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E5-85-39

N.E.Mazepa, S.I.Serdyukova

THE STABILITY INVESTIGATION OF SOME DIFFERENCE BOUNDARY PROBLEM WITH THE APPLICATION OF SYMBOLIC COMPUTATION SYSTEM

1985

The fluxon motion in the long system with microinhomogeneities is described[1] by the equation

$$\phi_{tt} = \phi_{xx} - (1 - \mu\delta(x - x_0))\sin\phi - a\phi_t, \quad -\ell \le x \le \ell.$$

The initial and boundary conditions are given

$$\phi(x, 0) = f(x), \quad \phi_t(x, 0) = g(x), \quad \phi_x(-\ell, t) = \phi_x(\ell, t) = 0.$$

Changing variables $u = \phi_x$, $v = \phi_t$ we get the system

$$u_t = v_x, \quad v_t = u_x - (1 - \mu\delta(x - x_0)) \sin(f(x) + \int_0^t v(x,t)\,dt) - av,$$

$$u(x, 0) = f'(x), \quad v(x, 0) = g(x), \quad u(-\ell, t) = u(\ell, t) = 0.$$

The problem is solved numerically. We use the Rusanov[2] scheme of the third order accuracy. The lacking values u, v in points at the limit are found by using the scheme of the second order accuracy. The computing formulas are given below. The strong exponential instability near the boundary was observed in computations. We change the computing formulas near the boundary by the Lax-Wendroff scheme. The strong oscillations near the boundary have disappeared. We would like to exclude such a type of instability in advance. In principle, the question of the linear difference boundary problem stability had been solved in[3,4]. But the verification of the stability criterion is a laborious work. We try to use the symbolic computation system REDUCE[5]. However all our efforts in this way were unavailing until we simplified essentially the problem by applying the algebraic methods. As usual we consider more simple model problem to investigate the stability. The computation shows that the same instability phenomenon is observed for the wave equation. The corresponding difference problem is

$$\frac{u^{(1)}_{\nu+(1/2)} - \frac{u^n_{\nu+1} + u^n_\nu}{2}}{(\tau/3)} = \frac{v^n_{\nu+1} - v^n_\nu}{h}, \qquad \frac{v^{(1)}_{\nu+(1/2)} - \frac{v^n_{\nu+1} + v^n_\nu}{2}}{(\tau/3)} = \frac{u^n_{\nu+1} - u^n_\nu}{h};$$

$$\nu \ge 1$$

$$\frac{u_\nu^{(2)}-u_\nu^{n}}{(2\tau/3)} = \frac{v^{(1)}_{\nu+(1/2)} - v^{(1)}_{\nu-(1/2)}}{h}, \quad \frac{v_\nu^{(2)}-v_\nu^{n}}{(2\tau/3)} = \frac{u^{(1)}_{\nu+(1/2)} - u^{(1)}_{\nu-(1/2)}}{h} \qquad \nu \ge 2$$

Here τ, h are the net steps in t, x correspondingly, $u_1^{(2)} = 0$, $v_1^{(2)}$ is found from approximation of $u_t = v_x$ using four points at the limit

$$\frac{\dfrac{u_1^{(2)}+u_2^{(2)}}{2} - \dfrac{u_1^{n}+u_2^{n}}{2}}{(2\tau/3)} = \frac{\dfrac{v_2^{(2)}+v_2^{n}}{2} - \dfrac{v_1^{(2)}+v_1^{n}}{2}}{h}.$$

