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ON A CONTINUOUS TIME ANALOGUE OF SEMIRECURRENT EVENTS

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1. INTRODUCTION

In connection with the study of some properties of counters with prolonging dead time there has appeared a class of events/1/, which has been named semirecurrent events. Semirecurrent events have many possibilities of application and their main properties have been described in the paper/2/ and some limit properties have been investigated in/3/.

So, following/2/ we suppose that during the k-th experiment, $k = 1,2,\ldots,$ the condition A^k either may be fulfilled or not. The fulfillment of A^k at the n-th trial, $n = 1,2,\ldots,$ we denote by A^k_n and its non-fulfillment by $\bar{A}^k_n$. The events of a probability space $(\Omega, \mathcal{G}, P)$, $\{A^k_n : n, k > 1\}$ are said to be semirecurrent if, for any integers i_j with

$$1 \leq i_0 < i_1 < \ldots < i_n, \; n \geq 1, \tag{1.1}$$

we have

$$P(A^k_{i_1} \ldots A^k_{i_n} \mid A^k_{i_0}) = P(A^{k+i_0}_{i_1 - i_0} \ldots A^{k+i_0}_{i_n - i_0}). \tag{1.2}$$

Certainly we may put $P(A^k_0) = 0, \; k \geq 1.$

Denote by ν_k, $k \geq 1$ an integer-valued random variable saying that $\nu_k = n$ if and only if the condition A^k is fulfilled for the first time in the k-th experiment at the n-th trial, and put $P^k_n = P(\nu_k = n) = P(\bar{A}^k_1 \ldots \bar{A}^k_{n-1} A^k_n)$. Using (1.2) we may prove that

$$P^k_1 = P(A^k_1), \; P^k_n = P(A^k_n) - \sum_{j=1}^{n-1} P(A^k_j) P^{k+j}_{n-j}, \quad n \geq 2. \tag{1.3}$$

An important case arises if there is an integer m such that $P(A^m_n) = P(A^{m+1}_n) = \ldots$ for any $n \geq 1$. In this case we say that the semirecurrent events are m-semirecurrent. If $m = 1$, then we obtain recurrent events in the sense of Feller/4/, and if $m = 2$, then they are names recurrent events with delay/4/.

A sequence of non-negative numbers, $\{u^k_n : n \geq 0, k \geq 1\}$, is said to be semirecurrent if, for any $k \geq 1$.

$$u^k_0 = 1, \tag{1.4}$$

and if there is a sequence of non-negative numbers $\{f^k_n\}_{n=1}^{\infty}$ with

$$\sum_{n=1}^{\infty} f^k_n \leq 1 \tag{1.5}$$

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such that

$$u_n^k = f_n^k + \sum_{j=1}^{n-1} u_j^k f_{n-j}^{k+j}, \quad n \geq 1. \tag{1.6}$$

From the results of the paper [2] there follows that (1.6) is equivalent to

$$u_n^k = f_n^k + \sum_{j=1}^{n-1} f_j^k u_{n-j}^{k+j}, \quad k, n \geq 1. \tag{1.7}$$

It is clear that if $\{A_n^k : k \geq 1, n \geq 0\}$ are semirecurrent events, then $\{P(A_n^k); k \geq 1, n \geq 0\}$ is a semirecurrent sequence. Indeed, if we put $f_n^k = P(\nu_k = n)$ then using (1.3) we have to obtain (1.6).

The sequence $\{u_n^k : k \geq 1, n \geq 0\}$ is far from being an arbitrary sequence of numbers between 0 and 1. Its behaviour is restricted by inequalities which are consequence of (1.2). The necessary and sufficient condition is given in Theorem 2.4 from [2].

Finally, we say that a semirecurrent sequence, $\{u_n^k : n \geq 0, k \geq 1\}$. is m-semirecurrent if there is an integer m such that $u_n^m = u_n^{m+1} = \ldots$ for any $n \geq 1$.

