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HETEROPHASE RANDOM FIELDS.
Existence and Ergodicity

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INTRODUCTION

This and subsequent papers are devoted to a detailed elaboration of the results announced in [15]. The notion of a heterophase random field is the result of the author's attempt to provide a rigorous meaning to the concept of a heterophase system (intensively studied in physically oriented literature, cf. [9] and the references therein). Intuitively, one can imagine a heterophase system as a certain "mixture" of different pure phases corresponding to a given potential. For a large class of potentials we characterized pure phases in terms of exponential bounds on probabilities of large deviations for sums of random variables forming a mixing stationary random field on the integer lattice [14]. This result may be interpreted as characterizing random fluctuations in configurations typical of pure phases. From this and the empirical evidence gathered in [9] we may conclude that configurations, physically interpreted as "heterophase" ones, are very unlikely to occur as the result of random fluctuations in configurations of pure phases.

We postpone a detailed discussion of basic features of heterophase systems to a subsequent paper. In the present one we shall concentrate ourselves on some purely mathematical constructions related to the existence and ergodicity of stationary heterophase random fields.

1. NOTATION

Let S , T be countable discrete sets, S being considered as the set of values to be taken on by the random fields, and T is the set labeling the sites (unless otherwise stated, $T = \mathbf{Z}^d$ - the d -dimensional integer lattice, $d \geq 1$). Let

$$\mathfrak{A} = \{V \subset T : 0 < |V| < \infty\}; \quad |V| = \mathrm{card}(V). \tag{1.1}$$

$$V(t) = \{u \in T : 0 \leq u^j < t^j , \quad 1 \leq j \leq d\}, \tag{1.2}$$

$$\tilde{V}(t) = \{u \in T : -t^j < u^j < t^j , \quad 1 \leq j \leq d\}. \tag{1.3}$$

where $\{t^1, \dots, t^d\} > 0$ in (1.2) and (1.3). We let $t \to \infty$ mean that $\min\{t^1, \dots, t^d\} \to \infty$. Sometimes we shall deal with convergences $V_k \uparrow T$, $V_k \in \mathcal{A}$ (note that $t \to \infty$ means, essentially, convergence to the "positive octant"). To this end, a sequence $\{V_k\}_{k=1}^{\infty} \subset \mathcal{A}$ is said to be strongly regular if there exist $\delta > 0$ and a positive integer N such that

(i) $V_k \subset \tilde{V}((k+1, \dots, k+1))$ (cf. (1.3)),

(ii) any V_k is a disjoint union of at most N sets of the form $\tilde{V}(t)$, and

(iii) $|V_k|/k^d \geq d$, $k = 1, 2, \dots$

(cf. /3/ ; infinite-volume limits of this type are met in Pitt's pointwise ergodic theorem for actions of Z^d /8/).

A mapping $\phi : T \to S$ is said to be a configuration and is denoted by $\phi = (\phi(t); t \in T)$. The space S^T of all configurations is a totally disconnected complete separable metric space (compact if $|S| < \infty$) relative to the product topology. In particular, the Borel σ-field on S^T is but the usual product σ-field, viz.

$$\mathcal{S}^T = \bigoplus_{t \in T} (\mathcal{S})_t ; \quad (\mathcal{S})_t = 2^S \text{ for any } t \in T. \tag{1.4}$$

Consequently, $\mathcal{S}^T$ is generated by the family of all finite-dimensional (f.d.) sets $E \subset S^T$, i.e., of all sets expressible in the form

$$E = \{\phi \in S^T : (\phi(t);\ t \in V) \in C\}, \quad V \in \mathcal{A}, C \subset S^V. \tag{1.5}$$

If $\bar{\phi} \in S^V$ and $C = \{\bar{\phi}\}$ then the f.d. set E of the form (1.5) is called elementary. The additive group (T , +) naturally acts as the group of shift homeomorphisms $\mathcal{T}^{(S)} = (\tau_t^{(S)} ; t \in T)$ on S^T :

$$(\tau_t^{(S)} \phi)(u) = \phi(u + t); \quad \phi \in S^T; \quad t, u \in T. \tag{1.6}$$