Let $\alpha = \tau/h$. Then for $\nu \ge 3$

$$u_\nu^{n+1} = u_\nu^n - \frac{1}{12}(u^n_{\nu+2} - 4u^n_{\nu+1} + 6u^n_\nu - 4u^n_{\nu-1} + u^n_{\nu-2}) +$$

$$+ \frac{\alpha}{24}(-2v^n_{\nu+2} + 7v^n_{\nu+1} - 7v^n_{\nu-1} + 2v^n_{\nu-2}) + \frac{3\alpha}{8}(v^{(2)}_{\nu+1} - v^{(2)}_{\nu-1}),$$

$$v_\nu^{n+1} = v_\nu^n - \frac{1}{12}(v^n_{\nu+2} - 4v^n_{\nu+1} + 6v^n_\nu - 4v^n_{\nu-1} + v^n_{\nu-2}) +$$

$$+ \frac{\alpha}{24}(-2u^n_{\nu+2} + 7u^n_{\nu+1} - 7u^n_{\nu-1} + 2u^n_{\nu-2}) + \frac{3\alpha}{8}(u^{(2)}_{\nu+1} - u^{(2)}_{\nu-1}).$$

The initial data u_ν^0, v_ν^0 are given. Let u_ν^n, v_ν^n are known. The computation of u_ν^{n+1}, v_ν^{n+1} passes three steps. On the layer $t = (n+1/3)\tau$ the values u, v are computed in the half-integer points by Lax scheme. On the following layer $t = (n+2/3)\tau$ the values u, v are computed in the integer points by the cross scheme. The lacking $v_1^{(2)}$ is found from the approximation $u_t = v_x$ using four points at the limit. At last the values u, v on the $(n+1)$-th layer are found by the formulae above providing the third order accuracy /2/. The lacking u_2^{n+1}, v_2^{n+1} are found from the second order accuracy approximation:

$$u_2^{n+1} = u_2^n + \frac{\alpha}{8}(v_3^n - v_1^n) + \frac{3\alpha}{8}(v_3^{(2)} - v_1^{(2)}),$$

$$v_2^{n+1} = v_2^n + \frac{\alpha}{8}(u_3^n - u_1^n) + \frac{3\alpha}{8}(u_3^{(2)} - u_1^{(2)}). \tag{1}$$

v_1^{n+1} is found from the approximation of $u_t = u_x$ using four points at the limit

$$\frac{u_2^{n+1}+u_1^{n+1}}{2} - \frac{u_2^{n}+u_1^{n}}{2} = \alpha\left(\frac{v_2^{n+1}+v_2^{n}}{2} - \frac{v_1^{n+1}+v_1^{n}}{2}\right).$$

This computational algorithm is called below "case I". In "case II" the Lax-Wendroff approximation is used instead of (1):

$$u_2^{n+1} = u_2^n + \frac{\alpha}{2}(v_3^n - v_1^n) + \frac{\alpha^2}{2}(u_3^n - 2u_2^n + u_1^n),$$

$$v_2^{n+1} = v_2^n + \frac{\alpha}{2}(u_3^n - u_1^n) + \frac{\alpha^2}{2}(v_3^n - 2v_2^n + v_1^n).$$

Let G be the operator of transition from layer to layer. In this work we calculate the spectra of the operator G in both "cases". In "case I" we found the spectrum point outside the unit circle $z^* = -1{,}063\ldots$. It causes the strong instability, which was observed in computations. In "case II" there is a unique spectrum point $z^* = 0$. In this case the initial boundary value problem is stable.

We begin to state the algebraic problem defining the spectrum of the operator G. For $\nu \ge 3$ the considering difference operator is

$$w_\nu^{n+1} = \sum_{\ell=-2}^{2} A_\ell w^n_{\nu+\ell}, \quad w_\nu^n = \begin{pmatrix} u_\nu^n \\ v_\nu^n \end{pmatrix},$$

$$A_2 = \begin{Vmatrix} -\frac{1}{12} + \frac{\alpha^2}{8} & -\frac{\alpha}{12} + \frac{\alpha^3}{12} \\ -\frac{\alpha}{12} + \frac{\alpha^3}{12} & -\frac{1}{12} + \frac{\alpha^2}{8} \end{Vmatrix}, \quad A_1 = \begin{Vmatrix} \frac{1}{3} & \frac{2\alpha}{3} - \frac{\alpha^3}{6} \\ \frac{2\alpha}{3} - \frac{\alpha^3}{6} & \frac{1}{3} \end{Vmatrix},$$

