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ON A DISCRETE MODIFIED M/GI/c/∞ QUEUE

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1. INTRODUCTION

During the last few years the use of discrete queueing systems with finitely or infinitely many servers has been increasing. The discrete systems are used as mathematical models of, for example, mass servicing machines electronic machines, transport problems, communication channels [1], automated filmless blob-length measurements in track chambers in high-energy physics [2], particle counters [3], etc.

For a modified queue we suppose that the service times and interarrival times of all customers served during any busy period are independent random variables with not necessarily identical distribution functions. Modified M/GI/1 queue has been investigated by Yeo [4] and Welch [5], modified GI/M/1 and GI/GI/1 queues by Pakes [6,7], and GI/M/1 by Shanthikumar [8]. Modified GI/GI/∞ has been studied, on the particle counter language, in [3].

The joint distribution of the busy and idle periods, for the GI/M/1 queue, has been derived by Kalashikov [9]. For the discrete modified counter with prolonging dead time, the joint distribution of the dead time and idle period is obtained in [3].

An important class of discrete queueing systems appears in a process of automated measurement systems when the interarrival times, T_k, have the geometric distribution

$$P(T_k = nh) = (1-p)p^{n-1}, \quad n \geq 1, \; k \geq 1, \tag{1.1}$$

where $0 < p < 1$, and $h > 0$ is a discretization step.

For example, the measurement in track chambers in high-energy physics leads to this model [2]. Along the particle trajectory we may observe a chain of streamers which are described as circles having centres on a trajectory. The number of streamer centres is a homogeneous Poisson process, and our task is to determine the blob and gap lengths. The actual measurements are performed using the scanning apparatus, so that the experimental data on the blob-length measurements have discrete values. Interpreting the blob and the gap as the busy and idle periods we obtain a discrete queueing system M/GI/∞ with (1.1). Some discrete queueing models with finitely many servers and with (1.1) may appear in communication channels [1].

In the present note we derive the busy period probability law for a discrete modified M/GI/c/∞ queue for any $1 \leq c \leq \infty$. First we concentrate on the discrete modified M/GI/c queue with finitely many servers. Then we shall continue, in more detail, with the cases of single-server and two-server queues, and with the queue having infinitely many servers. We note that the formulae presented are computationally convenient for practical use, and the computational process may be simply programmed for computer, too. Some remarks on computing, and on particular cases of queues will be done in Part 5.

2. DISCRETE MODIFIED M/GI/c/∞ QUEUE

Suppose that a queue is idle before the moment $t=0$ and let customers arrive at discrete instants $0 \leq \tau_1 < \tau_2 < \ldots < \infty$, which are multiples of a step $h > 0$, into a queueing system with c $(1 \leq c \leq \infty)$ available servers, and with a waiting room having infinitely many places (if $1 \leq c < \infty$). Let χ_k, $k \geq 1$, be a service time of the k-th customer, and let $T_k = \tau_{k+1} - \tau_k$, $k \geq 1$, be the interarrival time between the arrivals of the k+1-st and the k-th customers. The busy period, B^c, is the time interval during which at least one server is busy. The idle period, I^c, is the time interval during which no customer is served. The sum, $C^c = B^c + I^c$, of the busy period and the successive idle period is said to be a cycle.

For the discrete modified queue we suppose that

$$P(\tau_1 = nh) = (1-p)p^n, \quad n \geq 0, \tag{2.1}$$

$$P(T_k = nh) = (1-p)p^{n-1}, \quad n \geq 1, \; k \geq 1, \tag{2.2}$$

and the first busy period is produced by the sequence of service times $\{\chi_k\}_{k=1}^{\infty}$. $\{\chi_k\}_{k=1}^{\infty}$ is assumed to be a sequence of independent positive random variables, independent of the input process $\{T_k\}_{k=1}^{\infty}$ and τ_1, and with the distribution laws

$$P(\chi_k = nh) = h_k(n), \quad n \geq 1, \; k \geq 1, \tag{2.3}$$

where $\sum_{n=1}^{\infty} h_k(n) = 1$, $k \geq 1$. Moreover, we suppose that any successive busy period is resumed with the initial conditions, independently of previous periods, so that, the sequence of the busy periods (idle periods, cycles, respectively) are i.i.d. random variables. This discrete modified queue will be denoted by $\xi^c = (p; h_1, h_2, \ldots)$.

