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ON THE NONRELATIVISTIC LIMIT OF SOLUTIONS OF QUASIPOTENTIAL RADIAL EQUATIONS

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In work [1] we have elaborated a systematical approach for investigation of the relativistic quasipotential radial equations for the case of S-wave.

$$[-H_0^{rad} + 2E_q - V(r;E_q)]\,\psi(q,r) = 0, \tag{1}$$

where $H_0^{rad} = 2c^2 \operatorname{ch}(\frac{i}{c}\frac{d}{dr})$, $E_q = c\sqrt{c^2+q^2}$.

In this work with the help of the approach we receive the approximate and exact solutions of equation (1) which, as $c\to\infty$, turn into Jost solutions of equation

$$[\frac{d^2}{dr^2} - V(r;E_q) + q^2]\,\psi(q,r) = 0. \tag{2}$$

Marking in $F^{\pm}(q,c,r)$ and $E^{\pm}(q,r)$ Jost solutions of equations (1) and (2) respectively we have by definition

$$\lim_{c\to\infty} F^{\pm}(q,c,r) = E^{\pm}(q,r).$$

For enough large positive integer N equation (1) can be approximately presented as the system of equations

$$\frac{dx}{dr} = A(q,c,N)\vec{x}(r) + V(r;E_q)B(q,c,N)\vec{x}(r), \tag{3a}$$

$$\psi(q,r) = \sum_{k=1}^{n} x_k(r), \tag{3b}$$

where $n = 2+4n$, $\vec{x}(r)$ – n-dimensional vector, $A(q,c,N)$ and $B(q,c,N)$ matrices of a special form. Matrix $A(q,c,N)$ is defined by the following

$$A(q,c,r) = \begin{bmatrix} A_0(q,c,N)c & 0 & 0 \\ 0 & A_1(q,cN)c & 0 \\ 0 & 0 & A_2(q,c,N)c \end{bmatrix}, \quad A_0(q,c,N) = \begin{bmatrix} i\alpha & 0 \\ 0 & -i\alpha \end{bmatrix}$$

where $A_2(q,c,N) = -A_1(q,c,N)$ and $A_1(q,c,N)$ is the $(2N\times 2N)$-diagonal matrix with elements $i\alpha + 2\pi$, $-i\alpha + 2\pi, \ldots, i\alpha + 2k\pi$, $-i\alpha + 2k\pi, \ldots$ and besides $i\alpha = i\alpha(q,c) = \arccos\sqrt{(\frac{q^2}{c})^2 + 1}$.

Matrix $B(q,c,N)$ is the $(n\times n)$-matrix with the same columns, the elements of which $B_s(q,N)$ are defined from the correlations

$$\gamma(q,c,N) = \frac{\alpha}{2(\sqrt{q^2+c^2}-c)} \prod_{k=1}^{N} \frac{(4k^2\pi^2+\alpha^2)^2}{16k^2\pi^2(k^2\pi^2+\alpha^2)},$$

$$B_1(q,c,N) = \frac{\gamma(q,N)}{2i} = -B_2(q,N),$$

$$B_{2k-1}(q,c,N) = \omega_k(q,c,N)B_1(q,c,N) = -B_{2k}(q,c,N),$$

$$\omega_k(q,c,N) = \prod_{m=0}^{k-1} \frac{(N-m)^2(N+m+1)\pi^2 - \alpha^2(N-m)}{(N+m+1)(\alpha^2 + (N+m+1)^2\pi^2)}, \quad k = 1,\ldots,N.$$

The exact presentation of equation (1) in the form of a system of equations of an infinite order is received as $N\to\infty$.

It is obvious that in free case $V(r;E_q)=0$ the solutions $\exp(\pm i\alpha(q,c)cr)$ of equation (1) degenerate to Jost solutions of equations (2), i.e.,

$$\lim_{c\to\infty} \exp(\pm i\alpha(q,c)cr) = \exp(\pm iqr). \tag{4}$$

<u>Theorem 1.</u> Suppose potential $V(r;E_q)$ belongs to class $L^1[0,\infty]$, i.e.,

$$\int_0^\infty |V(r;E_q)|\,dr < \infty. \tag{5}$$

Then the solutions of integral equations

$$\begin{aligned} F^{\pm}(q,c,r) = {} & \exp(\pm i\alpha cr) - \int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)V(t;E_q)F^{\pm}(q,c,t)\,dt - \\ & - \int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(2k\pi c(r-t))V(t;E_q)\times \\ & \times F^{\pm}(q,c,t)\,dt + \\ & + \int_0^r \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(-2k\pi c(r-t))V(t;E_q)F^{\pm}(q,c,t)\,dt \end{aligned} \tag{6}$$

exist and are the solutions of system of equations (3) degenerating to Jost solutions of equation (2). Moreover integral equations (6) are solved by the method of successive approaches.

