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S.I.Serdyukova

THE DISCONTINUITY FLOATING IN ACROSS COMPUTATION OF A SINGULAR HYPERBOLIC EQUATION

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The motion of fluxons in the long system with microinhomogeneity is described by the equation[1]

$$\varphi_{tt} = \varphi_{xx} - \left(1 - \mu\,\delta(x - x_0)\right) \sin\varphi, \quad -\ell \le x \le \ell,$$

$$\varphi_x(-\ell) = \varphi_x(\ell) = 0.$$

The well formed fluxon is taken as initial data. The relation on the discontinuity is true

$$\varphi_x(x_0 + 0) - \varphi_x(x_0 - 0) = -\mu \sin\varphi(x_0).$$

Thus, if $\varphi(x_0) \neq k\pi$, the first derivative has the discontinuity. In numerical simulation the initial equation is transformed in the equivalent system. Let $u = \varphi_x$, $v = \varphi_t$, then we get

$$u_t = v_x, \quad v_t = u_x - \left(1 - \mu\,\delta(x - x_0)\right) \sin\left(\varphi(x,0) + \int_0^t v(x,\xi)\, d\xi\right),$$

$$u(-\ell) = u(\ell) = 0.$$

The problem is solved numerically. The Wendroff-Lax scheme and the Rusanov scheme are used[3]. The discontinuity in $x = x_0$ is clearly seen in computing, but it is floating. To investigate this phenomenon we find the asymptotic of the numerical solution of the model problem

$$u_t = u_x - \mu\,\delta(x) \sin u,$$

$$u(x,0) = \begin{cases} 0, & x < 0, \\ 1, & x \ge 0. \end{cases} \qquad (1)$$

The solution of this problem is the sum of stationary step function

$$u_1(x,\mu) = \begin{cases} 0, & x < 0, \\ \mu \sin 1, & x \ge 0. \end{cases}$$

and the step function with discontinuity which is moving along the characteristic

$$u_2(x,t,\mu) = \begin{cases} 0, & x+t<0, \\ 1-\mu\sin 1, & x+t\ge 0. \end{cases}$$

We conclude from asymptotic behaviour that the numerical solution approximates

$$u_1(x,\tilde{\mu}) + u_2(x,t,\tilde{\mu}), \quad \tilde{\mu} \ne \mu .$$

To compute correctly we have to take the fictitious μ^* in the case of the Wendroff-Lax scheme,

$$\mu^* = \frac{\mu\sin 1}{\sin(1-\mu\sin 1)},$$

in the case of Rusanov scheme

$$\mu^* = \frac{\mu\sin 1}{\sin(1-\mu(1+C_0)\sin 1)},$$

where C_0 depends on the scheme parameters. Below we give the explicit formula for C_0.

Let (1) is approximated by the Wendroff-Lax scheme

$$u_j^{n+1} = u_j^n + \frac{\alpha}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{\alpha^2}{2}\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right) - \mu\alpha\delta_{0,j}\sin u_j^n, \quad n\ge 0, \quad j=0,\pm 1,\pm 2,\dots \tag{2}$$

$$u_j^0 = \begin{cases} 0, & j<0, \\ 1, & j\ge 0, \end{cases} \qquad \delta_{0,j} = \begin{cases} 1, & j=0, \\ 0, & j\ne 0. \end{cases}$$

Here α is the ratio of net steps; in the real computation $\alpha = \frac{1}{2}$. After the Fourier transformation $v^n = \sum_j u_j^n e^{ij\varphi}$ we have

$$v^{n+1} = \left(1-2\alpha^2\sin^2\frac{\varphi}{2} - i\alpha\sin\varphi\right)v^n - \mu\alpha\sin u_0^n = f(e^{i\varphi})v^n - \mu\alpha\sin\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} v^n(e^{i\varphi})\,d\varphi\right), \quad v^0 = \frac{1}{(1-e^{i\varphi})}. \tag{3}$$