For a given queue $\xi^c = (p; h_1, h_2, \ldots)$ it is convenient to consider a sequence of discrete modified queues, $\{\xi_k^c\}_{k=1}^{\infty}$, where $\xi_k^c = (p; h_k, h_{k+1}, \ldots)$, $k \geq 1$. Define, for any ξ_k^c, the corresponding busy periods, B_k^c, idle periods, I_k^c, and cycles, C_k^c, respectively. For simplicity we put $h = 1$.

Due to the known properties of the geometric input process, the idle periods have the geometric distribution law with the same parameter p, that is, for $P_k^{I^c}(n) = P(I_k^c = n)$, we have

$$P_k^{Ic}(n) = (1-p)p^{n-1}, \quad n \geq 1, \ k \geq 1. \tag{2.4}$$

Moreover, the busy and idle periods are independent random variables.

Denote by A an event that the busy period begins from $t = 0$. Due to (2.1), we have $P(A) = 1 - p$. We define $p_k(j) = P(\chi_1^k = j, A)$, for $k, j \geq 1$. Therefore

$$p_k(j) = h_k(j)\,(1-p), \tag{2.5}$$

and for $j = 0$ we put

$$p_k(0) = p, \quad k \geq 1. \tag{2.6}$$

We denote the conditional probability in question, $P(B_k^c = n \mid A)$, by $P_k^c(n)$, and the joint distribution, $P(B_k^c = n, A)$, by $PP_k^c(n)$. Clearly

$$P_k^c(n) = PP_k^c(n)/(1-p), \quad n \geq 1, \ k \geq 1. \tag{2.7}$$

Let $W_k^c(n,j) = P(B_k^c = n, \ \chi_1^k = j, A)$, $n \geq 1$, $1 \leq j \leq n$, $k \geq 1$. Then

$$PP_k^c(n) = \sum_{j=1}^{n} W_k^c(n,j), \quad n \geq 1, \ k \geq 1. \tag{2.8}$$

Let now $n \geq 1$ and $1 \leq c < \infty$ be given (the queue with infinitely many servers will be treated in Part 4). For any i, $1 \leq i \leq n \wedge c$, where $x \wedge y = \min(x,y)$, and , for any $1 \leq j_1 \leq n$, $0 \leq j_s \leq n-s+1$ $(2 \leq s \leq i)$, we define $A_k^c(n; j_1,\dots,j_i)$ as the conditional probability of $B_k^c = n$ under the condition that, for any $1 \leq s \leq i$, at the time $t = s$ either a customer arrives and his service time is j_s (if $j_s \geq 1$) or no customer arrives (if $j_s = 0$). Hence

$$W_k^c(n,j) = p_k(j)\, A_k^c(n;j), \quad 1 \leq j \leq n, \ n \geq 1, \ k \geq 1, \tag{2.9}$$

and if $1 \leq i \leq n \wedge c$, then

$$A_k^c(n;j_1,\dots,j_i) = \sum_{j_{i+1}=0}^{n-i} p_{k+1}(j_{i+1})\, A_k^c(n;j_1,\dots,j_i,j_{i+1}). \tag{2.10}$$

Using the properties of the conditional probability and the independence of the dead time from the idle period we may prove the following relationships for $A_k^c(n;j_1,\dots,j_i)$.

$$\left.\begin{aligned} A_k^c(1;1) &= p, \\ PP_k^c(1) &= W_k^c(1,1) = p_k(1)p. \end{aligned}\right\} \tag{2.11}$$

Let $n \geq 2$ and suppose that we know all $A_k^c(m; j_1,\dots,j_v)$ for any $1 \leq v \leq m \wedge c$, $1 \leq m < n$, $k \geq 1$. Then the process of evaluating $A_k^c(n;j_1,\dots,j_i)$, $1 \leq i \leq n \wedge c$, will be (algorithmically) divided into five steps.

(I) Existence of "gaps": There is an integer u, $2 \leq u \leq i$, with $j_u = 0$, such that $\max(j_1, j_2+1,\dots,j_{u-1}+u-2) < u$. Then

$$A_k^c(n;j_1,\dots,j_i) = 0. \tag{2.12}$$

In the following let there be no "gaps".

