the operators by suitable algebraic objects. For details, (see Ref. 1., sections 3.7-3.9).

$$\bar{a} \to q$$
$$a \to p$$
$$\breve{X} \to X \; . \qquad (15)$$

We get the formulae

$$\tau(X_{ij}) = -q^X_j p^X_i - q^Y_j p^Y_i + q^X_{q+i} p^X_{q+j} + q^Y_{q+i} p^Y_{q+j} + X_{ij}$$

$$\tau(Y_{ij}) = -q^Y_j p^X_i + q^X_j p^Y_i + q^Y_{q+i} p^X_{q+j} - q^X_{q+i} p^Y_{q+j} + Y_{ij}$$

$$\tau(X_{i,q+j}) = q^X_{q+i} p^X_j - q^X_{q+j} p^X_i + q^Y_{q+1} p^Y_j p^X_i - q^Y_i p^X_j +$$
$$+ q^Y_{q+i} p^Y_j - q^Y_{q+j} p^Y_i + X_{i,q+j}$$

$$\tau(Y_{i,q+j}) = -q^Y_{q+i} p^X_j - q^Y_{q+j} p^X_i + q^Y_{q+1} p^X_j p^X_i +$$
$$+ q^X_{q+i} p^Y_j + q^X_{q+j} p^Y_i - q^Y_{q+1} p^Y_j p^Y_i + Y_{i,q+j}$$

$$\tau(X_{q+i,j}) = -q^X_j p^X_{q+i} + q^X_i p^X_{q+j} - q^Y_j p^Y_{q+i} + q^Y_i p^Y_{q+j} +$$
$$+ q^Y_{q+1} p^Y_{q+1} p^X_{q+i} - q^Y_{q+i} p^X_{q+j} + X_{q+i,j}$$

$$\tau(Y_{q+i,j}) = -q^Y_j p^X_{q+i} - q^Y_i p^X_{q+j} + q^X_j p^Y_{q+i} + q^X_i p^Y_{q+j} +$$
$$+ q^Y_{q+1} p^X_{q+j} p^X_{q+i} + q^Y_{q+j} p^Y_{q+i} + Y_{q+i,j}$$

$$\tau(X_{i\alpha}) = -q^X_\alpha p^X_i - (q^Y_\alpha + q^Y_{q+1} p^X_\alpha) p^Y_i - q^X_{q+i} p^X_\alpha - q^Y_{q+i} p^Y_\alpha + X_{i\alpha}$$

$$\tau(Y_{i\alpha}) = -(q^Y_\alpha + q^Y_{q+1} p^X_\alpha) p^X_i + q^X_\alpha p^Y_i + q^Y_{q+i} p^X_\alpha - q^X_{q+i} p^Y_\alpha + Y_{i\alpha}$$

$$\tau(X_{q+i,\alpha}) = -q^X_\alpha p^X_{q+i} - (q^Y_\alpha + q^Y_{q+1} p^X_\alpha) p^Y_{q+i} - q^X_i p^X_\alpha - q^Y_i p^Y_\alpha + X_{q+i,\alpha}$$

$$\tau(Y_{q+i,\alpha}) = -(q^Y_\alpha + q^Y_{q+1} p^X_\alpha) p^X_{q+i} + q^X_\alpha p^Y_{q+i} + q^Y_i p^X_\alpha - q^X_i p^Y_\alpha + Y_{q+i,\alpha}$$

$$\tau(X_{\alpha\beta}) = -q^X_\beta p^X_\alpha + q^X_\alpha p^X_\beta - q^Y_\beta p^Y_\alpha + q^Y_\alpha p^Y_\beta + X_{\alpha\beta}$$

$$\tau(Y_{\alpha\beta}) = -q^Y_\beta p^X_\alpha - q^Y_\alpha p^X_\beta - q^Y_{q+1} p^X_\beta p^X_\alpha + q^X_\beta p^Y_\alpha +$$
$$+ q^X_\alpha p^Y_\beta + q^Y_{q+1} p^Y_\beta p^Y_\alpha + Y_{\alpha\beta}$$

$$\tau(X_{11}) = \sum_{k=2}^{q} (q^X_k p^X_k + q^Y_k p^Y_k + q^X_{q+k} p^X_{q+k} + q^Y_{q+k} p^Y_{q+k}) +$$
$$+ \sum_{\alpha=2q+1}^{n+1} (q^X_\alpha p^X_\alpha + q^Y_\alpha p^Y_\alpha) + 2 q^Y_{q+1} p^Y_{q+1} + X_{11}$$