We let $\mathcal{P}(S)$, $\mathfrak{M}(S)$ and $\mathcal{E}(S)$ denote the sets of all probability measures of all $\mathcal{T}^{(S)}$ -invariant, and of all $\mathcal{T}^{(S)}$ -invariant and ergodic probability measures on $(S^T, \mathcal{S}^T)$, respectively. A random field on T taking values in S is defined to be a family $X = (X_t ; t \in T)$ of random variables defined on a common probability space $((\Omega, \mathcal{F}, \mu)$, say) and taking values in S . By regarding X as a random element of S^T, i.e.,

$$\pi_X : \Omega \to S^T, \quad \pi_X(\omega) = (X_t(\omega); t \in T), \tag{1.7}$$

we let $\pi_X(\mu) = \mathrm{dist}(X) \in \mathcal{P}(S)$ denote the distribution of X (defined via Kolmogorov consistency theorem /7/). If $\mathrm{dist}(X) \in \mathfrak{M}(S) (\subset \mathcal{E}(S))$, then X itself is called stationary (stationary and ergodic).

If $P \in \mathcal{P}(S)$ and if $f : S^T \to R^1$ is a bounded measurable function, we put

$$\langle f \rangle_P = E_P f = \int f(\mathcal{S}) P(d\mathcal{S}). \tag{1.8}$$

2. HETEROPHASE RANDOM FIELDS

Let I be a countable discrete space (interpreted as indexing the surfields considered). Let Γ denote the set of all probability vectors $\gamma = (\gamma_i)_{i \in I}$ with at least two positive entries (interpreted as concentrations of subfields).

Suppose we are given a set $\{X^{(i)} : i \in I\}$ of S-valued random fields defined on a common probability space (Ω , $\mathcal{F}$, μ). Let $P^{(i)} = \mathrm{dist}(X^{(i)})$, $i \in I$. An S-valued random field $X = (X_t ; t \in T)$ defined on (Ω , $\mathcal{F}$, μ) is said to be a heterophase random field composed of phases $\{X^{(i)} : i \in I\}$ with concentrations $\gamma = (\gamma_i)_{i \in I}$, in symbols, $X \in \mathcal{H}(X^{(i)}, \gamma_i, I)$, if for any strongly regular sequence $\{V_k\}_{k=1}^{\infty} \subset \mathcal{A}$ the following is true:

$$\mu[\bigcap_{i \in I} \{\omega \in \Omega : \lim_{k \to \infty} |V_k|^{-1} \times \times |\{t \in V_k : X_t(\omega) = X_t^{(i)}(\omega)\}| = \gamma_i\}] = 1. \tag{2.1}$$

Remark 1. It seems more natural to define first the set of "heterophase" configurations, and then define a heterophase random field as a measure concentrated on such configurations. However, given $i \in I$, the pieces of configurations typical of $X^{(i)}$ may come from different typical configurations of that field, and this would lead to serious measurability problems. Furthermore, there is some evidence /9/ ,pp.297-299, in favour of the hypothesis that the domain structure disappears in the infinite volume limit, and thus

$$\gamma_i = \mathrm{Prob}[X_t = X_t^{(i)}], \quad t \in T. \tag{2.2}$$

However, it should be noted that the latter fact strongly depends upon mixing properties which have been rigorously proved just in a few special cases.

Remark 2. Let supp $P^{(i)}$ denote the support of $P^{(i)} = \mathrm{dist}(X^{(i)})$, $i \in I$, (cf. /7/). Though formally correct, (2.1) allows a resonable interpretation only when

$$\mathrm{supp}\, P^{(i)} \cap \mathrm{supp}\, P^{(j)} = \emptyset; \quad i, j \in I, \quad i \neq j. \tag{2.3}$$

This condition will be more or less automatically satisfied when the subfields $X^{(i)}$ will be (different) stationary and ergodic fields or (different) extremal Gibbs fields.