(II) Existence of "busy periods stuck together": There is an integer u, $2 \leq u \leq i$, such that $\max(j_1, j_2+1,\dots,j_{u-1}+u-2) = u-1$. Then

$$A_k^c(n; j_1,\dots,j_i) = A_{k+u-1-(u)_0^*}^c(n-u-1; j_u,\dots,j_i), \tag{2.13}$$

where $(u)_0^*$ denotes the number of zeros in $\{j_1,\dots,j_u\}$.

Now, let there be no "busy periods stuck together".

(III) Existence of "zeros": There are two possible cases. First, we suppose that there is an integer u, $2 \leq u \leq i$, with $(j_1 \geq 2)$ $j_2 \geq 1,\dots, j_{u-1} \geq 1$, and $j_u = 0$. Then

$$\begin{aligned} &A_k^c(n;j_1,\dots,j_{u-1},0,j_{u+1},\dots,j_i) = \\ &= A_{k+(u)_1^*}^c(n;j_1-1,\dots,j_{u-1}-1,j_{u+1},\dots,j_i), \end{aligned} \tag{2.14}$$

where $(u)_1^*$ denotes the number of units in $\{j_1,\dots,j_u\}$.

Second, let $(j_1 \geq 2)$ $j_2,\dots,j_{i-1} \geq 1$, $j_i = 0$. Then

$$A_k^c(n;j_1,\dots,j_{i-1},0) = A_{k+(i)_1^*}^c(n-1;j_1-1,\dots,j_{i-1}-1). \tag{2.15}$$

In the following let there be no "zeros".

(IV) Let $1 \leq i \leq n \leq c$. Here we may assume without loss of generality that

$$j_1 \geq j_2 + 1 \geq \dots \geq j_i + i - 1. \tag{*}$$

Indeed, if there is an integer u, $1 \leq u < c \wedge n$, such that $u-1+j_u < u+j_{u+1}$, then

$$A_k^c(n;j_1,\dots,j_i) = A_{k+\delta(j_u)}^c(n;j_1,\dots,j_{u-1},j_{u+1}+1,j_u-1,j_{u+2},\dots,j_i), \tag{2.16}$$

where $\delta(x) = 0$, if $x \neq 1$, otherwise $\delta(x) = 1$. Applying finitely many times this argument we get (*).

Therefore let (*) hold. Assume that in the ordered i-tuple $(j_1, j_2+1,\dots,j_i+i-1)$ there are s "stairs", that is, there exist s indices $t_1,\dots,t_s \in \{1,\dots,i\}$ such that $t_s = 1$ and

$$j_{t_s} = \dots = j_{t_{s-1}} + t_{s-1} - 2 > j_{t_{s-1}} + t_{s-1} - 1 = \dots = j_{t_{s-2}-1} + t_{s-2} - 2 >$$

$$> \dots > j_{t_2} + t_2 - 1 = \dots = j_{t_1-1} + t_1 - 2 > j_{t_1} + t_1 - 1 = \dots = j_i + i - 1.$$

It is convenient to put $t_{s+1} = n$. Define recursively $\tau^0 = 0$, $\tau^v = \tau^{v-1} + t_{v+1} - t_v$, for $1 \leq v \leq s$. Then

$$\begin{aligned} A_k^c(n;j_1,\dots,j_i) = {} & pA_{k+\delta(j_i)}^c(n-1;j_1-1,\dots,j_i-1) + \\ & A_{k+\delta(j_i)+1}^c(n-1;j_1-1,\dots,j_i-1) \sum_{u=1}^{j_i-1} p_{k+1}(u) + \\ & \sum_{v=1}^{s} \sum_{u=1}^{t_{v+1}-t_v} p_{k+1}(\tau^{v-1}+u)\, A_{k+\delta(j_i)+1}^c(n-1; j_1-1,\dots,j_{t_v-1}-1, \\ & j_{t_v}-1+u, j_{t_v+1}-1,\dots, j_i-1). \end{aligned} \tag{2.17}$$

(V) In the following we shall deal with the case $1 \leq i \leq c < n$. For our aim it is sufficient to consider only the case when $i = c$. Indeed, if $1 \leq i < c$, then using (2.10) $c-i$-times we get the case $i = c$.