$$\tau(Y_{11}) = \sum_{k=2}^{q} (q^Y_k p^X_k - q^X_k p^Y_k + q^Y_{q+k} p^X_{q+k} - q^X_{q+k} p^Y_{q+k}) +$$
$$+ \sum_{\alpha=2q+1}^{n+1} \frac{1}{2} q^Y_{q+1} (p^Y_\alpha p^Y_\alpha + p^X_\alpha p^X_\alpha) - p^X_\alpha p^Y_\alpha + p^Y_\alpha p^X_\alpha) + Y_{11}$$

$$\tau(X_{12}) = q_{12}$$

$$\tau(X_{21}) = \sum_{k=2}^{q} (\tau(X_{2k}) + q^X_k p^X_{12}) p^X_k + (\tau(Y_{2k}) + q^Y_k p^X_{12} - q^X_k p^Y_{12}) p^Y_k +$$
$$+ \sum_{k=2}^{q} (X_{2q+k} p^X_{q+k} + Y_{2,q+k} p^Y_{q+k}) - \tau(X_{11}) p^X_{12} - \tau(Y_{11}) p^Y_{12} +$$
$$+ \sum_{\alpha=2q+1}^{n+1} \left(- \frac{q^X_{q+2}(p^X_\alpha + p^Y_\alpha)}{2} + X_{2\alpha} p^X_\alpha + Y_{2\alpha} p^Y_\alpha \right) + 2 q^Y_{q+2} p^Y_{q+1} \; .$$

4. Discussion

In the present paper, we construct, using the general method of Ref. 1, the explicit forms of boson realisations for algebras u(q,n+1-q), which are the real forms of the complex algebras gl(n+1,ℂ). These realizations are defined by means of (2n-1) - boson pairs and generators of the subalgebra $gl(1,\mathbb{C})^R \oplus u(q-1,n-q)$. Another class of realisations has been described by Havlíček and Lassner /5/, but these are not expressed as polynomials. Our construction proves that in the case of algebras u(q,n+1-q) we can construct the polynomial realisations too.

Appendix A:

Using the relations (1) we can compute commutation relations in the basis (2). In this appendix we give their explicit form:

$$[X_{ij},X_{kl}] = \delta_{jk}X_{il} - \delta_{il}X_{kj} \qquad [X_{ij},X_{k\alpha}] = \delta_{jk}X_{i\alpha}$$
$$[X_{ij},Y_{kl}] = \delta_{jk}Y_{il} - \delta_{il}Y_{kj} \qquad [X_{ij},Y_{k\alpha}] = \delta_{jk}Y_{i\alpha}$$
$$[Y_{ij},Y_{kl}] = -\delta_{jk}X_{il} + \delta_{il}X_{kj} \qquad [Y_{ij},Y_{k\alpha}] = -\delta_{jk}X_{i\alpha}$$
$$[Y_{ij},X_{k\alpha}] = \delta_{jk}Y_{i\alpha}$$

$$[X_{ij},X_{k,q+l}] = \delta_{jk}X_{i,q+l} - \delta_{jl}X_{i,q+k} \qquad [X_{ij},X_{q+k,\alpha}] = -\delta_{ik}X_{q+j,\alpha}$$
$$[X_{ij},Y_{k,q+l}] = \delta_{jk}Y_{i,q+l} + \delta_{jl}Y_{i,q+k} \qquad [X_{ij},X_{q+k,\alpha}] = -\delta_{ik}Y_{q+j,\alpha}$$
$$[Y_{ij},Y_{k,q+l}] = -\delta_{jk}X_{i,q+l} - \delta_{jl}X_{i,q+k} \qquad [Y_{ij},Y_{q+k,\alpha}] = -\delta_{ik}X_{q+j,\alpha}$$
$$[Y_{ij},X_{k,q+l}] = \delta_{jk}Y_{i,q+l} - \delta_{jl}Y_{i,q+k} \qquad [Y_{ij},X_{q+k,\alpha}] = +\delta_{ik}Y_{q+j,\alpha}$$

$$[X_{ij},X_{q+k,l}] = -\delta_{il}X_{q+k,j} + \delta_{ik}X_{q+l,j} \qquad [X_{ij},X_{\alpha\beta}] = 0$$
$$[X_{ij},Y_{q+k,l}] = -\delta_{il}Y_{q+k,j} - \delta_{ik}Y_{q+l,j} \qquad [X_{ij},Y_{\alpha\beta}] = 0$$
$$[Y_{ij},Y_{q+k,l}] = +\delta_{il}X_{q+k,j} + \delta_{ik}X_{q+l,j} \qquad [Y_{ij},Y_{\alpha\beta}] = 0$$
$$[Y_{ij},X_{q+k,l}] = -\delta_{il}Y_{q+k,j} + \delta_{ik}Y_{q+l,j} \qquad [Y_{ij},X_{\alpha\beta}] = 0$$