Proof. We mark the two first columns of matrix $A(q, N)$ in $\vec{h}(+i\alpha cr)$. Suppose further

$$U_0(q, c, N) = \begin{bmatrix} A_0(q, c, N)c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$U_1(q, c, N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_1(q, c, N)c & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$U_2(q, c, N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_2(q, c, N)c \end{bmatrix}.$$

Then $A(q, c, N) = U_0(q, c, N) + U_1(q, c, N) + U_2(q, c, N)$.

With the help of direct check-up it is easy to be convinced of that the solutions of integral equations

$$\begin{aligned} \vec{x}^{\pm}(q, r) = \vec{h}(\pm i\alpha cr) &- \int_r^\infty \exp(U_0(q, c, N)(r-t)B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)dt - \\ &- \int_r^\infty \exp(U_1(q, c, N)(r-t)\, B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)\,dt + \\ &+ \int_0^\infty \exp(U_2(q, c, N)(r-t))B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)\,dt \end{aligned} \tag{7}$$

are the solutions of equation (3a). Substituting (7) into (3b) and putting $\sum_{k=1}^{n} x_k(q, r) = F^{\pm}(q, c, r)$ we receive (6).

Let a be such value of r that satisfies the equality

$$\int_0^a |V(r; E_q)|\,dr = \int_a^\infty |V(r; E_q)|\,dr.$$

Now we prove that equations (6) have the solutions which can be found by the method of successive approaches.

We put

$$F_0^{\pm}(q, c, r) = \exp(\pm i\alpha cr),$$

$$\begin{aligned} F_{m+1}^{\pm}(q, c, r) = &-\int_r^\infty \gamma(q, c, r)\sin\alpha c(r-t)V(t; E_q)F_m^{\pm}(q, c, t)dt - \\ &- \int_r^\infty \gamma(q, c, N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q, c, N)\exp(2k\pi c(r-t))V(t; E_q)F_m^{\pm}(q, c, t)dt + \\ &+ \int_0^r \gamma(q, c, N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q, c, N)\exp(-2k\pi c(r-t))V(t; E_q)F_m^{\pm}(q, c, t)dt \end{aligned}$$

It is easy to be convinced of the justice of the following appraisals

$$|F_0^{\pm}(q, c, r)| = 1$$

$$|F_1^{\pm}(q, c, r)| \le \begin{cases} \gamma(q, c, N)\,(3M+2)\int_0^r |V(t; E_q)|\,dt, & r \ge a \\ \gamma(q, c, N)(3M+2)\int_r^\infty |V(t; E_q)|\,dt, & r \le a, \end{cases}$$

where $M = \sum_{k=1}^{N} \omega_k(q, c, N)$.

Let us assume that for any

$$|F_m^{\pm}(q, c, r)| \le \begin{cases} \dfrac{\gamma^m(q, c, N)\,(3M+2)^m}{m!}\left[\int_0^r |V(t; E_q)|\,dt\right]^m & r \ge a, \\[2ex] \dfrac{\gamma^m(q, c, N)\,(3M+2)^m}{m!}\left[\int_r^\infty |V(t; E_q)|\,dt\right] & r \le a. \end{cases} \tag{8}$$

Then for $r \ge a$ we have

$$\begin{aligned} |F_{m+1}^{\pm}(q, c, r)| \le \frac{\gamma^{m+1}(q, c, N)\,(3M+2)^m}{m!}\Bigg\{ &\int_r^\infty |V(t; E_q)|\left[\int_0^t |V(s; E_q)|\,ds\right]^m dt + \\ &+ M\left(\int_r^\infty |V(t; E_q)|\left[\int_0^t |V(s; E_q)|\,ds\right]^m dt\right) + \\ &+ M\left(\int_0^r |V(t; E_q)|\left[\int_t^\infty |V(s; E_q)|\,ds\right]^m dt + \int_0^r |V(t; E_q)|\left[\int_0^t |V(s; E_q)|\,ds\right]^m\right)\Bigg\} \le \\ &\le \frac{\gamma^{m+1}(q, c, N)\,(3M+2)^{m+1}}{(m+1)!}, \end{aligned}$$

since $\int_0^r |V(t; E_q)|\,dt \ge \int_r^\infty |V(t; E_q)|\,dt$ for $r \ge a$.

