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AN ABELIAN AND TAUBERIAN THEOREM FOR THE STIELTJES TRANSFORM OF GENERALIZED FUNCTIONS

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1. Introduction

To nominate sum rules for Green functions, for instance, for Bose fields on the basis of dispersion relations

$$\Delta_F(q) = \int_0^\infty \frac{\rho(\sigma^2)\, d\sigma^2}{q^2 - \sigma^2 + i\varepsilon}$$

the explanation of correct asymptotic relations between the behaviour of $\rho(\sigma^2)$ for $\sigma^2 \to \infty$ and the behaviour of Δ for $-q^2 \to \infty$ is needed in the distributional case. Mathematically this problem consists in studying Abelian and Tauberian theorems for the Stieltjes transform of generalized functions.

The Stieltjes transform /12/ of ordinary functions $f(t)$ with $(1+t)^{-\rho} f(t) \in \mathcal{L}(0,\infty)$, $\rho \in R$, is defined by

$$\mathcal{S}_{[f]}(z) = \mathcal{S}_{[f];\rho}(z) = \int_0^\infty \frac{f(t)}{(z+t)^\rho}\, dt.$$

Here we consider only the case $\rho > 0$. This has been extended to generalized functions in different ways. The resulting theories differ in the test function spaces which they employ and consequently differ in the classes of generalized functions for which they define Stieltjes transform.

Abelian theorems are called theorems which connect the asymptotic behaviour of a function or a generalized function in the neighbourhood of infinity or of zero with the asymptotic behaviour of its Fourier transform or other integral transform at infinity or at zero or at some other point. Theorems inverse to Abelian are called Tauberian ones.

Karamata /4/ derives such theorems for the Stieltjes transform of ordinary functions. This theory has been developed further by Stadtmüller /10/ and other peoples. For Stieltjes transform of generalized functions in the sense given by Benedetto /1/, Pandey /8/ and Pathak /9/ Abelian theorems were described by Lavoine and Misra /6,7/, Carmichael and Milton /2/ and Takači /12/. A Tauberian theorem in the case $\rho = 1$ was given by Krasnikov and Tshetyrkin /5/ where the Tauberian condition consists in the assumption that $f(t)$ is a non-negative measure. Another approach for non-negative measures was investigated by Pilipovič and Stancovič /11/.

In the present note we describe an Abelian and Tauberian theorem for the Stieltjes transform of generalized functions in the approach given by Erdélyi /3/. The Tauberian condition is more general than in the case /5/ and /11/.

2. Asymptotical behaviour of distributions

Before we can discuss Abelian and Tauberian theorems, we need a suitable definition for the asymptotic behaviour of generalized functions. A very useful method to describe the asymptotic behaviour of a distribution is the so-called quasiasymptotics /13/ given by Zavialov.

Definition 2.1:
Let $f(t) \in S'_+$ and $\gamma(k)$ be a regular varying function. We say that the distribution $f(t)$ has a quasiasymptotics at infinity with respect to $\gamma(k)$ if there exists the limit

$$\lim_{k \to \infty} \frac{f(kt)}{\gamma(k)} = F(t) \neq 0 \quad \text{in} \quad S'_+ .$$

The positive and continuous on $R_+ = (0,\infty)$ function $\gamma(k)$ is regular varying if for any $a > 0$ there exists the limit

$$\lim_{k \to \infty} \frac{\gamma(ak)}{\gamma(k)} = C(a) \neq 0$$

and the convergence is uniform with respect to any compact set of numbers a in R_+.

It is not difficult to see that $C(a) = a^{\gamma}$ for some real γ, and then we call the function $\gamma(k)$ regular varying of order γ. In this case in definition 2.1 $F(t) = C\,\theta_{\gamma+1}(t)$ in S'_+ where $\theta_{\gamma}(t)$ is the canonical kernel of fractional differentiation

$$\theta_{\gamma}(t) = \begin{cases} \theta(t)\,\dfrac{t^{\gamma-1}}{\Gamma(\gamma)} & \gamma > 0 \\ \theta'_{\gamma+1}(t) & \gamma \leq 0 \end{cases}$$

$\theta(t)$ is the Heviside step function.