Denote, for $A_k^c(n;j_1,\dots,j_c)$,

$$j = \min(j_1,\ j_2-1,\dots,j_c+c-1), \qquad s = \min\{t:\ j_t+t-1 = j\}.$$

First we suppose that $j > c$. Hence $j_1,\dots,j_c \geq 2$, and the $c+1$-st custome finds all servers busy, so that, it may be served only if the service of the s-th customer will be finished. Therefore

$$A_k^c(n;j_1,\dots,j_c) = pA_k^c(n-1;j_1-1,\dots,j_c-1) + \sum_{u=1}^{n-j} p_{k+c}(u)\, A_{k+1}^c(n-1;$$
$$j_1-1,\dots,j_{s-1}-1,j_s-1+u,j_{s+1}-1,\dots,j_c-1). \qquad (2.18)$$

Second, let $j \leq c$. The previous four steps guarantee $j_1 \geq 2, j_2,\dots, j_c \geq 1$. Hence, the $c+1$-st customer finds at least one server idle, consequently, it may be served immediately after its arrival. Therefore

$$A_k^c(n;j_1,\dots,j_c) = pA_{k+s^*}^c(n-1;j_1-1,\dots,j_{s-1}-1,c-s+1,\ j_{s+1}-1,\dots,j_c-1)+$$
$$+\sum_{u=1}^{n-c} p_{k+c}(u)\, A_{k+s^*+1}^c(n-1;j_1-1,\dots,j_{s-1}-1,c-s+u,j_{s+1}-1,\dots,j_c-1), \qquad (2.19)$$

where s^* denotes the number of the units in the set $\{j_1,\dots,j_c\} - \{j_1,\dots,j_s\}$.

All five steps prove the following theorem.

THEOREM 1 . The busy period probability law of the discrete modified queue $\mathcal{G}_k^c = (p;\ h_k,h_{k+1},\dots)$, $k \geq 1$, for any $1 \leq c < \infty$, is given by formula (2.7), where $PP_k^c(n)$ is algorithmically calculated from (2.8) through (2.19).

COROLLARY 1.1. The probability law of the cycle, $P_k^{Cc}(i) = P(C_k^c = i)$, of the discrete modified queue $\mathcal{G}_k^c = (p;\ h_k,h_{k+1},\dots)$, $k \geq 1$, $1 \leq c < \infty$, is given by

$$P_k^{Cc}(i) = \sum_{\substack{n+m=i\\ n,m\geq 1}} P_k^c(n)\, P_k^{Ic}(m), \qquad (2.20)$$

where $P_k^{Ic}(m)$ is calculated by (2.4).

Some practical remarks on the actual computation of $A_k^c(n;j_1,\dots,j_i)$, $1 \leq i \leq n \wedge c$, will be done in Part 5.

3. DISCRETE MODIFIED SINGLE SERVER AND TWO-SERVER

Here we concentrate on the discrete modified single $\mathcal{G}_k^1 = (p;\ h_k,h_{k+1},\dots)$, $k \geq 1$. For this case it is clear that it is necessary to evaluate only $A_k^1(n;j)$, for any $1 \leq j \leq n$, and any $k \geq 1$. Consequently, the formulae (2.13) and (2.18) have simpler forms. Summarizing this we have, for $PP_k^1(n)$ and $A_k^1(n;j)$, $k \geq 1$, the following recursive relationships.

$$\left.\begin{aligned} A_k^1(1;1) &= p,\\ PP_k^1(1,1) &= p_k(1)p. \end{aligned}\right\} \qquad (3.1)$$

If $n \geq 2$, then

$$A_k^1(n;1) = PP_{k+1}^1(n-1). \qquad (3.2)$$

If $2 \leq j \leq n-1$, then

$$A_k^1(n;j) = pA_k^1(n-1;j-1) + \sum_{i=1}^{n-j} p_k(i)\, A_{k+1}^1(n-;j+i-1), \qquad (3.3)$$

$$A_k^1(n;n) = pA_k^1(n-1;n-1) = p^n. \qquad (3.4)$$

Hence, for any $n \geq 1$, we have

$$PP_k^1(n) = \sum_{j=1}^{n} p_k(j)\, A_k^1(n;j), \qquad (3.5)$$

$$P_k^1(n) = PP_k^1(n)/(1-p). \qquad (3.6)$$

THEOREM 2. The busy period probability law of the discrete modified single server $\mathcal{G}_k^1 = (p;\ h_k,h_{k+1},\dots)$, $k \geq 1$, is given by (3.6), where $PP_k^1(n)$ and $A_k^1(n;j)$ are calculated from (3.1) through (3.5).