$$A_0 = \begin{Vmatrix} \frac{1}{2} - \frac{\alpha^2}{4} & 0 \\ 0 & \frac{1}{2} - \frac{\alpha^2}{4} \end{Vmatrix},$$

$$A_{-1} = \begin{Vmatrix} \frac{1}{3} & -\frac{2\alpha}{3} + \frac{\alpha^3}{6} \\ -\frac{2\alpha}{3} + \frac{\alpha^3}{6} & \frac{1}{3} \end{Vmatrix}, \quad A_{-2} = \begin{Vmatrix} -\frac{1}{12} + \frac{\alpha^2}{8} & \frac{\alpha}{12} - \frac{\alpha^3}{12} \\ \frac{\alpha}{12} + \frac{\alpha^3}{12} & -\frac{1}{12} + \frac{\alpha^2}{8} \end{Vmatrix}.$$

Let D is characteristic matrix

$$D(\ell^{i\phi}) = \sum_{\ell=-2}^{2} A_\ell \cdot e^{i\ell\phi}. \tag{2}$$

If $\alpha \leq 1$, the corresponding Cauchy problem is stable in L_2: D is transformed to the diagonal matrix and its eigenvalues satisfy the relation $|\lambda(\ell^{i\phi})| \leq 1$. Moreover the Cauchy problem is stable in C [6,7]. Then for $|z| > 1$ the characteristic equation

$$\mathrm{Det} \left\| \sum_{\ell=-2}^{2} A_\ell \kappa^\ell - zI \right\| = 0 \qquad (3)$$

has no solution equal to 1 in absolute value: $|\kappa_i(z)| \neq 1$, $|z| > 1$, $i = 1,\ldots,8$. For $|z| \geq 1$ the solutions are separated on two nonintersecting sets $|\kappa_i| \leq 1$ and $|\kappa_i| \geq 1$. As $|\kappa_i(z)| \neq 1$, $|z| > 1$, the eigenvalues can't go from one set to another. If for some z_0, $|z_0| > 1$, $|\kappa_i(z_0)| < 1$, then $|\kappa_i(z)| < 1$ for all z outside the unit circle. The asymptotic behaviour at infinity [3] shows that exactly four $|\kappa_i| < 1$ and exactly four $|\kappa_i| > 1$. For $|z| \geq 1$ there is determined [8] nonsingular analytical similarity transformation T(z), which reduces the resolvent matrix

$$M(z) = \left\| \begin{matrix} -A_2^{-1} \cdot A_1 & -A_2^{-1} \cdot (A_0 - zI) & -A_2^{-1} \cdot A_{-1} & -A_2^{-1} \cdot A_{-2} \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{matrix} \right\|$$

to the block form. The eigenvalues $|\kappa_i| \leq 1$ correspond to the upper block; and $|\kappa_i| \geq 1$ to the lower block. In our computation $\alpha = 1/2$. The spectra are calculated for $\alpha = 1/2$. The characteristic equation (3) has the form $\mathfrak{A}^2(\kappa) = \mathfrak{B}^2(\kappa)$:

$$[-5(\kappa^2 + \kappa^{-2}) + 32(\kappa + \kappa^{-1}) + 42 - 96z]^2 = [-3(\kappa^2 - \kappa^{-2}) + 30(\kappa - \kappa^{-1})]^2 \qquad (4)$$

The equation (4) is evidently equivalent to two equations

$$\mathfrak{A}(\kappa) = \mathfrak{B}(\kappa), \quad \mathfrak{A}(\kappa) = -\mathfrak{B}(\kappa).$$