3. Stieltjes transform of generalized functions

The way presented by Erdélyi /3/ to generalize conventional Stieltjes transform to distributions which may be called the method of adjoints is based on the following idea. Erdélyi employs a test function space $M_{\alpha,\beta}$ which is mapped by the Stieltjes transform into another test function space $M_{a,b}$ continuously. The adjoint mapping then defines the Stieltjes transform $\mathcal{S}[f]$ of elements of the dual of $M_{a,b}$. The transform thus defined is no longer a numerical-valued function but a distribution, an element of $M'_{\alpha,\beta}$. For conventional functions with suitable integrability properties the double integral

$$\int_0^\infty \int_0^\infty \frac{f(x)\,\phi(t)}{(x+t)^{\varrho}}\, dx\, dt$$

can be evaluated in two different ways showing that $\langle \mathcal{S}[f], \phi \rangle = \langle f, \mathcal{S}[\phi] \rangle$ and this relation is the basis for application of the method of adjoints to Stieltjes transform.
For infinitely differentiable complex-valued functions $\phi(t)$ on R_+ and $a,b \in R$ Erdélyi defines

$$\mu_{a,b,k}(\phi) = \sup_{t \in R_+} t^{1-a+k}(1+t)^{a-b} \left| \phi^{(k)}(t) \right| . \tag{3.1}$$

The test function space $M_{a,b}$ is given by

$$M_{a,b} = \left\{ \phi \in C^{\infty}(R_+) : \mu_{a,b,k}(\phi) < \infty \quad \text{for all} \quad k \in Z_+ \right\}$$

with the topology generated by the seminorms (3.1). $M_{a,b}$ is a complete countable multinormed barreled space. Erdélyi has proved the following statement:

Theorem 3.1 /3/ :
The Stieltjes transform maps $M_{\alpha,\beta}$ continuously into $M_{a,b}$ if α,β,a,b satisfy the following conditions :

$$\alpha > 0 \ , \ \beta < \rho$$
$$a \leq 1 \ , \ a \leq 1+\alpha-\rho \ \text{ and } \ a < 1 \ \text{ if } \ \alpha = \rho \tag{3.2}$$
$$b \geq 1-\rho \ , \ b \geq 1+\beta-\rho \ \text{ and } \ b > 1-\rho \ \text{ if } \ \beta = 0 \ .$$

The proof follows from the estimate

$$\mu_{a,b,k}(\mathcal{S}_{[\phi]}) \leq C\, \mu_{\alpha,\beta,0}(\phi)$$

true under conditions (3.2) where C depends on k,a,b,α,β .
Now let $f \in M'_{a,b}$. For each $\phi \in M_{\alpha,\beta}$ we have $\mathcal{S}_{[\phi]} \in M_{a,b}$ so that

$$\langle \mathcal{S}_{[f]} \ , \phi \rangle = \langle f, \mathcal{S}_{[\phi]} \rangle \tag{3.3}$$

defines the Stieltjes transform $\mathcal{S}_{[f]} \in M'_{\alpha,\beta}$ of f .
In this approach the so-called real inversion theorem for the Stieltjes transform is also true in the distributional case. Define for $n \in Z_+$ and $x > 0$ the differential operator L_n by

$$L_n = L_{n,\rho,x} = \frac{(-1)^n \Gamma(\rho)}{n!\,\Gamma(n+\rho-1)} \left(\frac{d}{dx}\right)^n x^{2n+\rho-1} \left(\frac{d}{dx}\right)^n \ .$$

This operator commutes with the Stieltjes operator $\mathcal{S}$ as follows

$$x^{\rho-1}\, \mathcal{S}_{[L_t \phi(t)]}(x) = L_x\, \mathcal{S}_{[t^{\rho-1}\phi(t)]}(x) \ . \tag{3.4}$$

Erdélyi proved that for $\phi \in M_{\alpha,\beta}$ $\mathcal{S}_{[L_n\phi]} \longrightarrow \phi$ and $L_n\,\mathcal{S}_{[\phi]} \longrightarrow \phi$ as $n \longrightarrow \infty$ where the convergence takes place in the topology of the space $M_{\alpha,\beta}$ under additional conditions.
The exact result gives