2. MARKOV CHAINS

Following K.L.Chung [5] by a Markov chain in discrete time we shall mean a sequence $\mathfrak{X} = \{X_n\}_{n=0}^{\infty}$ of random variables, taking values in a countable state space S, and having the property that, for any n and any $j \in S$

$$P(X_n = j \mid X_0, \ldots, X_{n-1}) = P(X_n = j \mid X_{n-1}). \tag{2.1}$$

In the sequel we shall assume that the one-step transition probability

$$p_{ij}(k) = P(X_k = j \mid X_{k-1} = i) \tag{2.2}$$

depends, in general, on k, $k = 0,1,2,\ldots$. So that, we have a Markov process, non-homogeneous, in general.

Under these conditions the joint distributions of the X_n are completely determined by the transition probabilities $p_{ij}(k)$ $i, j \in S$, $k = 0,1,\ldots,$ and the initial distribution

$$p_i = P(X_0 = i), \quad i \in S, \tag{2.3}$$

via

$$P(X_0 = i_0, X_1 = i_1, \ldots, X_n = i_n) = p_{i_0} p_{i_0 i_1}(1) \ldots p_{i_{n-1} i_n}(n-1), \tag{2.4}$$

for $i_0, i_1, \ldots, i_n \in S$, $n \geq 1$.

The numbers p_i and $p_{ij}(k)$ satisfy the conditions

$$p_i \geq 0, \quad \sum_{i \in S} p_i = 1, \tag{2.5}$$

$$p_{ij}(k) \geq 0, \quad \sum_{j \in S} p_{ij}(k) = 1, \tag{2.6}$$

for any $k \geq 1$, and conversely, the Kolmogorov theorem [6] shows us that there is a stochastic process $\mathfrak{X} = \{X_n\}_{n=0}^{\infty}$ satisfying (2.1)-(2.4) whenever (2.5) and (2.6) are satisfied.

It is convenient to denote by ${}^kP_i$, $i \in S$, $k = 0,1,\ldots$, the probability measure conditional on $\{X_k = i\}$, so that

$$P = \sum_{i \in S} p_i^k \, {}^kP_i, \tag{2.7}$$

where $p_i^k = P(X_k = i)$.

For our aim it is convenient to consider, for a given Markov process $\mathfrak{X} = \{X_n\}_{n=0}^{\infty}$, a whole class of Markov processes $\mathfrak{X}^k = \{X_n^k\}_{n=0}^{\infty}$, $k \geq 1$, where $X_n^k = X_{k+n-1}$. So that $\mathfrak{X} = \mathfrak{X}^1$. From (2.7) we obtain that

$${}^kP_i(X_1^k = i_1, \ldots, X_n^k = i_n) = p_{ii_1}(k) \ldots p_{i_{n-1} i_n}(k+n-1), \tag{2.8}$$

so that ${}^kP_i$ depends only on $p_{ij}(n)$, and not on the distribution of X_k.

If (2.8) is summed over all values of $i_1, \ldots, i_{n-1}$, we obtain the n-step transition probabilies

$$p_{ij}^{(n)}(k) = P(X_n^k = j \mid X_0^k = i) = {}^kP_i(X_n^k = j). \tag{2.9}$$

They are easily generated using the Chapman - Kolmogorov equation

$$\begin{aligned} p_{ij}^{(n+m)}(k) &= \sum_{r \in S} p_{ir}^{(n)}(k) p_{rj}^{(m)}(k+n) = \\ &= \sum_{r \in S} p_{ir}^{(m)}(k) p_{rj}^{(n)}(k+m), \quad k \geq 1, \ i, j \in S, \end{aligned} \tag{2.10}$$

where $p_{ij}^{(1)}(k) = p_{ij}(k)$.