We transform these equations to the form

$$\kappa^4 - \kappa^3 - (21 - 48z)\kappa^2 - 31\kappa + 4 = 0, \qquad (5)$$

$$4\kappa^4 - 31\kappa^3 - (21 - 48z)\kappa^2 - \kappa + 1 = 0. \qquad (6)$$

The eigenvector associated with $\kappa(z)$ is

$$\mathcal{E} = (\kappa^3 \cdot E(\kappa),\ \kappa^2 \cdot E(\kappa),\ \kappa \cdot E(\kappa),\ E(\kappa))^T,$$

where

$$E(\kappa) = \left\| \begin{matrix} \frac{1}{32}(\kappa^2 - \kappa^{-2}) - \frac{5}{16}(\kappa - \kappa^{-1}) \\ -\frac{5}{96}(\kappa^2 + \kappa^{-2}) + \frac{1}{3}(\kappa + \kappa^{-1}) + \frac{7}{16} - z \end{matrix} \right\| .$$

If for some z there are no multiple eigenvalues, then $\mathcal{E}_i(z) = \mathcal{E}(\kappa_i(z))$ are linearly independent and

$$T^{-1}(z) = \| \mathcal{E}_1(z), \mathcal{E}_2(z), \ldots, \mathcal{E}_8(z) \| .$$

Remind that $|\kappa_i(z)| < 1$, $|z| > 1$, $i = 1,\ldots,4$. Consider matrices

$$T_{11}^{-} = \left\| \begin{matrix} \kappa_1^3 \cdot E_1 & \kappa_2^3 \cdot E_2 & \kappa_3^3 \cdot E_3 & \kappa_4^3 \cdot E_4 \\ \kappa_1^2 \cdot E_1 & \kappa_2^2 \cdot E_2 & \kappa_3^2 \cdot E_3 & \kappa_4^2 \cdot E_4 \end{matrix} \right\| ,$$

$$T_{21}^{-} = \left\| \begin{matrix} \kappa_1 \cdot E_1 & \kappa_2 \cdot E_2 & \kappa_3 \cdot E_3 & \kappa_4 \cdot E_4 \\ E_1 & E_2 & E_3 & E_4 \end{matrix} \right\| .$$

"Case I" is connected with the boundary matrices

$$A = \left\| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\alpha} & -1 & 0 & 1 \end{matrix} \right\| ,$$

$$B = \left\| \begin{matrix} \frac{\alpha^2}{8} & \frac{\alpha^3}{12} & 0 & \frac{11\alpha}{16} - \frac{\alpha^3}{4} \\ \frac{\alpha^3}{12} & \frac{\alpha^2}{8} & \frac{\alpha}{2} - \frac{\alpha^3}{6} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \right\| ,$$

$$C = \left\| \begin{matrix} 1-\frac{3\alpha^2}{8} & -\frac{3\alpha}{4}+\frac{\alpha^3}{4} & \frac{\alpha^2}{4} & \frac{\alpha}{16}-\frac{\alpha^3}{12} \\ \frac{\alpha^3}{12} & 1-\frac{\alpha^2}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{\alpha} & 1 & 0 & -1 \end{matrix} \right\|$$

"Case II" is connected with the matrices

$$A = \left\| \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\alpha} & -1 & \frac{1}{\alpha} & 1 \end{matrix} \right\|, \quad B = \left\| \begin{matrix} 0 & 0 & \frac{\alpha^2}{2} & \frac{\alpha}{2} \\ 0 & 0 & \frac{\alpha}{2} & \frac{\alpha^2}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \right\|,$$

$$C = \left\| \begin{matrix} 1-\alpha^2 & 0 & \frac{\alpha^2}{2} & -\frac{\alpha}{2} \\ 0 & 1-\alpha^2 & -\frac{\alpha}{2} & \frac{\alpha^2}{2} \\ 0 & 0 & 0 & 0 \\ \frac{1}{\alpha} & 1 & \frac{1}{\alpha} & -1 \end{matrix} \right\|.$$

If for some z there are no multiple eigenvalues, then z is spectrum point if and only if [3]

$$\operatorname{Det} \| B T_{11}^{-}(z) + (C - zA) T_{21}^{-}(z) \| = 0. \qquad (7)$$