3. EXISTENCE OF STATIONARY HETEROPHASE RANDOM FIELDS

In this section we assume that the subfields $X^{(i)}$, $i \in I$ are stationary, i.e.,

$$\{P^{(i)} : i \in I\} \subset \mathfrak{M}(S). \tag{3.1}$$

Let

$$(S^T)^I = \prod_{i \in I} (S^T)_i \; ; \quad (S^T)_i = S^T \quad \text{for} \quad i \in I. \tag{3.2}$$

$$(\mathcal{S}^T)^I = \bigotimes_{i \in I} (\mathcal{S}^T)_i \; ; \quad (\mathcal{S}^T)_i = \mathcal{S}^T \quad \text{for} \quad i \in I. \tag{3.3}$$

The natural action of T on $(S^T)^I$ will be denoted by $\tilde{\mathcal{T}}^{(S)} = (\tilde{\tau}_t^{(S)};\ t \in T)$, where

$$\tilde{\tau}_t^{(S)}[(\phi^{(i)}(u^{(i)}))_{i \in I}] = (\phi^{(i)}(u^{(i)} + t))_{i \in I} \tag{3.4}$$

for $(\phi^{(i)})_{i \in I} \in (S^T)^I$ and $t, u^{(i)} \in T$. A set $\{X^{(i)} : i \in T\}$ of S-valued random fields is said to be jointly stationary if there exists a probability measure $Q = \mathrm{dist}(\{X^{(i)} : i \in I\})$ on $((S^T)^I, (\mathcal{S}^T)^I)$ such that

$$Q|(S^T)_i = \mathrm{dist}(X^{(i)}), \quad i \in I \qquad \text{(cf. (3.2))}, \tag{3.5}$$

$$Q\tilde{\tau}_t(S) = Q \quad \text{for all} \quad t \in T \qquad \text{(cf. (3.4))}, \tag{3.6}$$

Theorem 1. Let $\{X^{(i)} : i \in I\}$ be a jointly stationary set of random fields. Let $P^{(i)} = \mathrm{dist}(X^{(i)})$, $i \in I$. For any $\gamma = (\gamma_i)_{i \in I} \in \Gamma$,

$$\mathfrak{M}(S) \cap \mathcal{H}(P^{(i)}, \gamma_i, I) \neq \emptyset. \tag{3.7}$$

where $\mathcal{H}(P^{(i)}, \gamma_i, I) = \mathcal{H}(X^{(i)}, \gamma_i, I)$ in case when $P^{(i)} = \mathrm{dist}(X^{(i)})$, $i \in I$.

Proof. By redefining the basis probability space $(\Omega, \mathcal{F}, \mu)$ if necessary we define on that space an I-valued random field $Z = (Z_t : t \in T)$, stochastically independent of the set $\{X^{(i)} : i \in I\}$, where the joint distribution of the latter set, Q, satisfies (3.5) and (3.6). Hence, if $\lambda = \mathrm{dist}(Z) \in \mathcal{P}(I)$, then

$$\mathrm{dist}(\{X^{(i)} : i \in I\}, Z) = Q \otimes \lambda. \tag{3.8}$$

where $\otimes$ denotes the usual product of measures [7]. Following [2] define on $(\Omega, \mathcal{F}, \mu)$ a random field $X = (X_t\ ;\ t \in T)$ by

$$X_t(\omega) = X_t^{(Z_t(\omega))}(\omega); \quad \omega \in \Omega, \quad t \in T. \tag{3.9}$$

That is, X is a d-dimensional analogue of a composite source [2], to be called a composite random field. Let $J = 2^I$. Since S^T and I^T are complete separable metric spaces, we may identify $(S^T \times I^T, \mathcal{S}^T \otimes \mathcal{J}^T)$ with the space $((S \times I)^T, (\mathcal{S} \otimes \mathcal{J})^T)$. Consider the pair field

$$(Z, X) = ((Z_t, X_t);\ t \in T); \quad p = \mathrm{dist}(Z, X). \tag{3.10}$$

Under the above identification we may suppose that $p \in \mathcal{P}(I \times S)$ and $\mathcal{S}^T \subset (\mathcal{J} \times \mathcal{S})^T$. Consequently, given p and λ there exists a regular conditional probability distribution