In the sequel we show that any Markov chain generates a semirecurrent sequence. For that, for any $k \geq 1$, consider the return of the sequence X_n^k to a fixed state $a \in S$, that is, we observe the set of integers for which $X_n^k = a$. There may be a finite or infinite number of them, and they may be written in ascending order as $0 = \eta_0^k < \eta_1^k < \ldots \eta_{\kappa_k}^k$, where $\kappa_k < \infty$ is the total number of returns of the process $\mathfrak{X}^k$ to its initial state a. For $n \geq 1$.

$$^kP_a(\kappa_k \geq 1, \eta_1^k = n) = {}^kP_a(X_1^k, \ldots, X_{n-1}^k \neq a, X_n^k = a) =$$

$$= \sum_{i_1 \ldots i_{n-1} \neq a} p_{ai_1}(k) p_{i_1 i_2}(k+1) \ldots p_{i_{n-1} a}(k+n-1) = f_n^k,$$

say. Clearly $\sum_{n=1}^{\infty} f_n^k = P(\kappa_k \geq 1) \leq 1$. For convenience we define $\eta_1^k = \infty$ if $\kappa_k = 0$, so that

$$f_\infty^k = 1 - \sum_{n=1}^{\infty} f_n^k. \tag{2.11}$$

If now we define

$$u_n^k = P(X_n^k = a \mid X_0^k = a), \tag{2.12}$$

with the convention $u_n^k = 1$, then, for $n \geq 1$,

$$u_n^k = {}^kP_a(X_n^k = a) = \sum_{j=1}^{n} {}^kP_a(X_n^k = a, \eta_1^k = j) =$$

$$= \sum_{j=1}^{n} f_j^k \, {}^kP_a(X_n^k = a \mid \eta_1^k = j).$$

Since the event $\{\eta_1^k = j\}$ depends only on $X_1^k, \ldots, X_j^k$ and implies $\{X_j^k = a\}$, (2.1) gives

$$^kP_a(X_n^k = a \mid \eta_1^k = j) = P(X_n^k = a \mid X_j^k = a) = u_{n-j}^{k+j}.$$

From (1.7) we have that u_n^k defined by (2.12) forms a semirecurrent sequence.

An important result of K.L.Chung (recorded by Feller [4]) says that any 1-semirecurrent sequence arises from a homogeneous Markov chain via (2.12). The following result shows that this result is true for semirecurrent sequence, too, and this gives an answer to the question posed in [2] concerning the characterization of semirecurrent sequences.

Theorem 2.1. If $\{u_n^k : k \geq 1, n \geq 0\}$ is any semirecurrent sequence, then there exists a Markov chain $\mathfrak{X}$ taking values in a countable state space S and a state $a \in S$ such that, for all n, (2.12) holds. Moreover, there exists a sequence of semirecurrent events, $\{A_n^k : k \geq 1, n \geq 0\}$ such that $u_n^k = P(A_n^k)$.

Proof. Put $S = \{0,1,2,\ldots\}$ and $g_n^k = 1 - \sum_{j=1}^{n} f_j^k$. For any $k \geq 1$, we define the transition probability matrix, $\boldsymbol{P}(k) = \{p_{ij}(k) : i,j \in S\}$, as follows.

$$\boldsymbol{P}(k) = \begin{pmatrix} \boldsymbol{A}(k) & \boldsymbol{O}_k \\ \boldsymbol{O}_\infty & \boldsymbol{I}_\infty \end{pmatrix}, \tag{2.13}$$

where $\boldsymbol{A}(k)(\boldsymbol{O}_k)$ is a square (zero) matrix with k rows, and $\boldsymbol{O}_\infty(\boldsymbol{I}_\infty)$ is the infinite square zero (identical) matrix, the matrix $\boldsymbol{A}(k)$ has the following form