In ptinciple, there is the direct way to solve the spectrum problem. κ_i are the solutions of the fourth order polynomials (5), (6) with the parameter z. Using the Ferrary method [9] we can calculate $\kappa_i(z)$, $E_i(z)$, and Det(z) in (7). We get the algebraic equation with radicals of the second and third orders. Eliminated from radicals we get the polynomial equation. We tried to realise this algorithm on the computer EC-1060 without success. Then we simplified $E(\kappa)$. For $|z| > 1$ the equation (5) has exactly two solutions $|\kappa_i| < 1$, the equation (6) does the same. Let κ_1, κ_2 are the solutions of (5); and κ_3, κ_4, the solutions of (6). Notice, that $E(\kappa) = \| -\mathcal{A}/96, \ \mathcal{B}/96 \|^T$. So we can change E_1, E_2 by $\binom{1}{-1}$ and E_3, E_4 by $\binom{1}{1}$. But even such simplification gives no real possibility of calculating the polynomial equation. Let us simplify the problem further by the algebraic way. Notice that if κ_3, κ_4 are the solutions of (6), then $\kappa_3^* = \kappa_3^{-1}$, $\kappa_4^* = \kappa_4^{-1}$ are the solutions of (5). By analogy κ_1^*, κ_2^* are the solutions of (6). The following relations are true

$$\kappa_1 + \kappa_2 + \kappa_3^* + \kappa_4^* = 1, \quad \kappa_1^* + \kappa_2^* + \kappa_3 + \kappa_4 = \frac{31}{4},$$
$$\kappa_1\kappa_2\kappa_3^*\kappa_4^* = 4, \quad 48z - 21 = \kappa_1\kappa_2 + (\kappa_1 + \kappa_2)(\kappa_3^* + \kappa_4^*) + \kappa_3^*\kappa_4^*. \qquad (8)$$

We have for $|z| \geq 1$

$$\operatorname{Det} R = \operatorname{Det} \| B T_{11}^{-} + (C - zA) T_{21}^{-} \| = 96^{-4} (\kappa_1\kappa_2\kappa_3\kappa_4)^2 \operatorname{Det} R',$$

where $R' = \| R_1', R_2', R_3', R_4' \|$ with

$$R_i' \underset{(i=1,2)}{=} \left\| \begin{matrix} 2(-48z\kappa_i^* + 2\kappa_i^{*2} + 60\kappa_i^* + \kappa_i - 15) \\ 2(48z\kappa_i^* - 46\kappa_i^* - \kappa_i + 11) \\ -96z\kappa_i^{*2} \\ 96\kappa_i^*(z\kappa_i^* - 3z + \kappa_i^* + 1) \end{matrix} \right\|,$$

$$R_i' \underset{(i=3,4)}{=} \left\| \begin{matrix} 2(-48z\kappa_i^* + 4\kappa_i^{*2} + 27\kappa_i^* + 2\kappa_i + 15) \\ 2(-48z\kappa_i^* + 47\kappa_i^* + 2\kappa_i + 11) \\ -96z\kappa_i^{*2} \\ 96\kappa_i^*(-z\kappa_i^* - z - \kappa_i^* + 3) \end{matrix} \right\|$$

Let $x = \kappa_3^* + \kappa_4^*$, $y = \kappa_3 \cdot \kappa_4$. It follows from (8) that

$$x = \frac{1 - 31y}{1 - 4y^2}, \quad z = \frac{4y + x(1-x) + 1/y + 21}{48}.$$

The elementary transformation

$$R_i'' = (R_i' - R_{i-1}')/(\kappa_i^* - \kappa_{i-1}^*), \ i = 2, 4, \quad R_j'' = R_j' - R_{j+1}''\kappa_j^*, \ j = 1, 3,$$

reduces (7) to $z(\kappa_1\kappa_2\kappa_3\kappa_4)^2(\kappa_2^* - \kappa_1^*)(\kappa_4^* - \kappa_3^*) \operatorname{Det} R'' = 0$, where R'' is the function of x, y, z:

$$R'' = \left\| \begin{matrix} r''_{11} & (1-x) - \frac{1}{2y} - 15 & r''_{13} & 2xy - \frac{4}{y} + 15 \\ 48z - 46 + 4y & 11 - (1-x) & -48z + 47 - 2y & 11 + 2xy \\ \frac{31}{4} - xy & -\frac{1}{4y} & x & -\frac{1}{y} \\ r''_{41} & -\frac{(z+1)}{4y} & -zx - z - x + 3 & \frac{(z+1)}{y} \end{matrix} \right\| ,$$

$$r''_{11} = -48z + 2\left(\frac{31}{4} - xy\right) + 60 - 4y,$$

$$r''_{41} = z\left(\frac{31}{4} - xy\right) - 3z + \left(\frac{31}{4} - xy\right) + 1, \quad r''_{13} = -48z + 4x + 27 - 2y.$$

Using the REDUCE we calculate successively

$\mathrm{Det}\, R''(x, y, z)$, $\mathrm{Det}\, R''(x, y)$, $\mathrm{Det}\, R''(y)$:

$$\mathrm{Det}\, R''(y) = P(y)/Q(y), \quad P(y) = \sum_{i=0}^{18} a_i y^i = 0, \tag{9}$$

$(a_{18}, \ldots, a_0) = (-8978432, 330637312, -109246464,$

$1615520768, 1182309888, 3565248416, -5494303736,$

$-8523823072, 2527298746, -931032497, 224156783,$

$-12246666, 416032, 1760346, -603104, 62570, -2450, 23, 1).$

It is not difficult to understand that only $y = 0$, $y = \pm 1/2$ are the roots of $Q(y)$. $P(y)$ has no such roots. So we solve $P(y) = 0$ and have 18 points y_i, $z_i = z(y_i)$. As $y(z)$ is not one-valued function, we get the "false spectrum points": z_i, $|z_i| \geq 1$, is the spectrum point if and only if $y_i = \kappa_3(z_i) \cdot \kappa_4(z_i)$. Remind that κ_3, κ_4 are the solutions (6), $|\kappa_3| < 1$, $|\kappa_4| < 1$. So we found only one spectrum point in $|z| \geq 1$ $z^* = -1.063\ldots$. To be confident in this result we compute

$\mathrm{Det}\, R(-1{,}073) = 0{,}048986\ldots$, $\mathrm{Det}\, R(-1{,}053) = -0{,}048935\ldots$.

Note that $\mathrm{Det}\, R(z)$ is real or purely imaginary if z is real. The same equation (9) gives the spectrum points inside the unit circle $|z| < 1$ before we reach the range Ω, where (5) has only one solution κ_1 less than 1 in the absolute value. On the boundary Ω:

$x = (-10\cos 2\phi + 32\cos\phi + 26)/48$, $\quad y = (3\sin 2\phi - 30\sin\phi)/48$,

$-\pi \leq \phi \leq \pi$, $\kappa_2(x + iy) = e^{i\phi}$, -

the equation (9) is also true. We found no spectrum points on and inside the unit circle outside Ω. Inside Ω, we must take $E_2(z) = \binom{1}{1}$. Then we get $\mathrm{Det}\, R(z) = 96^{-4}\, \mathrm{Det}\, \| R_1, R_2, R_3, R_4 \|$,

$$R_1 = \left\| \begin{matrix} 2(-48z\kappa_1 + \kappa_1^3 - 15\kappa_1^2 + 60\kappa_1 + 2) \\ 2\kappa_1(48z - \kappa_1^2 + 11\kappa_1 - 46) \\ -96z \\ 96 \cdot (-3z\kappa_1 + z + \kappa_1 + 1) \end{matrix} \right\| ,$$

$$R_i = \left\| \begin{matrix} 2 \cdot (-48z\kappa_i + 2\kappa_i^3 + 15\kappa_i^2 + 27\kappa_i + 4) \\ 2\kappa_i(-48z + 2\kappa_i^2 + 11\kappa_i + 47) \\ -96z \\ 96 \cdot (-z\kappa_i - z + 3\kappa_i - 1) \end{matrix} \right\| \quad (i = 2, 3, 4)$$