$$[X_{i,q+j},X_{k,q+l}] = 0 \qquad [X_{i,q+j},X_{k\alpha}] = 0$$
$$[X_{i,q+j},Y_{k,q+l}] = 0 \qquad [X_{i,q+j},Y_{k\alpha}] = 0$$
$$[Y_{i,q+j},Y_{k,q+l}] = 0 \qquad [Y_{i,q+j},Y_{k\alpha}] = 0$$
$$[Y_{i,q+j},X_{k\alpha}] = 0$$

$$[X_{i,q+j},X_{q+k,l}] = \delta_{jk}X_{il} - \delta_{ik}X_{jl} + \delta_{il}X_{jk} - \delta_{jl}X_{ik} \qquad [Y_{i,q+j},Y_{q+k,l}] = -\delta_{jk}X_{il} - \delta_{ik}X_{jl} - \delta_{jl}X_{ik} - \delta_{il}X_{jk}$$

$$[X_{i,q+j},Y_{q+k,l}] = \delta_{jk}Y_{il} - \delta_{ik}Y_{jl} - \delta_{il}Y_{jk} + \delta_{jl}Y_{ik} \qquad [Y_{i,q+j},X_{q+k,l}] = \delta_{jk}Y_{il} - \delta_{ik}Y_{jl} - \delta_{jl}Y_{ik} + \delta_{il}Y_{jk}$$

$$[X_{i,q+j},X_{q+k,\alpha}] = \delta_{jk}X_{i\alpha} - \delta_{ik}X_{j\alpha} \qquad [X_{i,q+j},X_{\alpha\beta}] = 0$$

$$[X_{i,q+j},Y_{q+k,\alpha}] = \delta_{jk}Y_{i\alpha} - \delta_{ik}Y_{j\alpha} \qquad [X_{i,q+j},Y_{\alpha\beta}] = 0$$
$$[Y_{i,q+j},Y_{q+k,\alpha}] = -\delta_{jk}X_{i\alpha} - \delta_{ik}X_{j\alpha} \qquad [Y_{i,q+j},Y_{\alpha\beta}] = 0$$
$$[Y_{i,q+j},X_{q+k,\alpha}] = \delta_{jk}Y_{i\alpha} - \delta_{ik}Y_{j\alpha} \qquad [Y_{i,q+j},X_{\alpha\beta}] = 0$$

$$[X_{q+i,j},X_{q+k,l}] = 0 \qquad [X_{q+i,j},X_{k\alpha}] = \delta_{jk}X_{q+i,\alpha} - \delta_{ik}X_{q+j,\alpha}$$
$$[X_{q+i,j},Y_{q+k,l}] = 0 \qquad [X_{q+i,j},Y_{k\alpha}] = \delta_{jk}Y_{q+i,\alpha} - \delta_{ik}Y_{q+j,\alpha}$$
$$[Y_{q+i,j},Y_{q+k,l}] = 0 \qquad [Y_{q+i,j},Y_{k\alpha}] = -\delta_{jk}X_{q+i,\alpha} - \delta_{ik}X_{q+j,\alpha}$$
$$[Y_{q+i,j},X_{k\alpha}] = +\delta_{jk}Y_{q+i,\alpha} + \delta_{ik}Y_{q+j,\alpha}$$

$$[X_{q+i,j},X_{q+k,\alpha}] = 0 \qquad [X_{q+i,j},X_{\alpha\beta}] = 0 \qquad [Y_{q+i,j},X_{q+k,\alpha}] = 0$$
$$[X_{q+i,j},Y_{q+k,\alpha}] = 0 \qquad [X_{q+i,j},Y_{\alpha\beta}] = 0 \qquad [Y_{q+i,j},X_{\alpha\beta}] = 0$$
$$[Y_{q+i,j},Y_{q+k,\alpha}] = 0 \qquad [Y_{q+i,j},Y_{\alpha\beta}] = 0$$

$$[X_{i\alpha},X_{j\beta}] = \delta_{\alpha\beta}X_{i,q+j} \qquad [X_{i\alpha},X_{q+j,\beta}] = \delta_{ij}X_{\alpha\beta} + \delta_{\alpha\beta}X_{ij}$$
$$[X_{i\alpha},Y_{j\beta}] = -\delta_{\alpha\beta}Y_{i,q+j} \qquad [X_{i\alpha},Y_{q+j,\beta}] = \delta_{ij}Y_{\alpha\beta} - \delta_{\alpha\beta}Y_{ij}$$
$$[Y_{i\alpha},Y_{j\beta}] = \delta_{\alpha\beta}X_{i,q+j} \qquad [Y_{i\alpha},Y_{q+j,\beta}] = \delta_{ij}X_{\alpha\beta} + \delta_{\alpha\beta}X_{ij}$$
$$[Y_{i\alpha},X_{q+j,\beta}] = -\delta_{ij}Y_{\alpha\beta} - \delta_{\alpha\beta}Y_{ij}$$