For $r \le a$ we get

$$|F_{m+1}^{\pm}(q, c, r)| \le \frac{\gamma^{m+1}(q, c, N)(3M+2)^m}{m!}\left\{\int_r^a |V(t; E_q)|\left[\int_t^\infty |V(s; E_q)|\,ds\right]^m dt + \right.$$

Proof. We mark the two first columns of matrix $A(q, N)$ in $\vec{h}(+i\alpha cr)$. Suppose further

$$U_0(q, c, N) = \begin{bmatrix} A_0(q, c, N)c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$U_1(q, c, N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_1(q, c, N)c & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$U_2(q, c, N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_2(q, c, N)c \end{bmatrix}.$$

Then $A(q, c, N) = U_0(q, c, N) + U_1(q, c, N) + U_2(q, c, N)$.

With the help of direct check-up it is easy to be convinced of that the solutions of integral equations

$$\begin{aligned} \vec{x}^{\pm}(q, r) = \vec{h}(\pm i\alpha cr) &- \int_r^\infty \exp(U_0(q, c, N)(r-t)B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)dt - \\ &- \int_r^\infty \exp(U_1(q, c, N)(r-t)B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)dt + \\ &+ \int_0^\infty \exp(U_2(q, c, N)(r-t))B(q, c, N)V(t; E_q)\vec{x}^{\pm}(q, t)dt \end{aligned} \tag{7}$$

are the solutions of equation (3a). Substituting (7) into (3b) and putting $\sum_{k=1}^{n} x_k(q, r) = F^{\pm}(q, c, r)$ we receive (6).

Let a be such value of r that satisfies the equality

$$\int_0^a |V(r; E_q)| dr = \int_a^\infty |V(r; E_q)| dr.$$

Now we prove that equations (6) have the solutions which can be found by the method of successive approaches.

We put

$$F_0^{\pm}(q, c, r) = \exp(\pm i\alpha cr),$$

$$\begin{aligned} F_{m+1}^{\pm}(q, c, r) = &- \int_r^\infty \gamma(q, c, r)\sin\alpha c(r-t)V(t; E_q)F_m^{\pm}(q, c, t)dt - \\ &- \int_r^\infty \gamma(q, c, N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q, c, N)\exp(2k\pi c(r-t))V(t; E_q)F_m^{\pm}(q, c, t)dt + \\ &+ \int_0^r \gamma(q, c, N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q, c, N)\exp(-2k\pi c(r-t))V(t; E_q)F_m^{\pm}(q, c, t)dt \end{aligned}$$

It is easy to be convinced of the justice of the following appraisals

$$|F_0^{\pm}(q, c, r)| = 1$$

$$|F_1^{\pm}(q, c, r)| \le \begin{cases} \gamma(q, c, N)(3M+2)\int_0^r |V(t; E_q)| dt, & r \ge a \\ \gamma(q, c, N)(3M+2)\int_r^\infty |V(t; E_q)| dt, & r \le a, \end{cases}$$

where $M = \sum_{k=1}^{N} \omega_k(q, c, N)$.

Let us assume that for any

$$|F_m^{\pm}(q, c, r)| \le \begin{cases} \dfrac{\gamma^m(q, c, N)(3M+2)^m}{m!}\left[\int_0^r |V(t; E_q)| dt\right]^m & r \ge a, \\ \dfrac{\gamma^m(q, c, N)(3M+2)^m}{m!}\left[\int_r^\infty |V(t; E_q)| dt\right] & r \le a. \end{cases} \tag{8}$$

Then for $r \ge a$ we have

$$\begin{aligned} |F_{m+1}^{\pm}(q, c, r)| \le \frac{\gamma^{m+1}(q, c, N)(3M+2)^m}{m!}\Big\{ &\int_r^\infty |V(t; E_q)| \left[\int_0^t |V(s; E_q)| ds\right]^m dt + \\ &+ M\left(\int_r^\infty |V(t; E_q)| \left[\int_0^t |V(s; E_q)| ds\right]^m dt\right) + \\ &+ M\left(\int_0^r |V(t; E_q)| \left[\int_t^\infty |V(s; E_q)| ds\right]^m dt + \int_0^r |V(t; E_q)| \left[\int_0^t |V(s; E_q)| ds\right]^m\right)\Big\} \le \\ \le \frac{\gamma^{m+1}(q, c, N)(3M+2)^{m+1}}{(m+1)!}, \end{aligned}$$

since $\int_0^r |V(t; E_q)| dt \ge \int_r^\infty |V(t; E_q)| dt$ for $r \ge a$.