$$\nu : I^T \times \mathcal{S}^T \to [0, 1], \quad \nu = (\nu_\psi(\cdot);\ \psi \in I^T), \tag{3.11}$$

such that

$$\lambda\nu = p. \tag{3.12}$$

(cf. [7]). Here, $\lambda\nu \in \mathcal{P}(I \times S)$ is the probability measure uniquely determined by the properties that

$$\lambda\nu(E \times F) = \int_E \nu_\psi(F)\,\lambda(d\psi), \quad E \in \mathcal{J}^T; \quad F \in \mathcal{S}^T. \tag{3.13}$$

In other words, ν is a d-dimensional analogue of a discrete communication channel [1]. Of course, X is the channel output field, $\mathrm{dist}(X)$ being the output marginal of $\lambda\nu$. Let

$$C = \prod_{t \in V} C_t \subset S^V \qquad (V \in \mathfrak{A}\ ;\ \text{cf. (1.1)})$$

and let $E \in \mathcal{S}^T$ be the f.d. set determined by C and V (see (1.5)). Let $\psi \in I^T$ be arbitrary. Then the independence property (3.8) entails

$$\nu_\psi(E) = \mu[X_t \in C_t\ :\ t \in V | \psi] =$$

$$= \mu[X_t^{(\psi(t))} \in C_t\ :\ t \in V | \psi] =$$

$$= \mu[X_t^{(\psi(t))} \in C_t\ :\ t \in V].$$

where $\mu[\cdot|\psi] = \text{Prob}_\mu[\cdot|\mathfrak{J}^T](\psi)$ is the conditional probability given the σ-field $\mathfrak{J}^T \subset (\mathfrak{J}\times\mathfrak{S})^T$. In other words, ν obeys d-dimensional versions of being input historyless and nonanticipatory [5]. Since the subfields are jointly stationary, we get

$$\nu_\psi(\tau_{-t}^{(S)}E) = \mu[X_{\mu+t}^{(\psi(u+t))} \in C_\mu \; ; \; u \in V | \psi_V] =$$

$$= \mu[X_\mu^{(\psi(u+t))} \in C_u : u \in V] = \nu_{\tau_t^{(I)}\psi}(E).$$

where $\psi_V = (\psi(t); t \in V)$ and $(\tau_t^{(I)}; t \in T)$ is the action of T on I^T. Since the latter relations are true for any f.d. set, we get

$$\nu_\psi \tau_{-t}^{(S)} = \nu_{\tau_t}(I)_\psi \; . \quad \psi \in I^T, \quad t \in T. \tag{3.14}$$

i.e. ν is a stationary channel. Consequently, if $\kappa \in \mathfrak{M}(I)$, then $\kappa\nu \in \mathfrak{M}(I\times S)$ (cf. (3.13) with $\lambda = \kappa$), and $\text{dist}(X) \in \mathfrak{M}(S)$, where X is the output field of ν.

Thus, if Z is stationary, then X is as well. Suppose, in addition, that

$$\lambda = \text{dist}(Z) \in \mathcal{E}(I), \tag{3.15}$$

$$\mu[Z_0 = i] = \lambda\{\psi \in I^T : \psi(0) = i\} = \gamma_i \; , \quad i \in I, \tag{3.16}$$

where $\gamma = (\gamma_i)_{i\in I} \in \Gamma$. Let $\{V_k\}_{k=1}^\infty \subset \mathfrak{A}$ be any strongly regular sequence. (3.15), (3.16), and the pointwise ergodic theorem for Z [3,8] imply

$$\mu\{\omega \in \Omega : \lim_{k\to\infty} |V_k|^{-1} |\{t \in V_k : X_t(\omega) = X_t^{(i)}(\omega)\}| = \gamma_i\} =$$

$$= \mu\{\omega \in \Omega : \lim_{k\to\infty} |V_k|^{-1} |\{t \in V_k : Z_t(\omega) = i\}| = \gamma_i\} = 1.$$

Since I is countable, (2.1) follows from this.