When $\alpha\le 1$, the relation is true

$$|f(e^{i\varphi})|^2 = 1-4\alpha^2(1-\alpha^2)\sin^4\frac{\varphi}{2} \le 1 .$$

The system (3) is solved by application of the successive approximation method

$$v_{k+1}^n = \frac{f^n(e^{i\varphi})}{(1-e^{i\varphi})} - \mu\alpha\sum_{\xi=0}^{n-1} f^{\xi}(e^{i\varphi})\sin\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} v_k^{n-1-\xi}(e^{i\varphi})\,d\varphi\right).$$

Let $v_0^n=0$. Then v_1^n is the Fourier image of the difference step function for the equation $u_t=u_x$. Using the results from [4], we get

the representation of the absolute term of $v^{n-1-\xi}$:

$$u_{1,0}^{n-1-\xi} = 1+\varepsilon_{n-1-\xi} .$$

From this it follows that

$$v_2^n = \frac{f^n}{1-e^{i\varphi}} - \frac{\mu\alpha\sin 1}{(1-f)}(1-f^n) - \mu\alpha\sum_{\xi=0}^{n-1} f^{\xi}\left(\sin(1+\varepsilon_{n-1-\xi}) - \sin 1\right) = f^n v^0 - \frac{\mu\alpha\sin 1}{1-f}(1-f^n) + R_1^n(e^{i\varphi}). \tag{4}$$

In the following iteration we use

$$\int_{-\pi}^{\pi} R_1^n(e^{i\varphi})\,d\varphi = O(e^{-dn}).$$

Really, either ξ or $n-1-\xi$ is greater than $\frac{n-2}{2}$. But the deviation of the difference step function from the step function is exponentially small at the distance $O(n)$ from the characteristic[4] and the difference Green's function is exponentially small at $j=0$ if $\varphi=0$ is the unique defining point and the nonvertical characteristic corresponds to this defining point[5]. The following expansion is true

$$\frac{1}{1-f} = -\frac{e^{i\varphi}}{\alpha(1-e^{i\varphi})} - \frac{e^{i\varphi}(1-\alpha)}{\alpha(1+\alpha)}\sum_{k=0}^{\infty}\left(-\frac{1-\alpha}{1+\alpha}\right)^k e^{ik\varphi} = -\frac{1}{\alpha(1-e^{i\varphi})} + \frac{1}{\alpha} + \frac{1}{\alpha}\sum_{k=1}^{\infty}\left(-\frac{1-\alpha}{1+\alpha}\right)^k e^{ik\varphi}. \tag{5}$$

Thus, the absolute term of $(1-f)^{-1}$ is equal to 0 and hence $\mu\alpha(1-f)^{-1}$ gives no contribution to the following iteration. As result we obtain

$$v_2^n = (1-\mu\sin 1)\frac{f^n}{1-f} + \tilde{R}_1^n(e^{i\varphi}).$$

The summand $\tilde{R}_1^n(e^{i\varphi})$ gives the exponentially small addition to the following iteration. In the third step of iteration we have

$$v_3^n = f^n(1-e^{i\varphi})^{-1} - \mu\alpha\sin(1-\mu\sin 1)\frac{1-f^n}{1-f} - \mu\alpha\sum_{\xi=0}^{n-1} f^{\xi}\left\{\sin\left((1-\mu\sin 1)(1+\varepsilon_{n-1-\xi})\right) - \sin(1-\mu\sin 1)\right\} + R_2^n = \left(1-\mu\sin(1-\mu\sin 1)\right) f^n(1-e^{i\varphi})^{-1} + \tilde{R}_2^n ,$$

where $\tilde{R}_2^n$ gives again exponentially small addition to the next iteration. In the k-th step of iteration we obtain

$$v_k^n = \frac{f^n}{1-e^{i\varphi}} - \frac{\mu\alpha \sin \zeta_{k-1}}{1-f}\left(1-f^n\right) + R_{k-1}^n = \frac{1-\mu \sin \zeta_{k-1}}{1-e^{i\varphi}} f^n + \tilde{R}_{k-1}^n .$$