For $r \le a$ we get

$$|F_{m+1}^{\pm}(q, c, r)| \le \frac{\gamma^{m+1}(q, c, N)(3M+2)^m}{m!}\Big\{\int_r^a |V(t; E_q)| \left[\int_t^\infty |V(s; E_q)| ds\right]^m dt +$$

$$+\int_r^\infty |V(t;E_q)|[\int_0^t |V(s;E_q)|ds]^m dt + M(\int_r^a |V(t;E_q)|[\int_t^\infty V(s;E_q)|ds]^m dt +$$

$$+\int_r^\infty |V(t,E_q)|[\int_0^t |V(s,E_q)|ds]^m dt + M(\int_0^r |V(t;E_q)|[\int_t^\infty |V(s;E_q)|ds]^m dt)\Big\} \le$$

$$\le \frac{\gamma^{m+1}(q,c,N)(3M+2)^{m+1}}{(m+1)!}[\int_r^\infty |V(t;E_q)|dt]^{m+1},$$

since $\int_0^r |V(t;E_q)dt \le \int_r^\infty |V(t;E_q)|dt$ for $r \le a$.

So by inductive supposition appraisal (8) is satisfied for every natural number m and series

$$F^{\pm}(q,c,r) = \sum_{m=0}^{\infty} F^{\pm}(q,c,r)$$

uniformly converges on the semi-axis $[0,\infty)$, and their sums $F^{\pm}(q,c,r)$ are the solutions of equations (6).

As $c\to\infty$ from (4) we have

$$\lim_{c\to\infty} \gamma(q,c,N) = \lim_{c\to\infty} \frac{\alpha c}{2(c\sqrt{q^2+c^2}-c^2)} \prod_{k=1}^{N} \frac{(4k^2\pi^2+\alpha^2)^2}{16k^2\pi^2(k^2\pi^2+\alpha^2)} = \frac{1}{q}.$$

As a result integral equations (6) have the form

$$\lim_{c\to\infty} F^{\pm}(q,c,r) = E^{\pm}(q,r) = \exp(\pm iqr) - \int_r^\infty \frac{\sin q(r-t)}{q} V(t;E_q)E^{\pm}(q,t)dt.$$

Q.E.D.

Theorem 2. Suppose potential $V(r;E_q)$ satisfies condition (5). Then the solutions of integral equations

$$F^{\pm}(q,c,r) = \exp(\pm i\alpha cr) - \int_r^\infty \gamma(q,c)\sin\alpha c(r-t)V(t;E_q)F^{\pm}(q,c,t)dt -$$

$$+\int_r^\infty \gamma(q,c)\sin\alpha c(r-t)\sum_{k=1}^{\infty}\omega_k(q,c)\exp(2k\pi c(r-t))V(t;E_q)F^{\pm}(q,c,t)dt$$

$$+\int_0^r \gamma(q,c)\sin\alpha c(r-t)\sum_{k=1}^{\infty}\omega_k(q,c)\exp(-2k\pi c(r-t))V(t;E_q)F^{\pm}(q,c,t)dt,$$

where $\gamma(q,c) = \lim_{N\to\infty}\gamma(q,c,N)$, $\omega_k(q,c) = \lim_{N\to\infty}\omega_k(q,c,N)$, exist and are the solutions of equation (1) degenerating to Jost solutions of equation (2). Moreover integral equations (9) are solved by the method of successive approaches.