Remark 3. The result of Theorem 1 is not satisfactory for the following reason. It follows from ergodic decomposition of stationary random fields (cf., e.g., [10,11] and Section 19 of [13]; see also Section 4 below) that a stationary non-ergodic random field cannot be observable. In fact, quite loosely, such a field is concentrated on configurations typical of its ergodic components so that its own typical configurations can appear only with probability zero. Therefore it is of interest to know when there exist heterophase random fields which are both stationary and ergodic.

At the first step, we investigate the conditions under which the construction from the proof of Theorem 1 may lead to an ergodic random field. Call a set $\{X^{(i)} : i \in T\}$ of random fields defined on a common probability space jointly weak mixing if there is a Q on $((S^T)^I, (\mathfrak{S}^T)^I)$ (cf. (3.2), (3.3)) such that (3.5) is valid, and for any two f.d. sets $E, F \in (\mathfrak{S}^T)^I$,

$$\lim_{t\to\infty} \frac{1}{|V(t)|} \sum_{u\in V(t)} |Q(E \cap \tilde{\tau}_{-u}^{(S)} F| - Q(E)Q(F)| = 0. \tag{3.17}$$

Theorem 2. Let $\{X^{(i)} : i \in I\}$ be a jointly weak mixing set of random fields, and let $\text{dist}(X^{(i)}) = P^{(i)}, i \in I$. For any $\gamma = (\gamma_i)_{i\in I} \in \Gamma$ we have

$$\mathcal{E}(S) \cap \mathcal{H}(P^{(i)}, \gamma_i, I) \neq \emptyset. \tag{3.18}$$

Proof. Let Z satisfy $\lambda = \text{dist}(Z) \in \mathfrak{M}(I)$, let Q satisfy (3.5) and (3.17). Consider the corresponding channel (cf. (3.11)-(3.13)). Let

$$E = \{\phi \in S^T : \phi_V \in \prod_{t\in V} C_t\}, \quad \bar{E} = \{\phi \in S^T : \phi_W \in \prod_{t\in W} \bar{C}_t\}.$$

be arbitrary f.d. sets. Then (note that ν obeys the properties shown in the proof of Theorem 1)

$$\nu_\psi(\bar{E}) = \mu[X_t^{(\psi(t))} \in \bar{C}_t \; ; \; t \in W];$$

$$\nu_\psi(\tau_{-u}^{(S)}E) = \mu[X_t^{(\psi(t+u))} \in C_t; \; t \in V].$$

and a similar expression is valid for the probability $\nu_\psi(\tau_{-u}^{(S)} E \cap \bar{E})$. Since all these expressions depend only on the joint distribution Q, condition (3.17) implies

$$\lim_{t\to\infty} \frac{1}{|V(t)|} \sum_{u\in V(t)} |\nu_\psi(\tau_{-u}^{(S)}E \cap \bar{E}) - \nu_\psi(\tau_{-u}^{(S)}E)\nu_\psi(\bar{E})| = 0.$$

Thus, all measures ν_ψ, $\psi \in I^T$ are weakly mixing (though non-stationary) in the sense of [4]. But this is the same as to say that the channel is output weakly mixing [1]. By modifying slightly (to $d > 1$) the arguments of [1] we see that ν is ergodic, i.e., if $\lambda \in \mathcal{E}(I)$, then $\lambda\nu \in \mathcal{E}(I\times S)$ and $P \in \mathcal{E}(S)$, where $P = \text{dist}(X)$ (cf. (3.9)). If Z is chosen satisfying (3.15) and (3.16), then $P \in \epsilon(S)$ by the above reasoning and $P \in \mathcal{H}(P^{(i)}, \gamma_i, I)$. as follows from the proof of Theorem 1.