Theorem 3.2 /3/ :
For $f \in M'_{a,b}$, $\phi \in M_{\alpha,\beta}$ yields

$$\langle L_n\, \mathcal{S}_{[f]} \ , \phi \rangle = \langle f, \mathcal{S}_{[L_n\phi]} \rangle \longrightarrow \langle f, \phi \rangle$$

with

$$\alpha > 1-\rho \ , \ \alpha \geq a \ \text{ and } \ \alpha > 1 \ \text{ if } \ a = 1 \ , \ b \geq 1-\rho \tag{3.5}$$
$$\beta < 1 \ , \ \beta \leq b \ \text{ and } \ \beta < 1-\rho \ \text{ if } \ b = 1-\rho \ .$$

A crucial point in the proof is the inequality

$$\mu_{\alpha,\beta,k}(\mathcal{S}_{[L_n\phi]} - \phi) \leq$$
$$\leq \varepsilon_n \left(\mu_{\alpha,\beta,k+1}(\phi) + \mu_{\alpha,\beta,k}(\phi)\right) \tag{3.6}$$

with $\varepsilon_n \longrightarrow 0$ if $n \longrightarrow \infty$.

Suppose now once for all fixed ε , $0 < \varepsilon < 1$, $\varepsilon > 1-\rho$ and fix
$\alpha = 1-\varepsilon$, $\beta = \rho - 1 + \varepsilon$
$a = 2-\rho-\varepsilon$, $b = \varepsilon$.
The set of parameters a,b,α,β thus chosed satisfy the conditions (3.2) and (3.5).
To get an Abelian and Tauberian theorem, we, besides, need the additional

Lemma 3.3 :
The set $A = \left\{ \mathcal{S}_{[\psi]} : \psi \in M_{\alpha,\beta} \right\}$ is dense in the space $M_{a,b}$.

Proof:
Consider $f \in M'_{\alpha,\beta}$ and suppose $\langle f, \mathcal{S}_{[\psi]} \rangle = 0$ for every $\psi \in M_{\alpha,\beta}$. If $\phi \in M_{a,b}$, then we have $t^{\rho-1}\phi(t) \in M_{\alpha,\beta}$ and $\langle f, \mathcal{S}_{[t^{\rho-1}\phi(t)]} \rangle = 0$. Because $L_n\, t^{1-\rho}\psi(t)$ $M_{\alpha,\beta}$, then for all n we have

$$\langle f, \mathcal{S}_{[L_n t^{1-\rho} t^{\rho-1}\phi(t)]} \rangle = \langle f, \mathcal{S}_{[L_n\phi]} \rangle = 0 \ .$$

By theorem 3.2 $\langle f, \mathcal{S}_{[L_n\phi]} \rangle$ converges to $\langle f, \phi \rangle$ if n runs to infinity so that $\langle f, \phi \rangle = 0$ for every $\phi \in M_{a,b}$ and $f = 0$ in $M'_{a,b}$. This means that the set A is dense in the space $M_{a,b}$.

4. The main theorem

Theorem 4.1 :

Let $f \in M'_{a,b}$ and $\gamma(k)$ be a regular varying function of order $\gamma > -1+\varepsilon$. The following statements are equivalent:

i) f has a quasiasymptotics at infinity with respect to $\gamma(k)$.

ii) $\mathcal{S}_{[f]}$ has a quasiasymptotics at infinity with respect to $k^{1-\rho}\gamma(k)$ and

$$\left\{ \frac{1}{k^{1-\rho}\gamma(k)} L_n \mathcal{S}_{[f]}(kx) \ : \ k \geqslant k_o \right\} \tag{4.1}$$

is bounded in $M'_{a,b}$ uniformly for $k \geqslant k_o$ independent of $n \in Z_+$.

Proof:

We have

$$\lim_{k\to\infty} \frac{1}{\gamma(k)} \langle f(kt), \phi(t)\rangle = \langle g(t), \phi(t)\rangle$$

for $\phi \in M_{a,b}$. If $\psi \in M_{\alpha,\beta}$ then $\mathcal{S}_{[\psi]} \in M_{a,b}$ so that

$$\langle g(t), \mathcal{S}_{[\psi]}(t)\rangle = \lim_{k\to\infty} \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[\psi]}(t)\rangle$$

$$= \lim_{k\to\infty} \frac{1}{k^{1-\rho}\gamma(k)} \langle \mathcal{S}_{[f]}(kx), \psi(x)\rangle$$

and by definition 3.3 yields

$$\lim_{k\to\infty} \frac{1}{k^{1-\rho}\gamma(k)} \langle \mathcal{S}_{[f]}(kx), \psi(x)\rangle = \langle \mathcal{S}_{[g]}(x), \psi(x)\rangle$$