For the discrete modified queue with two servers, $\mathcal{G}_k^2 = (p;\ h_k,h_{k+1},\dots)$, $k \geq 1$, the general formulae from Part 2 reduce to the following form.

$$\left.\begin{aligned} A_k^2(1;1) &= p,\\ PP_k^2(1) &= p_k(1)\ p. \end{aligned}\right\} \qquad (3.7)$$

$$\left.\begin{aligned} A_k^2(2;1) &= PP_{k+1}^2(1),\\ A_k^2(2;2) &= p^2 + PP_{k+1}^2(1),\\ PP_k^2(2) &= p_k(1)\ A_k^2(2;1) + p_k(2)\ A_k^2(2;2), \end{aligned}\right\} \qquad (3.8)$$

$$\left.\begin{aligned} A_k^2(2;1,0) &= 0, \qquad & A_k^2(2;1,1) &= p,\\ A_k^2(2;2,0) &= p, \qquad & A_k^2(2;2,1) &= p. \end{aligned}\right\} \qquad (3.9)$$

Now let $n \geq 2$. Then

$$\left.\begin{aligned} A_k^2(n;1,0) &= 0,\\ A_k^2(n;1,i) &= A_k^2(n-1;i), \qquad 1 \leq i \leq n-1. \end{aligned}\right\} \qquad (3.10)$$

If $2 \leq j \leq n$, then

$$A_k^2(n;j,0) = A_k^2(n-1;j-1). \qquad (3.11)$$

For $1 \leq i \leq n-1$, there are two possible cases.

First, let $i + 1 \leq j$, then

$$A_k^2(n;j,i) = pA^2_{k+\delta(i)}(n-1;j-1,i-1) + \sum_{u=1}^{n-i-1} p_{k+2}(u)\, A^2_{k+1}(n-1;j-1,i-1+u), \qquad (3.12)$$

where $\delta(x) = 1$, if $x = 1$, otherwise $\delta(x) = 0$.

Second, let $i+i > j$. Then

$$A_k^2(n;j,i) = pA^2_{k+\delta(j-1)}(n-1;i,j-2) + \sum_{u=1}^{n-j} p_{k+2}(u) A^2_{k+1}(n-1;i,j-2+u). \qquad (3.13)$$

Hence, for any $n \geq 2$, we have

$$PP_k^2(n) = \sum_{j=1}^{n} \sum_{i=0}^{n-1} p_k(j)\, p_{k+1}(i)\, A_k^2(n;j,i), \qquad (3.14)$$

and finally

$$P_k^2(n) = PP_k^2(n)/(1-p), \qquad (3.15)$$

and this proves the following theorem.

THEOREM 3. The busy period probability law of the discrete modified two-server queue $\mathcal{G}_k^2 = (p;\, h_k, h_{k+1}, \dots)$, $k \geq 1$, is given by (3.15) using formulae from (3.7) through (3.14).

4. DISCRETE MODIFIED M/GI/∞ QUEUE

The method developed in the second part for the discrete modified M/GI/c/∞ queue, where $1 \leq c < \infty$, may be used for the discrete modified queue with infinitely many servers. Obviously here we do not need the waiting room because any customer finds at least one idle server. In the following we shall see that in order to determine $P_k^\infty(n) = P(B_k^\infty = n \mid A)$ it is necessary to evaluate only $A_k^\infty(n;j)$, for any $1 \leq j \leq n$. Therefore the step (IV) and (2.17) have simpler form, and we obtain the next formulae, for $\mathcal{G}_k^\infty = (p;\, h_k, h_{k+1}, \dots)$, $k \geq 1$.

$$P_k^\infty(n) = PP_k^\infty(n)/(1-p), \quad n \geq 1, \qquad (4.1)$$

$$PP_k^\infty(n) = \sum_{j=1}^{n} W_k^\infty(n,j), \quad n \geq 1. \qquad (4.2)$$

$$W_k^\infty(n,j) = p_k(j)\, A_k^\infty(n;j), \quad 1 \leq j \leq n, \quad n \geq 1. \qquad (4.3)$$

$$\left.\begin{aligned} A_k^\infty(1;1) &= p, \\ PP_k^\infty(1) &= W_k^\infty(1,1) = p_k(1)\, p. \end{aligned}\right\} \qquad (4.4)$$