The absolute terms expansion of $R_{k-1}^n, \tilde{R}_{k-1}^n$ are exponentially small. ζ_k is defined recursively: $\zeta_1 = 1-\mu \sin 1$, $\zeta_{k+1} = 1-\mu \sin \zeta_k$. If $\mu < 1$, the succession $\{\zeta_k\}$ converges to the solution of the equation $\zeta = 1-\mu \sin \zeta$. In the limit we have the function

$$\tilde{v}^n = \frac{f^n}{1-e^{i\varphi}} - \frac{\mu\alpha \sin \zeta}{1-f}\left(1-f^n\right).$$

It will be shown below that $\tilde{u}_\nu^n$ approximates u_ν^n. Particularly, in the vicinity of $\nu = 0$ the obtained approximation differs from the exact solution by the exponentially small value of n. The direct substitution shows that $\tilde{v}^n$ is the Fourier image of the solution of the following problem

$$\tilde{u}_\nu^{n+1} = \tilde{u}_\nu^n + \frac{\alpha}{2}\left(\tilde{u}_{\nu+1}^n - \tilde{u}_{\nu-1}^n\right) + \frac{\alpha^2}{2}\left(\tilde{u}_{\nu+1}^n - 2\tilde{u}_\nu^n + \tilde{u}_{\nu-1}^n\right) -$$
$$-\mu\alpha\, \delta_{0,\nu} \sin \zeta, \qquad \tilde{u}_\nu^0 = \begin{cases} 0, & \nu < 0, \\ 1, & \nu \geq 0. \end{cases}$$

It follows from the expansion (5) that

$$\tilde{u}_0^n = \zeta\left(1+\varepsilon_n\right) + \mu w_n \sin \zeta = \zeta + \sigma_n ,$$

where w_n is the Couchy's problem solution in $\nu = 0$ with such initial data

$$u_\nu^0 = \begin{cases} 0, & \nu < 0, \\ \left(-\frac{1-\alpha}{1+\alpha}\right)^\nu, & \nu \geq 0. \end{cases}$$

It is naturally to seek the correction to $\tilde{u}_\nu^n$ as the solution of the nonhomogeneous problem with zero initial data

$$\Delta_\nu^{n+1} = \Delta_\nu^n + \frac{\alpha}{2}\left(\Delta_{\nu+1}^n - \Delta_{\nu-1}^n\right) + \frac{\alpha^2}{2}\left(\Delta_{\nu+1}^n - 2\Delta_\nu^n + \Delta_{\nu-1}^n\right) -$$
$$-\mu\alpha\, t_n\, \delta_{0,\nu} ; \quad n \geq 0, \quad \nu = 0, \pm 1, \pm 2, \ldots$$

Denote the difference Green's function in $\nu = 0$ by γ_n. The succession $\{t_n\}$ must satisfy the system

$$t_0 = \sin 1 - \sin \zeta,$$
$$t_n = \sin\left(\zeta + \sigma_n - \mu\alpha \sum_{\xi=0}^{n-1} \gamma_\xi\, t_{n-1-\xi}\right) - \sin \zeta, \quad n \geq 1,$$

which can be transformed to the form

$$\frac{t_n}{\cos \zeta} + \frac{\sin \zeta}{\cos^3 \zeta}\, t_n^2\left(1+\rho(t_n)\right) = \sigma_n - \mu\alpha \sum_{\xi=0}^{n-1} \gamma_\xi\, t_{n-1-\xi} .$$

The corresponding linear system solution is

$$\tilde{t}_n = \sum_{\xi=0}^{n-1} \sigma_\xi \cdot \mathcal{F}_{n-\xi} .$$

Using the methods of the complex variable function theory we investigate the asymptotic behaviour of $\mathcal{F}_n$ for $n \to \infty$. Consider the approximate linear system

$$\frac{\tilde{t}_n}{\cos \zeta} = \sigma_n - \mu\alpha \sum_{\xi=0}^{n-1} \gamma_\xi\, \tilde{t}_{n-1-\xi}, \qquad n \geq 0. \tag{6}$$