$$\boldsymbol{A}(k) = \begin{bmatrix} f_1^k/g_0^k & g_1^k/g_0^k & 0 & \ldots & 0 \\ f_2^{k-1}/g_1^{k-1} & 0 & g_2^{k-1}/g_1^{k-1} & \ldots & 0 \\ \vdots & & & & \\ f_k^1/g_{k+1}^1 & 0 & 0 & \ldots & g_k^1/g_{k+1}^1 \end{bmatrix} \tag{2.14}$$

here we define 0/0 as 1/2. We note the elements of $\boldsymbol{A}(k)$ in (2.14) are well defined. Then, for $a = 0$, we have

$$^kP_0(X_1^k, \ldots, X_{n-1}^k \neq 0, X_n^k = 0) = {}^kP_0(X_1^k = 1, X_2^k = 2, \ldots, X_{n-1}^k =$$

$$= n-1, X_n^k = 0) = p_{01}(k) p_{12}(k+1) \ldots p_{n-2,n-1}(k+n-2) p_{n-1,0}(k+n-1) =$$

$$= \frac{g_1^k}{g_0^k} \frac{g_2^k}{g_1^k} \cdots \frac{g_{n-1}^k}{g_{n-2}^k} \frac{f_n^k}{g_{n-1}^k} = f_n^k,$$

so that, $p_\infty^{(n)}(k) = u_n^k$.

For the second part of the assertion, define, for any $k \geq 1$ and $1 \leq j_1 < j_2 \ldots < j_n$, $n \geq 1$, functions $\Phi_k(j_1, \ldots, j_n) = P(X_{j_1}^k \neq a, \ldots, X_{j_n}^k \neq a, X_0^k = a)$ where $\mathfrak{X} = \{X_n\}_{n=0}^{\infty}$ is a Markov chain from the first part of the Theorem. The functions Φ_k fulfill the conditions of Theorem 2.4 of [2], hence, there exist semirecurrent events, $\{A_n^k : k \geq 1, n \geq 0\}$, say, with $u_n^k = P(A_n^k)$. Q.E.D.

Theorem 2.2. If $\{u_n^k : k \geq 1, n \geq 0\}$ and $\{v_n^k : k \geq 1, n \geq 0\}$ are two semirecurrent sequences, then $\{w_n^k : k \geq 1, n \geq 0\}$ where $w_n^k = u_n^k v_n^k$, $k \geq 1$, $n \geq 0$, is a semirecurrent sequence, too.

Proof. According to Theorem 2.1, there exists a Markov chain $\mathfrak{X}$ on a state space S, and a state $a \in S$ such that $u_n^k = {}^kP_a(X_n^k = a)$.

Analogously, there also exists a Markov chain $\mathcal{Y}$, on a state space S' and independent of $\mathcal{X}$, and a state $b \in S'$ such that $v_n^k = {}^kP_b(Y_n^k = b)$. Then $Z_n = (X_n, Y_n)$ defines a Markov chain on $S \times S'$, for which ${}^kP_{a,b}(X_n^k = a, Y_n^k = b) = u_n^k v_n^k$. Q.E.D.

Note. In the paper [7] there is shown that for any 2-semirecurrent sequence $\{u_n^k: k \geq 1, n \geq 0\}$ there exists a homogeneous Markov process $\mathcal{X} = \{X_n\}_{n=0}^{\infty}$ with a countable state space S such that

$$u_n^1 = P(X_n = a | X_0 = b), \quad u_n^k = P(X_n = a | X_0 = a),$$

for all $n \geq 1$, $k \geq 2$, and some pair $a \neq b$ of states from S. Theorem 2.1 shows us that these u_n^k may be also determined as diagonal elements of appropriate transition matrices of a nonhomogeneous Markov chain.