Let $n \geq 2$. Then

$$A_k^\infty(n;1) = PP_k^\infty(n-1), \qquad (4.5)$$

and, for any $2 \leq j \leq n$, we have

$$A_k^\infty(n;j) = pA_k^\infty(n-1;j-1) + A_{k+1}^\infty(n-1;j-1) \sum_{i=1}^{j-1} p_{k+1}(i) + \sum_{i=j}^{n-1} W_{k+1}^\infty(n-1,i) \qquad (4.6)$$

(here, as usually, the sum over the empty set is defined as 0).

This proves the following theorem.

THEOREM 4. The busy period probability law of the discrete modisfied queue $\mathcal{G}_k^\infty = (p;\, h_k, h_{k+1}, \dots)$, $k \geq 1$, is given by (4.1), where $PP_k(n)$ is evaluated from (4.2) through (4.6).

We note that the cycle probability law of the discrete modified queue $\mathcal{G}_k^\infty = (p;\, h_k, h_{k+1}, \dots)$, $k \geq 1$, is given by (2.20), where we put $c = \infty$.

Theorem 4 generalizes the analogous result from /3/ concerning the discrete modified queue $\mathcal{G}^\infty = (p;\, h_1, h_2, \dots)$, where $h_1 = h_2 = \dots$.

5. CONCLUSION

We see that the actual computation of the busy period probability law is relatively simple in the case when we have either only few servers (for example, $c = 1,2$) or infinitely many servers. In other cases the result of Theorem 1 may be simply programmed for a computer. Here we note only that the following relationships hold.

If $1 \leq i \leq n \leq c < \infty$, then

$$A_k^c(n;j_1,\dots,j_i) = A_k^{c+1}(n;j_1,\dots,j_i), \qquad (5.1)$$

$$A_k^c(n;j) = A_k^{c+1}(n;j) = A_k^\infty(n;j), \qquad (5.2)$$

$$P_k^c(n) = P_k^{c+1}(n) = P_k^\infty(n), \qquad (5.3)$$

and they enable us to simplify the computation for a queue with a large number of available servers.

If $h_k(j)$ $(k \geq 1)$ are non-zero only for few integers j, then the calculation is simple, too. Indeed, it suffices to evaluate, for example, for $W_k^c(n,j)$ only a few of them (that is, only $W_k^c(n,j)$ with $h_k(j) > 0$). Analogically we proceed for other quantities $A_k^c(n;j_1,\dots,j_i)$.

We say that the discrete modified queue $\mathcal{G}^c = (p;\, h_1, h_2, \dots)$, $1 \leq c \leq \infty$, is of order m, if $h_m = h_{m+1} = \dots$. If $m = 1$, then we obtain the usual (non-modified) queue, and in this case all above formulae do not depend on the subscripts k.

If $m > 1$, then the computation of the busy period probability laws for the queues $\mathcal{G}_k^c = (p;\, h_k, h_{k+1}, \dots)$, $1 \leq k \leq m$, $1 \leq c \leq \infty$, may be organized so that first of all we calculate all necessary expressions for $k = m$, then we continue for $k = m-1$, etc., for $k = 1$.

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О дискретной модифицированной системе M/GI/c/∞

Измерение ионизации треков в стримерных камерах в физике высоких энергий ставит задачу определения длины дискретизированного трека. Эта физическая задача может быть решена в рамках дискретных систем обслуживания с конечным или бесконечным числом обслуживающих каналов. В настоящей заметке определяется распределение периода занятости дискретной модифицированной системы M/GI/c/∞, где функции распределения времен обслуживания, в общем случае, могут быть различны.

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On a Discrete Modified M/GI/c/∞ Queue

The measurement of track ionization in streamer chambers in high-energy physics leads to the determination of the length of the discretized track. This physical problem may be solved within the framework of the theory of discrete queueing systems with finitely or infinitely many servers, and with the geometric input process. In the present note we derive the busy period distribution of the discrete modified M/GI/c/∞-queue, $1 \leq c \leq \infty$, where the distribution functions of service times may be different, in general.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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