4. ERGODIC DECOMPOSITION OF HETEROPHASE RANDOM FIELDS

Throughout this section we are given a fixed $P \in \mathfrak{M}(S) \cap \mathcal{H}(P^{(i)}, \gamma_i, I)$ (cf. Theorem 1). We wish to prove (3.18) without impossing the stronger conditions of Theorem 2 upon the class of subfields $\{X^{(i)} : i \in I\}$. Instead of this, we would like to use the ergodic decomposition of stationary random fields [10,12] and show that almost all ergodic components of a field $P \in \mathfrak{M}(S) \cap \mathcal{H}(P^{(i)}, \gamma_i, I)$ are again in $\mathcal{H}(P^{(i)}, \gamma_i, I)$. First note that if $\mathrm{dist}(X) = P$, then, according to (2.1)

$$X \in \mathcal{H}(X^{(i)}, \gamma_i, I) \quad \text{iff} \quad \mu(E^X) = 1. \tag{4.1}$$

where E^X abbreviates the event in (2.1).

A configuration $\phi \in S^T$ is said to be quasiregular if for any elementary f.d. set E there exists the limit

$$\lim_{t \to \infty} \frac{1}{|V(t)|} \sum_{u \in V(t)} \mathbb{1}_E(\tau_\mu^{(S)} \phi) = P_\phi(E). \tag{4.2}$$

Let R denote the set of all <u>regular</u> configurations, i.e., of all quasiregular configurations ϕ such that P_ϕ uniquely extends to a measure $P_\phi \in \mathcal{E}(S)$. Given $\phi \in R$, let

$$R(\phi) = \{\phi' \in R : P_{\phi'} = P_\phi\}. \tag{4.3}$$

Then $R \in \mathcal{S}^T$, $P(R) = 1$ for any $P \in \mathfrak{M}(S)$, $P \in \mathcal{E}(S)$ if and only if $P\{\phi \in R : P_\phi = P\} = 1$, and the ergodic decomposition formula reads (cf. (1.8))

$$\langle f \rangle_P = \int_R \langle f \rangle_{P_\phi} \, P(d\phi). \tag{4.4}$$

For technical reasons we require that the basic probability space $(\Omega, \mathcal{F}, \mu)$ is a Lebesgue space [6]; of course, we can always achieve this by completing a standard Borel space $(\Omega, \mathcal{F})$ [7] with respect to a probability measure μ on $\mathcal{F}$.

<u>Theorem 3</u>. Let $P \in \mathfrak{M}(S) \cap \mathcal{H}(P^{(i)}, \gamma_i, I)$. Then

$$P\{\phi \in R : P_\phi \in \mathcal{H}(P^{(i)}, \gamma_i, I)\} = 1. \tag{4.5}$$

<u>Proof</u>. We shall prove the theorem only in the case when there are finitely many ergodic components of P. On the one hand, this is the situation most frequently met in physical applications and, on the other hand, in this case the proof becomes much more transparent being not encumbered by the technical frame of Lebesgue spaces (as to the technical requisities needed for the general case consult [11,12]).

So, suppose that $P = \alpha P_\phi + (1-\alpha) P_\psi$, where $P_\phi, P_\psi \in R$, $0 < \alpha < 1$ and $\phi \notin R(\psi)$. Consequently, $P_\phi \neq P_\psi$, $R(\phi) \cap R(\psi) = \emptyset$. $P_\phi(R(\phi)) = P_\psi(R(\psi)) = 1$, $P(R(\phi)) = 1 - P(R(\psi)) = \alpha$. Let $(\Omega, \mathcal{F}, \mu)$ be a standard probability space and let $\pi_X : \Omega \to S^T$ be defined by (1.7). Let $P : S^T \times \mathcal{F} \to [0,1]$ denote the transition probability such that

$$P\{\eta \in R(\phi) \cup R(\psi) : P(\eta, \pi_X^{-1}\{\eta\}) = 1\} = 1. \tag{4.6}$$

$$\mu(F) = \int p(\eta, F \cap \pi_X^{-1}\{\eta\}) \, P(d\eta), \quad F \in \mathcal{F}. \tag{4.7}$$