for every $\psi \in M_{\alpha,\beta}$. This means that the quasiasymptotics of $\mathcal{S}_{[f]}$ at infinity with respect to $k^{1-\rho}\gamma(k)$ exists. Remark since $g(t) = C\,\theta_{\gamma+1}(t)$ and

$$\mathcal{S}_{[\theta_\gamma]}(x) = B(\gamma, \rho-\gamma)\,\theta_{\gamma+1-\rho}(x) \quad , \quad 0<\gamma<\rho \quad ,$$

where $B(i,j)$ is the usual Beta function so that

$$\mathcal{S}_{[g]}(x) = C'\,\theta_{\gamma+2-\rho}(x) \quad \text{in} \quad M'_{\alpha,\beta} \quad .$$

Property (4.1) follows from the existence of the quasiasymptotics and inequality (3.6) by careful estimates. Really,

$$\left| \frac{1}{k^{1-\rho}\gamma(k)} \langle L_n \mathcal{S}_{[f]}(kx), \phi(x)\rangle \right| = \left| \left\langle \frac{f(kt)}{\gamma(k)} , \mathcal{S}_{[L_n\phi]}(t)\right\rangle \right| .$$

Because of the existence of the quasiasymptotics the set

$$B = \left\{ \frac{f(kt)}{\gamma(k)} \ : \ k \geqslant k_o \right\}$$

is weakly bounded in $M'_{a,b}$. Since the space $M_{a,b}$ is barrelled, the set B is also strongly bounded in $M'_{a,b}$ so that

$$\left| \frac{1}{k^{1-\rho}\gamma(k)} \langle L_n \mathcal{S}_{[f]}(kx), \phi(x)\rangle \right| \leqslant \tag{4.2}$$

$$\leqslant C\,\mu_{a,b,p}(\mathcal{S}_{[L_n\phi]})$$

for some p . From inequality (3.6) we have

$$\mu_{a,b,p}(\mathcal{S}_{[L_n\phi]})$$

$$\leqslant \mu_{a,b,p}(\mathcal{S}_{[L_n\phi]} - \phi) + \mu_{a,b,p}(\phi)$$

$$\leqslant \varepsilon_p\{\mu_{a,b,p+1}(\phi) + p\,\mu_{a,b,p}(\phi)\} + \mu_{a,b,p}(\phi)$$

$$= \varepsilon_p\,\mu_{a,b,p+1}(\phi) + (p\,\varepsilon_p + 1)\,\mu_{a,b,p}(\phi) \tag{4.3}$$

Combining inequalities (4.3) and (4.2) we have

$$\left| \frac{1}{k^{1-\rho}\gamma(k)} \langle L_n \mathcal{S}_{[f]}(kx), \phi(x)\rangle \right| \leqslant$$

$$\leqslant C_1\,\mu_{a,b,p}(\phi) + C_2\,\mu_{a,b,p+1}(\phi)$$

uniformly for $k \geq k_o$ for all $n \in Z_+$, where constants C_1 and C_2 depend only on f and p.

On the other hand we start with

$$\lim_{k \to \infty} \frac{1}{k^{1-\rho} \gamma(k)} \langle \mathcal{S}_{[f]}(kx), \psi(x) \rangle \doteq \langle \mathcal{S}_{[g]}(x), \psi(x) \rangle$$

so that we have

$$\lim_{k \to \infty} \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[\psi]}(t) \rangle = \langle g(t), \mathcal{S}_{[\psi]}(t) \rangle .$$

By lemma 3.3 this means that the limit

$$\lim_{k \to \infty} \frac{1}{\gamma(k)} f(kt)$$

exists on a dense set of elements of the space $M_{a,b}$. If we show that the set

$$B = \left\{ \frac{1}{\gamma(k)} f(kt) \ : \ k \geq k_o \right\}$$

is bounded in $M'_{a,b}$ uniformly for $k \geq k_o$, then the quasiasymptotics exists by the theorem of uniform convergence. From (4.1) we have for every $\phi \in M_{a,b}$

$$\left| \frac{1}{k^{1-\rho} \gamma(k)} \langle L_n \mathcal{S}_{[f]}(kx), \phi(x) \rangle \right| \leq C(\phi)$$

for $k \geq k_o$ uniformly independent of $n \in Z_+$. Since

$$\frac{1}{k^{1-\rho} \gamma(k)} \langle L_n \mathcal{S}_{[f]}(kx), \phi(x) \rangle = \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[L_n \phi]}(t) \rangle$$

and using theorem 3.2 yields

$$\left| \frac{1}{\gamma(k)} \langle f(kt), \phi(t) \rangle \right| \leq C(\phi)$$

for every $\phi \in M_{a,b}$. This completes the proof of the theorem.