Remind that

$$\sigma_n = \zeta \cdot \varepsilon_n + \mu \sin \zeta \cdot w_n ,$$

where

$$\varepsilon_n = \frac{1}{2\pi i}\int_\Gamma \frac{f^n(u)\,du}{u(1-u)}, \quad w_n = \frac{1}{2\pi i}\int_\Gamma \frac{f^n(u)\,du}{u\left(1+\frac{1-\alpha}{1+\alpha}u\right)}, \tag{7}$$
$$\gamma_n = \frac{1}{2\pi i}\int_\Gamma \frac{f^n(u)\,du}{u}.$$

The contour Γ follows the unit circle, the pole $u = 1$ is being passed by a small arc inside the unit circle. f is the characteristic function

$$f(u) = 1 + \frac{\alpha}{2}\left(u^{-1} - u\right) + \frac{\alpha^2}{2}\left(u^{-1} - 2 + u\right),$$

which has two zeros

$$\frac{1+\alpha}{\alpha} \pm \sqrt{\left(\frac{1+\alpha}{\alpha}\right)^2 + \frac{1+\alpha}{1-\alpha}},$$

two poles $0, \infty$ and two stationary points

$$u_0^\pm = \pm i\sqrt{\frac{1+\alpha}{1-\alpha}}, \quad f\left(u_0^\pm\right) = \sqrt{1-\alpha^2}\, e^{\mp i \operatorname{Arcsin} \alpha} .$$

The initial contour Γ is deformed into the line of the steepest descent of the function $\ln f(u)$:

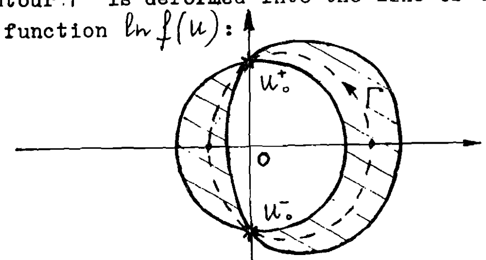

Fig. 1.

In Fig. 1 the line of the steepest descent is shown by the shaded line, the stationary points are marked by the asterisks, the domain

$$|f(u)| \leq |f(u_0^{\pm})|$$

is shaded. Using the method of the saddle point/6/, we find that if $\alpha = \frac{1}{2}$, $n \to \infty$, then

$$\gamma_n = \frac{2}{\sqrt{\pi n}} \left(\frac{\sqrt{3}}{2}\right)^n \cdot \left(\cos (n-1) \frac{\pi}{6}\right) \cdot \left(1 + O\left(\frac{1}{n}\right)\right),$$

$$\varepsilon_n = \frac{1}{\sqrt{\pi n}} \left(\frac{\sqrt{3}}{2}\right)^n \cdot \left(\sin n \frac{\pi}{6}\right) \cdot \left(1 + O\left(\frac{1}{n}\right)\right),$$

$$w_n = \sqrt{\frac{3}{\pi n}} \left(\frac{\sqrt{3}}{2}\right)^n \cdot \left(\cos n \frac{\pi}{6}\right) \cdot \left(1 + O\left(\frac{1}{n}\right)\right).$$

To study the asymptotic behaviour of $\mathcal{F}_n$ let us multiply both sides of the equation (6) by z^n and sum over n from 0 to ∞. In the line of the steepest descent $|f(u)| = |f(u_0^{\pm})|$. In order the row $\sum_0^{\infty} z^n f^n(u)$ should be convergent, it is necessary that $|z f(u_0^{\pm})| < 1$. Then we have (see (7))

$$T(z) = \cos s \left(\sum_0^{\infty} \sigma_n z^n - \mu \alpha z T(z) \sum_0^{\infty} \gamma_n z^n\right) =$$

$$= \cos s \left(S(z) - \mu \alpha \cdot T(z) \cdot B(z)\right), \text{ and hence}$$

$$T(z) = \frac{S(z)}{(\cos s)^{-1} + \mu \alpha B(z)}, \text{ where}$$

$$B(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{z\, du}{u(1 - z f(u))}.$$