By analogy with preceding we reduced the equation $\mathrm{Det}\, R = 0$ to $\mathrm{Det}\, R' = 0$, where R' is the function of $x = \kappa_1$ and z. We used the relations

$$\kappa_2 + \kappa_3 + \kappa_4 = \frac{31}{4} - \frac{1}{x}, \quad \kappa_2\kappa_3\kappa_4 = \frac{x}{4},$$

$$\kappa_2\kappa_3 + \kappa_2\kappa_4 + \kappa_3\kappa_4 = \frac{4(1-x)}{x}. \tag{10}$$

After a number of elementary transformations we have $\mathrm{Det}\, R'' = 0$, where

$$R'' = \left\| \begin{matrix} -48zx + x^3 - 13x^2 + 53x - 1 & 2 & 2\left(\frac{31}{4} - \frac{1}{x}\right) - 10 & 0 \\ 48zx - x^3 + 11x^2 - 46x & 2\left(\frac{31}{4} - \frac{1}{x}\right) + 11 & r''_{23} & \frac{x}{2} \\ 1 & 0 & 0 & 1 \\ (1 - 3z)x + 2z + 2 & 0 & 3 - z & 0 \end{matrix} \right\|$$

$$r''_{23} = -48z + 2\left(\frac{31}{4} - \frac{1}{x}\right)^2 - (1-x)\frac{x}{2} + 11\left(\frac{31}{4} - \frac{1}{x}\right) + 47.$$

From (10) and from the relation

$$\kappa_1(\kappa_2^* + \kappa_3^* + \kappa_4^*) + \kappa_2^*(\kappa_3^* + \kappa_4^*) + \kappa_3^*\kappa_4^* = -21 + 48z$$

we get $z = \frac{1}{48}\left(x(1-x) + \frac{4}{x}\left(\frac{31}{4} - \frac{1}{x}\right) + 21\right)$.

The determinant of $R''(x)$ is calculated by applying the REDUCE:

$$\mathrm{Det}\, R''(x) \equiv (-11x^9 + 80x^8 - 1544x^7 + 9067x^6 - 20695x^5 +$$

$$+ 22546x^4 - 12582x^3 + 3107x^2 - 368x + 16)/(4x^3).$$

We find 9 points x_i, $z_i = z(x_i)$. z_i is the spectrum point if $|x_i| < 1$ and $z_i \subset \Omega$. We find only one spectrum point inside Ω, $z^* = 0$. We did not consider yet "the multiple z"; for which the equation (5) has multiple solutions: $z_1 = 1.693...$, $z_{2,3} = 0.6649... \pm i0.2799...$, $z_4 = -0.0163...$. Notice that $(z_2, z_3, z_4) \subset \Omega$. When $z = z_1$ we choose

$$T^{-}_{11} = \left\| \begin{matrix} \kappa_1^3 & 3\kappa_1^2 & \kappa_3^3 & \kappa_4^3 \\ -\kappa_1^3 & -3\kappa_1^2 & \kappa_3^3 & \kappa_4^3 \\ \kappa_1^2 & 2\kappa_1 & \kappa_3^2 & \kappa_4^2 \\ -\kappa_1^2 & -2\kappa_1 & \kappa_3^2 & \kappa_4^2 \end{matrix} \right\|, \quad T^{-}_{21} = \left\| \begin{matrix} \kappa_1 & 1 & \kappa_3 & \kappa_4 \\ -\kappa_1 & -1 & \kappa_3 & \kappa_4 \\ 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 1 \end{matrix} \right\|$$

For $z = z_2, z_3, z_4$

$$T^{-}_{11} = \left\| \begin{matrix} \kappa_1^3 & \kappa_2^3 & \kappa_3^3 & 3\kappa_3^2 \\ -\kappa_1^3 & \kappa_2^3 & \kappa_3^3 & 3\kappa_3^2 \\ \kappa_1^2 & \kappa_2^2 & \kappa_3^2 & 2\kappa_3 \\ -\kappa_1^2 & \kappa_2^2 & \kappa_3^2 & 2\kappa_3 \end{matrix} \right\|, \quad T^{-}_{21} = \left\| \begin{matrix} \kappa_1 & \kappa_2 & \kappa_3 & 1 \\ -\kappa_1 & \kappa_2 & \kappa_3 & 1 \\ 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 \end{matrix} \right\|$$

The eigenvalues κ_2, κ_3, κ_4 are the solutions of (6). The direct computation shows that $\mathrm{Det}\, R(z_i) \neq 0$. So z_i are not the spectrum points.