3. SEMIRECURRENT EVENTS WITH CONTINUOUS TIME PARAMETER

In this section we shall deal with a generalization of a semirecurrent events to a continuous time parameter. A system of events $\{A_t^s: s, t > 0\}$ on the probability space $(\Omega, \mathcal{G}, P)$ is said to be semirecurrent (with a continuous time parameter t) if, for any $s > 0$ and

$$0 < t_0 < t_1 < \ldots < t_n, \quad n \geq 1, \tag{3.1}$$

we have

$$P(A_{t_1}^s \ldots A_{t_n}^s | A_{t_0}^s) = P(A_{t_1 - t_0}^{s+t_0} \ldots A_{t_n - t_0}^{s+t_0}). \tag{3.2}$$

It is evident, that, if $\{A_t^s: s, t > 0\}$ is a system of semirecurrent events with a continuous time parameter, then, for any $h > 0$,

$$\{B_n^k := A_{nh}^{kh}: k \geq 1, n \geq 0\}, \tag{3.3}$$

is a sequence of semirecurrent events.

Define a class of functions $\{p^s(t): s > 0\}$ of a real $t > 0$ via

$$p^s(t) = P(A_t^s). \tag{3.4}$$

Lemma 3.1. Let $\{A_t^s: s, t > 0\}$ be a system of semirecurrent events and let $s_0 > 0$ be given. Then $\{B_t^s: s, t > 0\}$ where $B_t^s = A_t^{s+s_0}$, is a system of semirecurrent events, too.

Proof. Let (3.1) hold, then, for any $s > 0$,

$$P(B_{t_1}^s \ldots B_{t_n}^s | B_{t_0}^s) = P(A_{t_1}^{s+s_0} \ldots A_{t_n}^{s+s_0} | A_{t_0}^{s+s_0}) =$$

$$= P(A_{t_1 - t_0}^{s+s_0+t_0} \ldots A_{t_n - t_0}^{s+s_0+t_0}) = P(B_{t_1-t_0}^{s+t_0} \ldots B_{t_n - t_0}^{s+t_0}).$$

Q.E.D.

If $p^{s_1}(t) = p^{s_2}(t)$ for any $s_1, s_2, t > 0$, $\{A_t^s: s, t > 0\}$ are said to be a system of recurrent events with a continuous time. An important case arises when there exists as $s_0 > 0$ such that, for any $s > s_0$,

$$p^s(t) = p^{s_0}(t), \quad t > 0. \tag{3.5}$$

Immediately we have the following result.

Corollary 3.2. If for a system of semirecurrent events $\{A_t^s: s, t > 0\}$ there is an $s_0 > 0$ such that (3.5) holds for any $s > s_0$ and any $t > 0$, then $\{B_t^s: s, t > 0\}$ in a system of semirecurrent events; where $B_t^s = A^{s+s_0}$, $s, t > 0$.

Now we give an example of a system of semirecurrent events. Let $\{X_t: t > 0\}$ by a (non-homogeneous) Poisson process with a rate function $\lambda(u)$ where $\lambda(u)$ is a non-negative, continuous function bounded on any finite interval. So that, we may assume that X_t denotes, for example, the number of particles arriving at the counter during the time interval $(0, t]$. Then $P(X_t = n) = e^{-\Lambda(0,t)} \Lambda(0,t)^n / n!$, $n = /, 1, 2, \ldots$, where $\Lambda(s,t) = \int_s^{s+t} \lambda(u)\,du$. Let $\xi_{s,t}$, $s, t > 0$, denote a random variable corresponding to the Poisson process $\{X_t: t > 0\}$ and denoting the number of particles arriving at the counter during the time interval $(s, s+t]$. Then $P(\xi_{s,t} = n) = e^{-\Lambda(s,t)} \Lambda(s,t)^n / n!$, $n = 0, 1, 2, \ldots$. If we put $A_t^s = \{\xi_{s,t} = 0\}$, then using familiar properties of a Poisson process $\{X_t: t > 0\}$ we may easily check that $\{A_t^s: s, t > 0\}$ is a system of semirecurrent events with

$$p^s(t) = e^{-\Lambda(0, s+t)} / e^{-\Lambda(0, s)}, \quad t > 0. \tag{3.6}$$

The functions $p^s(t)$ defined by (3.6) have an important property

$$\lim_{t \to 0} p^s(t) = 1 \tag{3.7}$$

uniformly in s on any finite interval. A system of functions $\{p^s(t): s > 0\}$ arising by (3.4) is said to be standard if (3.7) holds uniformly in s on any finite interval.