To study the asymptotic behaviour of $\mathcal{F}_n$ we find the singularities of $B(z)$. We have

$$B(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{du}{\frac{u}{z} - u - \frac{\alpha}{2}(1-u^2) + \frac{\alpha^2}{2}(u^2 - 2u + 1)} =$$

$$= \frac{1}{2\pi i} \int_{\Gamma} \frac{2\, du}{\alpha(1-\alpha)(u-u_1)(u-u_2)}.$$

Here u_1, u_2 are the solutions of

$$u^2 + 2bu - \frac{1+\alpha}{1-\alpha} = 0, \text{ where } b = \frac{\frac{1}{z} - 1 + \alpha^2}{\alpha(1-\alpha)}, \text{ so}$$

$$u_{1,2} = -b \pm \sqrt{b^2 + \frac{1+\alpha}{1-\alpha}}.$$

Hence $u_1 = u_2$, if and only if

$$b = \pm i \sqrt{\frac{1+\alpha}{1-\alpha}} = u_0^{\pm}, \quad (z_0^{\pm})^{-1} = f(u_0^{\pm}) = 1 - \alpha^2 \pm i\alpha\sqrt{1-\alpha^2}.$$

In both cases if $z \to z_0^{\pm}$ being inside $|z| \leq |z_0^{\pm}|$, $u_1(z_0^{\pm})$ are inside Γ. In particular, when $\alpha = \frac{1}{2}$ and $z^{-1} = (z_0^{\pm})^{-1}(1+\delta)$, $\delta > 0$, we have the picture shown in Figs. 2,3

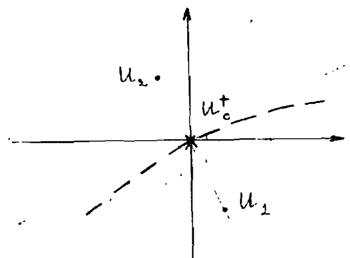

Fig.2.

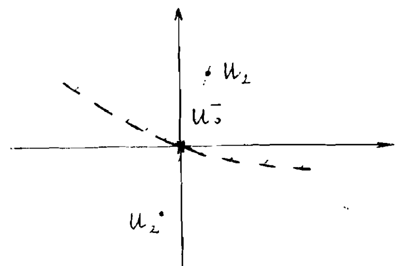

Fig. 3.

In Fig. 2 $\quad u_{1,2} = i\sqrt{3} \pm 2\sqrt{3\delta} \cdot e^{-\frac{i\pi}{3}} + O(\delta)$,

in Fig. 3 $\quad u_{1,2} = -i\sqrt{3} \pm 2\sqrt{3\delta} \cdot e^{\frac{i\pi}{3}} + O(\delta)$.

The corresponding residues $\mathrm{Res}^{\pm}(z)$ are

$$\mathrm{Res}^{+}(z) = \frac{2e^{\frac{i\pi}{3}}}{\sqrt{3\left(1 - \frac{z}{z_0^+}\right)}} \left(1 + O(z - z_0^+)\right),$$

$$\mathrm{Res}^{-}(z) = \frac{2e^{-\frac{i\pi}{3}}}{\sqrt{3\left(1 - \frac{z}{z_0^-}\right)}} \left(1 + O(z - z_0^-)\right).$$

Then we have

$$\mathcal{F}_n = \frac{\sqrt{3}}{2\pi i\mu}\int\limits_{|z|=|z_0^\pm|-\delta}\Big\{ e^{-\frac{i\pi}{3}}\sqrt{1-\frac{z}{z_0^+}}\left(1+O(z-z_0^+)\right) + e^{\frac{i\pi}{3}}\sqrt{1-\frac{z}{z_0^-}}\left(1+O(z-z_0^-)\right)\Big\}\, z^{-n-1}\, dz.$$

The initial contour is transformed into the contour Γ_1 for the first summand and the contour Γ_2 for the second summand, shown in Fig. 4