Proof. When $N\to\infty$ we have

$$\lim_{N\to\infty}\gamma(q,c,N) = \frac{\alpha}{2(\sqrt{q^2+c^2}-c)}\prod_{k=1}^{\infty}\frac{4k^2\pi^2+\alpha^2}{16k^2\pi^2(k^2\pi^2+\alpha^2)} = \frac{\alpha}{2(\sqrt{q^2+c^2}-c)}\prod_{k=1}^{\infty}\left(1-\frac{8k^2\pi^2\alpha^2-\alpha^4}{16k^4\pi^4+16k^2\pi^2\alpha^2}\right).$$

It is easy to verify that

$$\sum_{k=1}^{\infty}\left|\frac{8k^2\pi^2\alpha^2-\alpha^4}{16k^4\pi^4+16k^2\pi^2\alpha^2}\right| \le \frac{\alpha^2}{12}. \qquad (9)$$

Hence it follows that there exists the limit $\gamma(q,c,N)$ as $N\to\infty$.

It is obvious that $|\omega_k(q,c,N)| \le 1$ for any positive integer N. Moreover when $k\to\infty$ we have

$$\lim_{\substack{N\to\infty\\k\to\infty}}\omega(q,c,N) < \frac{p}{k^2},$$

where $p = \text{const}$. Hence $M = \sum_{k=1}^{\infty}\omega_k(q,c) < \infty$.

Further reasoning completely repeats the proof of theorem 1. Q.E.D.

Theorem 3. Suppose potential $V(r;E_q)$ satisfies condition

$$\int_0^\infty r|V(r;E_q)|dr < \infty \qquad (10)$$

then for any $n > 0$ the solutions of integral equations

$$F^{\pm}(q,c,r,\eta) = \exp(\pm i\alpha cr) - \int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)V(t;E_q)F^{\pm}(q,c,t,\eta)dt -$$

$$-\int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(2k\pi c(r-t))V(t;E_q)F^{\pm}(q,c,t,\eta)dt +$$

$$+\int_\eta^r \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(-2k\pi c(r-t))V(t;E_q)F^{\pm}(q,c,t,\eta)dt \qquad (11)$$

exist and are the solutions of system of equations (3) on the semi-axis $[0,\infty)$. Functions

$$F^{\pm}(q,c,r) = \lim_{\eta\to 0} F^{\pm}(q,c,r,\eta) \qquad (12)$$

also are the solutions of system (3) degenerating the Jost solutions of equation (2).

This statement remains just also in the limit $N \to \infty$.

Proof. From (10) and theorem 1 it follows that for any $\eta>0$ integral equations (11) are solved by the method of successive approaches on $[0,\infty)$. We consider the interval $0 \le r \le \eta$.

Suppose $F_0^\pm(q,c,r,\ ) = \exp(\pm i\alpha cr)$, and $F^\pm_{m+1}(q,c,r,\eta)$ is received with the help of substitution $F^\pm_m(q,c,t,q)$ instead of $F^\pm(q,c,t,\eta)$ into the right part of equation (11). Then we have the appraisals:

$$|F^\pm(q,c,r,\eta)| = 1$$

$$|F^\pm(q,c,r,\eta)| \le \alpha c\gamma(q,c,N)\{\int_r^\infty t|V(t;E\ )|\,dt + M\int_r^\infty t|V(t;E_q)|\,dt + M\int_r^\eta t\,t|V(t;E_q)|\,dt\},$$

where $M = \sum_{k=1}^{N} \omega_k(q,c,N)\exp(2k\pi c\eta)$.

We choose η such that

$$\int_r^\eta t|V(t;E_q)|\,dt \le \int_\eta^\infty t|V(t;E_q)\,dt.$$

In consequences we have

$$|F_1^\pm(q,c,r,\eta)| \le \alpha c\gamma(q,c,N)(2M+1)[\int_r^\infty t|V(t;E_q)|\,dt].$$

We propose further that for any number m we have the appraisal

$$|F_m^\pm(q,c,r,\eta)| \le \frac{(\alpha c\gamma(q,c,N)(2M+1))^m}{m!}[\int_r^\infty |V(t;E_q)|\,dt]^m. \qquad (13)$$

Then

$$|F^\pm_{m+1}(q,c,r,\eta)| \le \frac{(\alpha c\gamma(q,c,N)(2M+1))^m}{m!}\alpha c\gamma(q,c,N)\{\int_r^\infty t|V(t;E_q)| \times [\int_t^\infty s|V(s;E_q)|\,ds]^m dt + M\int_r^\infty t|V(t;E_q)|[\int_t^\infty s|V(s;E_q)|\,ds]^m dt + M\int_r^\eta t|V(t;E_q)| \times [\int_t^\infty s|V(s;E_q)|\,ds]\,dt \le \frac{(\alpha c\gamma(q,c,N)(2M+1))^{m+1}}{(m+1)!}[\int_r^\infty t|V(t;E_q)|\,dt]^{m+1}.$$