To show that our condition (4.1) is more general than the Tauberian condition by which $f(t)$ is a non-negative measure, at first we give a description of non-negative elements of M' with the help of Stieltjes transform. This is a straightforward verification of the classical ones given by Widder /14/. Remember that $f(t)$ is a non-negative element of M' if for every non-negative function $\phi(t) \geq 0$, $\phi \in M$ yields $\langle f, \phi \rangle \geq 0$.

Lemma 4.2 :
The distribution $f \in M'_{a,b}$ is a non-negative element if and only if $L_n \mathcal{S}_{[f]}$ are non-negative elements of $M_{a,b}$ for every $n \in Z_+$.

Proof:
Suppose $f \geq 0$. Then $\langle f, \phi \rangle \geq 0$ for all $\phi(t) \geq 0$ from $M_{a,b}$. Because of equality (3.4),

$$\mathcal{S}_{[L_n \phi]}(x) = x^{1-\rho} L_{n,x} \mathcal{S}_{[t^{\rho-1} \phi(t)]}(x)$$

$$= \frac{\Gamma(2n+\rho)}{n!\,\Gamma(n+\rho-1)} \int_0^\infty \frac{x^n t^{n+\rho-1}}{(x+t)^{2n+\rho}} \phi(t)\, dt$$

so that $\mathcal{S}_{[L_n \phi]}(x) \geq 0$ for all n only if $\phi(t) \geq 0$.
Now we have

$$\langle L_n \mathcal{S}_{[f]}, \phi \rangle = \langle f, \mathcal{S}_{[L_n \phi]} \rangle \geq 0$$

for every n if $\phi(t) \geq 0$.
On the other hand we start with $\langle L_n \mathcal{S}_{[f]}(x), \phi(x) \rangle \geq 0$ for all $\phi \in M_{a,b}$, $\phi(x) \geq 0$ and $n \in Z_+$. By theorem 3.2 this means $0 \leq \langle L_n \mathcal{S}_{[f]}, \phi \rangle = \langle f, \mathcal{S}_{[L_n \phi]} \rangle$ converges to $\langle f, \phi \rangle \geq 0$ so that $\langle f, \phi \rangle \geq 0$ for all $\phi(x) \geq 0$ from $M_{a,b}$. The lemma is proved.

Now we can prove that our Tauberian condition (4.1) is more general than the condition by which $f(t)$ is a non-negative measure.

Theorem 4.3 :

Let $f \in M'_{a,b}$ and $\gamma(k)$ be a regular varying function of order $\gamma > -1+\varepsilon$. Suppose that $\mathcal{S}_{[f]}$ has a quasiasymptotics at infinity with respect to $k^{1-\rho}\,\gamma(k)$ and suppose further that $f(t)$ is a non-negative element.

Then the condition (4.1) is valid.

Proof:

We have $f \in M'_{a,b}$, $\mathcal{S}_{[f]} \in M'_{\alpha,\beta}$. For $\phi \in M_{a,b}$ yield $L_n\phi \in M_{\alpha,\beta}$ and $\mathcal{S}_{[L_n\phi]} \in M_{a,b}$ so that

$$\frac{1}{k^{1-\rho}\,\gamma(k)} \langle \mathcal{S}_{[f]}(kx), L_n\phi(x) \rangle = \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[L_n\phi]}(t) \rangle$$

is well defined and the limit for $k \to \infty$ exists for every n .