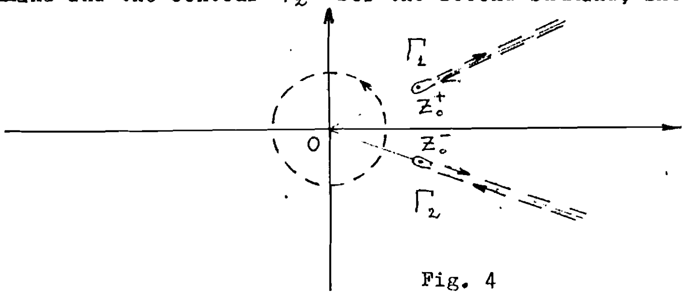

Fig. 4

We obtained

$$\mathcal{F}_n = \frac{-\sqrt{3}}{2\pi i\mu}\Big\{ e^{-\frac{i\pi}{3}}\int\limits_{\Gamma_1}\sqrt{1-\frac{z}{z_0^+}}\left(1+O(z-z_0^+)\right)z^{-n-1}\,dz + e^{\frac{i\pi}{3}}\int\limits_{\Gamma_2}\sqrt{1-\frac{z}{z_0^-}}\left(1+O(z-z_0^-)\right)z^{-n-1}\,dz\Big\}.$$

Change the variables $z=z_0^\pm(1+t)$, $u=nt$, then

$$\mathcal{F}_n = \frac{-\sqrt{3}}{2\pi i\mu}\Big\{ e^{-\frac{i\pi}{3}} f^n(u_0^+)\int\limits_C\sqrt{-t}\left(1+t\rho_1(t)\right)e^{-(n+1)\ln(1+t)}\,dt + e^{\frac{i\pi}{3}} f^n(u_0^-)\int\limits_C\sqrt{-t}\left(1+t\rho_2(t)\right)e^{-(n+1)\ln(1+t)}\,dt\Big\} = \frac{-\sqrt{3}\,n^{-\frac{3}{2}}}{2\pi i\mu}\cdot\Big\{e^{-\frac{i\pi}{3}}f^n(u_0^+)+e^{\frac{i\pi}{3}}f^n(u_0^-)\Big\}\int\limits_C\sqrt{-u}\,e^{-u}\,du\left(1+O\left(\frac{1}{n}\right)\right).$$

The contour C is shown in Fig. 5

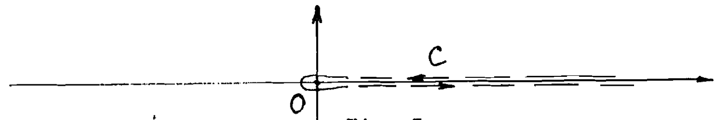

Fig. 5

It remains to use the Hunkel representation of the gamma-function. As a result we have proved that

$$\mathcal{F}_n = \frac{\sqrt{3}}{\mu}\,\frac{n^{-\frac{3}{2}}}{\sqrt{\pi}}\left(\frac{\sqrt{3}}{2}\right)^n\cos\left(\frac{n\pi}{6}-\frac{2\pi}{3}\right)\left(1+O\left(\frac{1}{n}\right)\right),\quad n\to\infty.$$

Therefore

$$t_n=\sum_{\xi=0}^{n}\sigma_\xi\cdot\mathcal{F}_{n-\xi}\cdot\left(1+O\left(\left(\frac{\sqrt{3}}{2}\right)^\xi\right)\right)=O\left(\left(\frac{\sqrt{3}}{2}\right)^n\right).$$

Hence we proved that

$$v^n=\frac{f^n}{1-e^{i\varphi}}-\frac{\mu\alpha\sin s}{1-f}\left(1-f^n\right)-\mu\alpha\sum_{\xi=0}^{n-1}f^\xi(e^{i\varphi})\cdot t_{n-\xi-1}.$$