Thus appraisal (13) is satisfied for every m and integral equations (11) are found by the method of successive approaches

on $[0,\infty)$. Moreover for $r \le \eta$ we have

$$|F^\pm(q,c,r,\eta)| < \exp\{\alpha c\gamma(q,c,N)(2M+1))[\int_r^\infty t|V(t;E_q)|\,dt\}.$$

The appraisal does not depend on η. Consequently there exists the limit $F^\pm(q,c,0) = \lim F^\pm(q,c,0,\eta)$. Hence it follows that correlation (12) is satisfied in some neighbourhood of the coordinate beginning $r = 0$.

Let a be a number of the neighbourhood, i.e., $F^\pm(q,c,a) = \lim_{\eta\to 0} F^\pm(q,c,a,\eta)$. We consider the integral equations

$$\bar F^\pm(q,c,N) = \exp(\pm i\alpha cr)(1 + p\exp(-2\pi cr)) - \int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)V(t;E_q)\bar F^\pm(q,c,t)\,dt - \int_r^\infty \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(2k\pi c(r-t))V(t;E_q)\bar F^\pm(q,c,t)\,dt + \int_a^r \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(-2k\pi c(r-t)V(t;E_q)\bar F^\pm(q,c,t)\,dt,$$

where p is an arbitrary number.

It is easy to be convinced (compare with the proof of theorem 2) of that for any number p the equations (14) can be solved by the method of successive approaches and their solutions satisfy the system of equations (3). Now we choose number p such that it satisfies the equality

$$\bar F^\pm(q,c,a) = \exp(\pm i\alpha ca)(1 + p\exp(-2\pi ca)) - \int_a^\infty \gamma(q,c,N)\sin\alpha c(r-t)V(t;E_q)\bar F^\pm(q,c,t)\,dt - \int_a^\infty \gamma(q,c,N)\sin\alpha c(r-t)\sum_{k=1}^{N}\omega_k(q,c,N)\exp(2k\pi c(r-t))V(t;E_q)\bar F^\pm(q,c,t)\,dt.$$

Then the solutions of equations (14) can be considered as the continuation of $F^\pm(q,c,r)$.

From equations (11) and (14) and theorem 1 it follows that when $c \to \infty$ the solutions $F^\pm(q,c,r)$ satisfy the equations

$$\lim_{c\to\infty} F^\pm(q,c,r) = E^\pm(q,r) = \exp(\pm iqr) - \int_r^\infty \frac{\sin q(r-t)}{q}V(t;E_q)E^\pm(q,t)\,dt.$$

To put the other way round solutions $F^\pm(q,c,r)$ degenerate to Jost solutions of equation (2).

Finally the passage to limit $N \to \infty$ is accomplished just as in proof of theorem 2.

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О нерелятивистском пределе решений
квазипотенциальных радиальных уравнений

Для случая S-волны найдены решения релятивистского квазипотенциального радиального уравнения

$$[2c\sqrt{q^2+m^2c^2} - 2mc^2 \mathrm{ch}(\frac{i\hbar}{mc}\frac{d}{dr}) - V(r;E_q)]\psi(q,r) = 0,$$

которые при $c \to \infty$ переходят в решения Йоста нерелятивистского радиального уравнения Шредингера

$$[\frac{d^2}{dr^2} - V(r;E_q) + q^2]\psi(q,r) = 0.$$

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Vu Xuan Minh, Zhidkov E.P., Kadyshevskij V.G. E5-85-394
On the Nonrelativistic Limit of Solutions
of Quasipotential Radial Equations

The solutions of the relativistic quasipotential radial equation

$$[2c\sqrt{q^2+m^2c^2} - 2mc^2 \mathrm{ch}(\frac{i\hbar}{mc}\frac{d}{dr}) - V(r;E_q)]\psi(q,r) = 0,$$

which, as $c \to \infty$, turn into Jost solutions of Schrödinger nonrelativistic radial equation

$$[\frac{d^2}{dr^2} - V(r;E_q) + q^2]\psi(q,r) = 0$$

are found for the case of S-wave.

The investigation has been performed at the Laboratory of Computing Techniques and Automation.

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