"Case II" is investigated in the same manner. We found only one spectrum point $z^* = 0$. Case II is more simple. In particular, for $z \subset \Omega$ the determinant of $R(z)$ is calculated by hand:

$$\mathrm{Det}\, R(z) = 48z(\kappa_3 - \kappa_2)(\kappa_4 - \kappa_2)(\kappa_4 - \kappa_3)(24 - 8z)\,\cdot$$

$$\cdot\,[\kappa_1^2 - (6 - 8z)\kappa_1 - 3] = 0.$$

So $z^* = 0$ is the spectrum point. $z = 2$ is outside Ω. The direct computation shows that "multiple z" are not the spectrum points. To investigate the last factor let subtract from (5) $6\kappa^3 - (36 - 48z)\kappa^2 - 18\kappa = 0$. As a result z disappeared, and we get

$$\kappa^4 - 7\kappa^3 + 15\kappa^2 - 13\kappa + 4 = (\kappa - 1)^3(\kappa - 4) = 0.$$

By our assumption $|\kappa_1| < 1$ in $\bar{\Omega}$. So there are no other spectrum points inside Ω.

We investigated the stability problem in both cases. In "case I" the spectrum point outside the unit circle $z^* = -1.063...$ causes the strong instability observed in computations. In "case II" there are no spectrum points on and outside the unit circle. The Cauchy problem is stable in C [6,7]. According to the Main Theorem [3] the considered initial boundary value problem is stable in L_2.

We thank G.S.Kazacha for useful discussions and help in computation of "the multiple points".

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Received by Publishing Department
on January 21, 1985.

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Мазепа Н.Е., Сердюкова С.И. E5-85-39
Исследование устойчивости одной разностной краевой задачи с применением системы аналитических вычислений

Исследуется явление неустойчивости, которое наблюдается вблизи границы при численном моделировании движения флюксонов в протяженной системе с микронеоднородностями по схеме Русанова. Приграничные точки считаются по несимметричной схеме второго порядка точности. Осцилляции экспоненциального типа вблизи границы исчезают,если приграничные точки считаются по схеме Вендрофа-Лакса. В этой работе для обоих случаев вычисляются спектры операторов перехода от слоя к слою. В первом случае найдена точка спектра вне единичного круга. Это приводит к сильной неустойчивости, которая и наблюдается в расчетах. Во втором случае нет точек спектра вне и на границе единичного круга. Рассматриваемая краевая задача устойчива. Мы использовали систему аналитических вычислений REDUCE. Однако все наши усилия были тщетны,пока мы существенно не упростили задачу алгебраическими методами.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Mazepa N.E., Serdyukova S.I. E5-85-39
The Stability Investigation of Some Difference Boundary Problem with the Application of Symbolic Computation System

The phenomenon of instability observed near the boundary in numerical modelling of the fluxon motion in the long system with microinhomogeneity by the Rusanov scheme is investigated. Lacking points at the limit are computed by nonsymmetrical approximation of the second order accuracy. Beatings of exponential type near the boundary disappear when we use the Lax-Wendroff scheme to compute the points at the limit. In this work we calculate the spectra of the operator of transition from layer to layer in both cases. In case I we found the spectrum point outside the unit circle.It causes the strong instability observed in computations. In case II there are no spectrum points on and outside the unit circle. The considered initial boundary value problem is stable. We used the symbolic computation system REDUCE.But all our efforts were unavailing until we simplified the problem essentially by applying the algebraic methods.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985