Analogously as in a disctete case we show that no system of functions of t, $\{p^s(t): s > 0\}$, with $0 \le p^s(t) \le 1$, $s, t > 0$ corresponds to a system of semirecurrent events via (3.4). The necessary and sufficient condition is the next result.

Theorem 3.3. Let $p^s(t): s > 0$ by a system of non-negative functions of $t > 0$, and write, for any $n \ge 1$,

$$\Phi^s(t_1,\ldots,t_n) = 1 - \sum_{1 \le j_1 \le n} p^s(t_{j_1}) + \sum_{1 \le j_1 < j_2 \le n} p^s(t_{j_1}) p^{s+t_{j_1}}(t_{j_2} - t_{j_1}) + \ldots + (-1)^n \sum_{1 \le j_1 < \ldots < j_n \le n} p^s(t_{j_1}) p^{s+t_{j_1}}(t_{j_2} - t_{j_1}) \ldots p^{s+t_{j_n}}(t_{j_n} - t_{j_{n-1}}),$$

whenever $0 \le t_1 < t_2 < \ldots < t_n$. Then there is a system of semirecurrent events $\{A_t^s: s, t > 0\}$ with (3.4) if and only if, whenever $n \ge 1$, and $0 < t_1 < \ldots < t_n$ we have

$$0 \le \Phi^s(t_1,\ldots,t_n) \le \Phi^s(t_1,\ldots,t_{n-1}). \tag{3.8}$$

Proof. The necessity of (3.8) follows from the simple observation that if $0 < t_1 < \ldots < t_n$, then $\Phi^s(t_1,\ldots,t_n) = P(\bar{A}^s_{t_1} \ldots \bar{A}^s_{t_n})$.

Conversely, suppose that (3.8) hold. For any $s > 0$ we have to construct a probability space $(\Omega_s, \mathcal{G}_s, P_s)$ and a system of events $\{\bar{A}^s_t : t > 0\} \subset \mathcal{G}_s$ such that $p^s(t) = P_s(\bar{A}^s_t)$, $t > 0$. To do this we must verify the Kolmogorov consistence conditions /6/. Hence, if we construct the direct probability space $(\Omega, \mathcal{G}, P) = \prod_{s>0} (\Omega_s, \mathcal{G}_s, P_s)$, then $A^s_t := \pi_s^{-1}(\bar{A}^s_t)$, where $\pi_s: \Omega \to \Omega_s$ is the s-th projection function forms a system of semirecurrent events in question. Q.E.D.

A simple corollary of the last Theorem is the following inequality. For any $s, t, u > 0$

$$p^s(t)p^{s+t}(u) \le p^s(t+u) \le 1 - p^s(t) + p^s(t)p^{s+t}(u). \tag{3.9}$$

Lemma 3.4. If $\{p^s(t): s > 0\}$ is a standard system of functions arising from some semirecurrent events via (3.4). Then $p^s(t) > 0$ for any $s, t > 0$.

Proof. Using (3.9) we have, for any $t > 0$ and $s > 0$,

$$p^s(t) \ge p^s(t/n)p^{s+t/n}(t/n)\ldots p^{s+(n-1)t/n}(t/n).$$

Using the property (3.7) we see that, for sufficiently large n, any term in the right-hand side of the last inequality is positive. Q.E.D.

Now we show that a system of non-negative functions $\{p^s(t): s > 0\}$ of $t > 0$ satisfying the conditions of Theorem 3.3 may appear in a different way as that described via a (non-homogeneous) Poisson process.