It is known[5] that when $n\to\infty$, the difference Green's function is $O(n^{-\frac{1}{3}})$ in the zone of the size $O(\sqrt{n})$ near the characteristic and exponentially small outside this zone. The difference step function is $O(1)$ in the characteristic vicinity[4]. Therefore $\tilde{u}_j^n$ approximates u_j^n for all j. In particular, in the vicinity of the discontinuity at $x=0$, the error Δ_j^n is exponentially small when $n\to\infty$. The approximation (1) by the Rusanov scheme is studied in the same manner

$$\begin{aligned} u_j^{n+1} &= u_j^n+\frac{\alpha}{12}\left(-u_{j+2}^n+8u_{j+1}^n-8u_{j-1}^n+u_{j-2}^n\right)+\\ &+\frac{\alpha^2}{8}\left(u_{j+2}^n-2u_j^n+u_{j-2}^n\right)+\frac{\alpha^3}{12}\left(u_{j+2}^n-2u_{j+1}^n+\right.\\ &\left.+2u_{j-1}^n-u_{j-2}^n\right)-\frac{\omega}{24}\left(u_{j+2}^n-4u_{j+1}^n+6u_j^n-4u_{j-1}^n+u_{j-2}^n\right)-\\ &-\mu\alpha\,\delta_{0,j}\sin u_0^n,\qquad u_j^0=\begin{cases}0, & j<0,\\ 1, & j\ge 0.\end{cases}\end{aligned}\tag{8}$$

As above, after the Fourier transformation we obtain the system which is solved by application of the successive approximation method. In the second iteration step we have

$$v_2^n=\frac{f^n}{1-e^{i\varphi}}-\frac{\mu\alpha\sin 1}{1-f}\left(1-f^n\right)+R_1^n\left(e^{i\varphi}\right).$$

In this case

$$f(e^{i\varphi}) = 1+\frac{\alpha}{12}\left(e^{2i\varphi}-8e^{i\varphi}+8e^{-i\varphi}-e^{-2i\varphi}\right)+\frac{\alpha^2}{8}\left(e^{2i\varphi}-2+e^{-2i\varphi}\right)+$$
$$+\frac{\alpha^3}{12}\left(-e^{2i\varphi}+2e^{i\varphi}-2e^{-i\varphi}+e^{-2i\varphi}\right)-\frac{\omega}{24}\left(e^{2i\varphi}-4e^{i\varphi}+6-4e^{-i\varphi}+e^{-2i\varphi}\right).$$

For $0\le\alpha\le 1$, $4\alpha^2-\alpha^4\le\omega\le 3$, $|f(e^{i\varphi})|\le 1$ /3/. Inside the L_2 stability region there is stability in C /7/. Computations were performed with $\alpha=\frac{1}{2}$, $\omega=2$. In difference from preceding $(1-f)^{-1}$ has the absolute term. In the following let $\alpha=\frac{1}{2}$, $\omega=2$, then

$$\frac{\alpha}{1-f} = -\frac{24\,e^{2i\varphi}}{(1-e^{i\varphi})(e^{3i\varphi}+27e^{i\varphi}-4)} = \sum_{-\infty}^{\infty} c_\ell\, e^{i\ell\varphi}.$$

The polynomial $z^3+27z-4$ has one real root $\delta=0{,}1480\ldots$ and two complex-conjugate $A, \bar{A}$, $A\cdot\bar{A}=4\delta^{-1}$. From this it follows, that

$$c_0 = -\frac{24\delta}{(1-\delta)(3\delta^2+27)} = -0{,}154\ldots .$$

The following iterations give

$$v_3^n = \frac{f^n}{1-e^{i\varphi}}-\mu\alpha\sin\left(1-\mu(1+c_0)\sin 1\right)\frac{1-f^n}{1-f}+R_2^n(e^{i\varphi}),$$

$$v_4^n = \frac{f^n}{1-e^{i\varphi}}-\mu\alpha\sin\left(1-\mu(1+c_0)\sin\left(1-\mu(1+c_0)\sin 1\right)\right)\frac{1-f^n}{1-f}+R_3^n(e^{i\varphi}),$$

etc. In the limit we obtain

$$\tilde{v}^n = \frac{f^n}{1-e^{i\varphi}}-\mu\alpha\sin\zeta\,\frac{1-f^n}{1-f},$$

where ζ is the solution of the equation

$$\zeta = 1-\mu(1+c_0)\sin\zeta .$$

By analogy with preceding it can be proved, that $\tilde{u}_j^n$ approximates u_j^n for all j. In order that $1-\mu^*\sin\zeta = 1-\mu\sin 1$, we have to take

$$\mu^* = \frac{\mu\sin 1}{\sin\zeta} = \frac{\mu\sin 1}{\sin\left(1-\mu(1+c_0)\sin 1\right)}.$$

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Принимается подписка на препринты и сообщения Объединенного института ядерных исследований.