(cf. [7]). We let $X(\phi)$, $X(\psi)$ denote the ergodic random fields with distributions P_ϕ, P_ψ and let $E^{X(\phi)}$, $E^{X(\psi)}$ denote the events appearing in (4.1) for these fields. Observe that

$$E^X \cap \pi_X^{-1}\{\eta\} = \begin{cases} E^{X(\phi)} \cap \pi_X^{-1}\{\eta\} & \text{if } \eta \in R(\phi). \\ E^{X(\psi)} \cap \pi_X^{-1}\{\eta\} & \text{if } \eta \in R(\psi). \end{cases} \tag{4.8}$$

Using (4.6)-(4.8) we get

$$1 = \mu(E^X) = \int_{R(\phi) \cup R(\psi)} p(\eta), E^X \cap \pi_X^{-1}\{\eta\}) P(d\eta) =$$

$$= \int_{R(\phi)} p(\eta) E^{X(\phi)} \cap \pi_X^{-1}\{\eta\}) P(d\eta) + \int_{R(\psi)} p(\eta, E^{X(\psi)} \cap \pi_X^{-1}\{\eta\}) P(d\eta) =$$

$$= \int_{R(\phi)} p(\eta, E^{X(\phi)} \cap \pi_X^{-1}\{\eta\}) [\alpha P_\phi + (1-\alpha) P_\psi](d\eta) +$$

$$+ \int_{R(\psi)} p(\eta, E^{X(\psi)} \cap \pi_X^{-1}\{\eta\}) [\alpha P_\phi + (1-\alpha) P_\psi](d\eta) =$$

$$= \int_{R(\phi)} p(\eta, E^{X(\phi)} \cap \pi_X^{-1}\{\eta\} \alpha \cdot P_\phi(d\eta) +$$

$$+ \int_{R(\psi)} p(\eta, E^{X(\psi)} \cap \pi_X^{-1}\{\eta\}) (1-\alpha) P_\psi(d\eta).$$

Hence

$$P_\phi\{\eta \in R(\phi) : p(\eta, E^{X(\phi)} \cap \pi_X^{-1}\{\eta\}) = 1\} = P_\psi\{\eta \in R(\psi) : p(\eta, E^{X(\psi)} \cap \pi_X^{-1}\{\eta\}) = 1\} = 1.$$

Since $X(\phi)$ has its configurations only in the set $R(\phi)$, $\pi_X | \pi_X^{-1}(R_\phi) = \pi_{X(\phi)}$, and similarly for $X(\psi)$. Using this fact together with (4.7) and the latter equalities we get

$$\mu(E^{X(\phi)}) = \int_{R(\phi)} p(\eta, E^{X(\phi)} \cap \pi_X^{-1}\{\eta\}) \bar{P}_\phi (d\eta) = 1 .$$

$$\mu(E^{X(\psi)}) = \int_{R(\psi)} p(\eta, E^{X(\psi)} \cap \pi_X^{-1}\{\eta\}) \bar{P}_\psi (d\eta) = 1 .$$

Since $E^{X(\phi)} = E^{X(\eta)}$ when $\eta \in R(\phi)$ and $E^{X(\psi)} = E^{X(\eta)}$ when $\eta \in R(\psi)$, we finally get

$$P\{\eta \in R(\phi) \cup R(\psi) : \mu(E^{X(\eta)}) = 1\} = 1 ,$$

and, with the aid of (4.1), the claimed (4.5).

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- abbreviated name of the Institute (JINR) and publication index,
- location of publisher (Dubna),
- year of publication,
- page number.

For example:

Kolpakov I.F. In: XI Intern. Symposium on Nuclear Electronics, JINR, D13-84-53, Dubna, 1984, p.26.

Savin I.A., Smirnov G.I. In: JINR Rapid Communications, N2-84, Dubna, 1984, p.3.

Шуян Ш. E5-85-483

Гетерофазные случайные поля. Существование и эргодичность

Вводится понятие гетерофазного случайного поля на d-мерной решетке. Устанавливается существование стационарного гетерофазного случайного поля и указываются условия его эргодичности.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Šujan Š. E5-85-483

Heterophase Random Fields. Existence and Ergodicity

The notion of a heterophase random field on the d-dimensional integer lattice is introduced. Existence of stationary heterophase random fields is established, and conditions for their ergodicity are given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985