So, let $\mathfrak{X} = \{X_t: t > 0\}$ be a Markov process with a countable state space S. That is, if $0 < t_1 < t_2 < \ldots < t_n$, $n \ge 1$, then

$$P(X_{t_n} = j | X_{t_1} \ldots X_{t_{n-1}}) = P(X_{t_n} = j | X_{t_{n-1}}),$$

for any $j \in S$.

Then for the transition probabilities

$$P_{ij}(s,t) := P(X_{s+t} = j | X_s = i), \quad i, j \in S, \quad s, t > 0, \tag{3.10}$$

we have the following properties

$$P_{ij}(s,t) \ge 0, \quad \sum_{j \in S} P_{ij}(s,t) = 1, \tag{3.11}$$

$$\sum_{k \in S} P_{ik}(s,u) P_{kj}(s+u, t-u) = P_{ij}(s,t), \quad i, j \in S, \tag{3.12}$$

for any $0 < u < t$, and $s > 0$,

$$P_{ij}(s,s) = \delta_{ij}, \quad i, j \in S, \quad s > 0, \tag{3.13}$$

where δ_{ij} denotes the Kronecker delta function.

Conversely, any system functions $\{P_{ij}(s,t): i, j \in S, s, t > 0\}$ with (3.11)-(3.13) determines a (non-homogeneous) Markov process $\{X_t: t > 0\}$ with continuous time whose transition probabilities are given functions $\{P_{ij}(s,t), i, j \in S\}$ of $s, t > 0$.

Now, fix a state $a \in S$. Then $\{p^s(t): s > 0\}$, where $p^s(t) := P(X_{s+t} = a | X_s = a)$, $t > 0$, is a system of functions fulfilling the conditions of Theorem 3.3. Indeed, put, for any $s > 0$,

$$\Phi^s(t_1,\ldots,t_n) = P(X_{s+t_1} \ne a, \ldots, X_{s+t_n} \ne a | X_s = a),$$

where $t_1 < \ldots < t_n$, $n \ge 1$. Consequently, there is a system of semirecurrent events, $\{A^s_t: s, t > 0\}$ say, such that $P(X_{s+t} = a | X_s = a) = P(A^s_t)$.

In this place we remark that the converse implication is not true, in general. From the paper /8, p.429/ there follows that not every functions, determined from recurrent events in continuous time, arise from a Markov chain. This is true only for discrete time case, see Theorem 2.2.

Theorem 3.4. If $\{p_1^s(t): s > 0\}$ and $\{p_2^s(t): s > 0\}$ are two systems of real functions of t corresponding to some systems of semirecurrent events, then a system function $\{p^s(t): s > 0\}$, where $p^s(t) = p_1^s(t)p_2^s(t)$, $t > 0$, corresponds to a system of semirecurrent events.

Proof. Construct independent semirecurrent events $\{A_t^s: s, t > 0\}$ and $\{B_t^s: s, t > 0\}$. Then $\{C_t^s: s, t > 0\}$, where $C_t^s = A_t^s \cap B_t^s$ is a system of semirecurrent events in question. Q.E.D.

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13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Двуреченский А. E5-86-352

О непрерывном по времени аналоге семирекуррентных событий

В некоторых стохастических моделях работы счетчиков частиц с мертвым временем продлевающегося типа появляются семирекуррентные события, дискретные по времени. Показано, что они описываются через переходные вероятности неоднородной цепи Маркова со счетным числом состояний. Семирекуррентные события с дискретным временем обобщаются на случай семирекуррентных событий, непрерывных по времени,и исследуются их основные свойства.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1986

Dvurečenskij A. E5-86-352

On a Continuous Time Analogue of Semirecurrent Events

In some stochastic models of the work of particle counters with prolonging dead time there appear the semirecurrent events with discrete time. In the present paper it is shown that they are described as transition probabilities of any non-homogeneous Markov chain with countable state space. The semirecurrent events with discrete time are generalized to the case of semirecurrent events with continuous time, and some of their main properties are investigated.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986