Установлена следующая стоимость подписки на 12 месяцев на издания ОИЯИ, включая пересылку, по отдельным тематическим категориям:

ИНДЕКС	ТЕМАТИКА	Цена подписки на год
1.	Экспериментальная физика высоких энергий	10 р. 80 коп.
2.	Теоретическая физика высоких энергий	17 р. 80 коп.
3.	Экспериментальная нейтронная физика	4 р. 80 коп.
4.	Теоретическая физика низких энергий	8 р. 80 коп.
5.	Математика	4 р. 80 коп.
6.	Ядерная спектроскопия и радиохимия	4 р. 80 коп.
7.	Физика тяжелых ионов	2 р. 85 коп.
8.	Криогеника	2 р. 85 коп.
9.	Ускорители	7 р. 80 коп.
10.	Автоматизация обработки экспериментальных данных	7 р. 80 коп.
11.	Вычислительная математика и техника	6 р. 80 коп.
12.	Химия	1 р. 70 коп.
13.	Техника физического эксперимента	8 р. 80 коп.
14.	Исследования твердых тел и жидкостей ядерными методами	1 р. 70 коп.
15.	Экспериментальная физика ядерных реакций при низких энергиях	1 р. 50 коп.
16.	Дозиметрия и физика защиты	1 р. 90 коп.
17.	Теория конденсированного состояния	6 р. 80 коп.
18.	Использование результатов и методов фундаментальных физических исследований в смежных областях науки и техники	2 р. 35 коп.
19.	Биофизика	1 р. 20 коп.

Подписка может быть оформлена с любого месяца текущего года.

По всем вопросам оформления подписки следует обращаться в издательский отдел ОИЯИ по адресу: 101000 Москва, Главпочтамп, п/я 79.

Сердюкова С.И. Е5-86-45

Размывание разрыва при сквозном счете одного сингулярного гиперболического уравнения

Исследуется размывание разрыва, которое наблюдается при численном моделировании динамики флюксонов в протяженной системе с микронеоднородностью. Построена асимптотика численного решения модельного сингулярного гиперболического уравнения $u_t = u_x - \mu\delta(x)\sin u$. В качестве начальных данных берется "ступенька" $u(x, 0) = 0$ при $x < 0$, $u(x, 0) = 1$ при $x \geq 0$. Из асимптотики следует, что численное решение аппроксимирует решение непрерывной задачи с $\tilde{\mu} \neq \mu$. Для корректного счета следует брать фиктивное μ^*, которое зависит от исходного μ и параметров разностной схемы. При доказательстве используются методы теории функций комплексного переменного, в частности, метод перевала.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1986

Serdyukova S.I. E5-86-45

The Discontinuity Floating in Across Computation of a Singular Hyperbolic Equation

The discontinuity floating observed in numerical simulation of the motion of fluxons in the long system with microinhomogeneity is investigated. The asymptotic behaviour of numerical solution of model singular hyperbolic equation $u_t = u_x - \mu\delta(x)\sin u$. has been found. We take the step function as initial data: $u(x, 0) = 0$, when $x < 0$; $u(x, 0) = 1$, when $x \geq 0$. It follows from asymptotic behaviour, that the numerical solution approximates the continuous problem solution with $\tilde{\mu} \neq \mu$. To compute correctly we have to take fictitious μ^*, depending on initial μ and difference scheme parameters. The methods of the complex variable function theory were used in the proof, in particular, the method of the saddle point.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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