$$[X_{i\alpha},X_{\beta\gamma}] = \delta_{\alpha\beta}X_{i\gamma} - \delta_{\gamma\alpha}X_{i\beta} \qquad [Y_{i\alpha},Y_{\beta\gamma}] = -\delta_{\alpha\beta}X_{i\gamma} - \delta_{\alpha\gamma}X_{i\beta}$$
$$[X_{i\alpha},Y_{\beta\gamma}] = \delta_{\alpha\beta}Y_{i\gamma} + \delta_{\alpha\gamma}Y_{i\beta} \qquad [Y_{i\alpha},X_{\beta\gamma}] = \delta_{\alpha\beta}Y_{i\gamma} - \delta_{\alpha\gamma}Y_{i\beta}$$
$$[X_{q+i,\alpha},X_{q+j,\beta}] = \delta_{\alpha\beta}X_{q+i,j} \qquad [X_{q+i,\alpha},X_{\beta\gamma}] = \delta_{\alpha\beta}X_{q+i,\gamma} - \delta_{\alpha\gamma}X_{q+i,\beta}$$
$$[X_{q+i,\alpha},Y_{q+j,\beta}] = -\delta_{\alpha\beta}Y_{q+i,j} \qquad [X_{q+i,\alpha},Y_{\beta\gamma}] = \delta_{\alpha\beta}Y_{q+i,\gamma} + \delta_{\alpha\gamma}Y_{q+i,\beta}$$
$$[Y_{q+i,\alpha},Y_{q+j,\beta}] = \delta_{\alpha\beta}X_{q+i,j} \qquad [Y_{q+i,\alpha},Y_{\beta\gamma}] = -\delta_{\alpha\beta}X_{q+i,\gamma} - \delta_{\alpha\gamma}X_{q+i,\beta}$$
$$[Y_{q+i,\alpha},X_{\beta\gamma}] = \delta_{\alpha\beta}Y_{q+i,\gamma} - \delta_{\alpha\gamma}Y_{q+i,\beta}$$

$$[X_{\alpha\beta},X_{\gamma\delta}] = \delta_{\beta\gamma}X_{\alpha\delta} - \delta_{\alpha\delta}X_{\gamma\beta} \qquad Y_{\alpha\beta},Y_{\gamma\delta} = -\delta_{\beta\gamma}X_{\alpha\delta} + \delta_{\alpha\delta}X_{\gamma\beta}$$
$$[X_{\alpha\beta},Y_{\gamma\delta}] = \delta_{\beta\gamma}Y_{\alpha\delta} - \delta_{\alpha\delta}Y_{\gamma\beta}$$

where $i,j,k,l=1,2,\ldots,q$ and $\alpha,\beta,\gamma,\delta = 2q+1,2q+2,\ldots,n+1$.

References

1. Burdík Č., J.Phys.A: Math.Gen. 1985, v.18, p.3101.
2. Burdík Č., J.Phys.A. Math.Gen. 1986, v.19 (in print).
3. Burdík Č., Czech.J.Phys.B. (in print).
4. Burdík Č., A New Class of Realizations of the Lie Algebra so(q,2n-q), JINR preprint, E5-86-662, Dubna, 1986.
5. Havlíček M., Lassner W., Rep.Math.Phys., 1977, v.12, p.1.
6. Exner P., Havlíček M and Lassner W., Czech.J.Phys., 1976, v.B26, p.1213.
7. Burdík Č., Czech.J.Phys.B (in print).

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SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
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5.	Mathematics
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7.	Heavy ion physics
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9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Бурдик Ч.

E5-86-772

Новый класс реализаций алгебры $u(q, n+1-q)$

В данной работе применяется метод построения бозонных реализаций алгебр $u(q, n+1-q)$, приведенный в /1/. Эти реализации описываются рекуррентными формулами, содержащими $(2n-1)$ бозонных пар и генераторов подалгебры $gl(1,C)^R \oplus u(q-1,n-q)$. Они антиэрмитовы и шуровские.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1986

Burdík Č.

E5-86-772

A New Class of Realizations of the Lie Algebra $u(q,n+1-q)$

The method of Ref. 1 is applied to the construction of boson realizations for Lie algebras $u(q,n+1-q)$, $q = 2,3,\dots, n$. These realizations are expressed by means by certain recurrent formulae in terms of $(2n-1)$ — boson pairs and generators of the subalgebra $gl(1,C)^R \oplus u(q-1, n-q)$. They are skew-Hermitean and Shurean.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986