Consider $\phi_0 = x^{-\varepsilon}\,(1+x)^{\rho-2+2\varepsilon} \in M_{\alpha,\beta}$. Because

$$\mathcal{S}_{[\phi_0]}(t) = \int_0^\infty \frac{x^{-\varepsilon}\,(1+x)^{\rho-2+2\varepsilon}}{(x+t)^{\rho}}\,dx > 0 \;, \quad t > 0$$

and $f(t)$ is non-negative, we have

$$\left| \frac{1}{k^{1-\rho}\,\gamma(k)} \langle \mathcal{S}_{[f]}(kx), L_n\phi(x) \rangle \right|$$

$$= \left| \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[L_n\phi]}(t) \rangle \right|$$

$$= \left| \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[\phi_0]}(t) \frac{\mathcal{S}_{[L_n\phi]}(t)}{\mathcal{S}_{[\phi_0]}(t)} \rangle \right|$$

$$\leq \left\{ \sup_{t \in R_+} \frac{\mathcal{S}_{[L_n\phi]}(t)}{\mathcal{S}_{[\phi_0]}(t)} \right\} \left| \frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[\phi_0]}(t) \rangle \right| \quad . \tag{4.4}$$

Since $\mathcal{S}_{[\phi_0]}(t)$ is continuous, monotonical decreasing for $t > 0$ and

$$\mathcal{S}_{[\phi_0]}(t) = O(t^{1-\rho-\varepsilon}) \;, \quad t \to +0 \;,$$

$$\mathcal{S}_{[\phi_0]}(t) = O(t^{-1+\varepsilon}) \;, \quad t \to +\infty \;,$$

we can estimate the first term of (4.4) by

$$\sup_{t \in R_+} \frac{\left|\mathcal{S}_{[L_n\phi]}(t)\right|}{\mathcal{S}_{[\phi_0]}(t)}$$

$$= \sup_{t \in R_+} \frac{t^{\rho-1+\varepsilon}\,(1+t)^{2-\rho-2\varepsilon}\,\left|\mathcal{S}_{[L_n\phi]}(t)\right|}{t^{\rho-1+\varepsilon}\,(1+t)^{2-\rho-2\varepsilon}\,\left|\mathcal{S}_{[\phi_0]}(t)\right|}$$

$$\leq C_1 \sup_{t \in R_+} \left| t^{\rho-1+\varepsilon}\,(1+t)^{2-\rho-2\varepsilon}\,\mathcal{S}_{[L_n\phi]}(t) \right|$$

$$= C_1\,\mu_{a,b,0}(\mathcal{S}_{[L_n\phi]}) \quad .$$

Using inequality (4.3) leads to

$$\sup_{t \in R_+} \frac{\mathcal{S}_{[L_n\phi]}(t)}{\mathcal{S}_{[\phi_0]}(t)}$$

$$\leq C_2\,\mu_{a,b,0}(\phi) + C_2'\,\mu_{a,b,1}(\phi) \quad . \tag{4.5}$$

Because of the existence of the quasiasymptotics of $\mathcal{S}_{[f]}$ the second term of (4.4)

$$\frac{1}{\gamma(k)} \langle f(kt), \mathcal{S}_{[\phi_0]}(t) \rangle$$

$$= \frac{1}{k^{1-\rho}\,\gamma(k)} \langle \mathcal{S}_{[f]}(kx), \phi_0(x) \rangle \leq C(\phi_0) \tag{4.6}$$

is uniformly bounded for $k \geq k_0$. Consequently, from inequalities (4.4), (4.5) and (4.6) it follows that

$$\left| \frac{1}{k^{1-\rho}\,\gamma(k)} \langle L_n\,\mathcal{S}_{[f]}(kx), \phi(x) \rangle \right| =$$

$$= \left| \frac{1}{k^{1-\rho}\,\gamma(k)} \langle \mathcal{S}_{[f]}(kx), L_n\phi(x) \rangle \right|$$

$$\leq C\{\mu_{a,b,0}(\phi) + \mu_{a,b,1}(\phi)\}$$

for every $\phi \in M_{a,b}$ where the constant C depends only on f and ϕ_0 . This proves the theorem.

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Одна Абелева и Тауберова теорема для преобразования Стильтеса обобщенных функций

С использованием техники квазиасимптотики описываются асимптотические соотношения для преобразования Стильтеса обобщенных функций. Условие тауберова типа, которое сформулировано в данной работе, является более общим по сравнению с предположением о том, что прообраз Стильтеса есть положительная мера.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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An Abelian and Tauberian Theorem for the Stieltjes Transform of Generalized Functions

Using the technique of quasiasymptotics, we describe the asymptotic relations for the Stieltjes transform of generalized functions. The Tauberian condition given here is more general than the assumption by which the Stieltjes original is a non-negative measure.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR

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