сообщения
объединенного
института
ядерных
исследований
дубна

E5-85-108

R.Gielerak

ON THE D-L-R EQUATIONS IN THE EUCLIDEAN FIELD THEORY AND STATISTICAL MECHANICS.

Euclidean Field Theory

1985

A.0. INTRODUCTION [1]

Let us denote by μ_0 a free field Gaussian measure, i.e., a Gaussian measure on the space $\{S'(R^d), \Sigma\}$ with mean 0 and covariance $S_0(x-y) = (-\Delta + m_0^2)^{-1}(x, y)$, where $-\Delta$ standards for the Laplace operator. The measure space $\{S'(R^d), \Sigma\}$ consists of a space of tempered distributions and its Borel σ-algebra Σ. As is well known it is a standard measure space, therefore all conventional tools of the probability theory apply to this case.

Let us assume that $\{U_\Lambda(\phi)\}$ forms on additive functional of the field μ_0 such that $\{\exp U_\Lambda(\phi)\}$ is then a multiplicative functional. For a bounded, regular set $\Lambda \subset R^d$, let us define a new measure on the space $\{S'(R^d), \Sigma\}$

$$\mu_\Lambda(d\phi) = Z_\Lambda^{-1} \exp(U_\Lambda(\phi))\mu_0(d\phi), \quad Z_\Lambda = \int_{S'(R^d)} \mu_0(d\phi)\exp U_\Lambda(\phi). \tag{A.1}$$

Let us denote by $\Sigma(\Lambda)$ local σ-algebras and by $E_\mu\{-|\Sigma(\Lambda)^c\}$ conditional expectation values of the given measure and a local σ-algebra $\Sigma(\Lambda)$. The Gibbsian approach to the Euclidean Field Theory may be formulated then as a problem of a detailed description of the set $\mathcal{G}^t(\{U_\Lambda\})$ consisting of all probability measures μ defined on $\{S'(R^d), \Sigma\}$ and such that for all bounded, regular sets $\Lambda \subset R^d$ the following relations hold:

$$\mu \circ E_{\mu_\Lambda} = \mu. \qquad \text{(DLR)} \tag{A.2}$$

For a given $\{U_\Lambda\}$ the elements of the set $\mathcal{G}^t(\{U_\Lambda\})$ will be called tempered Gibbs measures corresponding to the given interactions U_Λ. With the minimal technical assumptions made on $\{U_\Lambda\}$ it follows that one can apply general results from the theory of inverse limits of measure spaces [2] to establish several properties of the set $\mathcal{G}^t(\{U_\Lambda\})$ like nonemptiness, integral representations on the Dynkin-Martin boundaries $\partial\mathcal{G}^t(\{U_\Lambda\})$, etc.

A Gibbs measure $\mu \in \mathcal{G}^t(\{U_\Lambda\})$ will be called the regular (respectively, completely regular) Gibbs measure corresponding to the interactions U_Λ iff:

$$\underset{c \in R_+}{\exists} : \underset{f \in S(R^d)}{\forall} \int \mu(d\phi)\phi^2(f) \le c\,||f||^2_{-1} \tag{A.3}$$

(resp. $\exists_{C\in R_+} : \forall_{f\in S(R^2)} \int \mu(d\phi) e^{\phi(f)} \leq e^{C\|f\|^2_{-1}}$ (A.4))

By $\mathcal{G}^t_r(\{U_\Lambda\})$ (resp. $\mathcal{G}^t_{cr}(\{U_\Lambda\})$) we denote the set of regular (resp., completely regular) Gibbs measures from $\mathcal{G}^t(\{U_\Lambda\})$.

From the papers [3,4] it is well known that in the case of $\mu \in \mathcal{G}^t_r(\{U_\Lambda\})$ the following formulas for the conditional expectation values hold: for μ-almost every $\eta \in S'(R^d)$:

$$E_\mu\{F(\phi)\mid\Sigma(\Lambda^c)\}(\eta) = E_{\mu_\Lambda}\{F(\phi)\mid\Sigma(\Lambda^c)\}(\eta) = \int \mu^\eta_\Lambda(d\phi)\, F(\phi+\Psi^{\partial\Lambda}_\eta), \qquad (A.5)$$

where $\Psi^{\partial\Lambda}_\eta$ is a (unique) solution of the following Dirichlet problem:

$$\begin{aligned} (-\Delta+m^2)\,\Psi^{\partial\Lambda}_\eta(x) &= 0 \qquad \text{for} \quad x\in \operatorname{Int}\Lambda \\ \Psi^{\partial\Lambda}_\eta(x) &= \eta(x) \qquad \text{for} \quad x\in\partial\Lambda \end{aligned} \qquad (A.6)$$

By $\mu^\eta_\Lambda(d\phi)$ we denoted the conditioned (by η) measure:

$$\mu^\eta_\Lambda(d\phi) = (Z^\eta_\Lambda)^{-1} \exp(U_\Lambda(\phi+\Psi^{\partial\Lambda}_\eta))\,\mu^{\partial\Lambda}_0(d\phi), \qquad (A.7)$$

$$Z^\eta_\Lambda = \int \mu^{\partial\Lambda}_0(d\phi)\exp(U_\Lambda(\phi+\Psi^{\partial\Lambda}_\eta)), \qquad (A.8)$$

where $\mu^{\partial\Lambda}_0$ is the free field Gaussian measure with the Dirichlet boundary condition imposed on $\partial\Lambda$. In (A.6) we have to assume that $\partial\Lambda$ is C^1-piecewise.

Full set $\mathcal{G}^t_r(\{U_\Lambda\})$ can be obtained by taking convex combinations of the limits like $\mu^\eta_\infty = \omega - \lim_{\Lambda\uparrow R^d} \mu^\eta_\Lambda$. In the process of controlling these limits some information about the behaviour of $\Psi^{\partial\Lambda}_\eta$ is needed.

In the formulated below sequence of estimates, by $\{\Lambda_n\}$ we will always mean a sequence of bounded regular subsets of R^2 with C^1-piecewise boundaries. $\Lambda_n \uparrow R^2$ means that the sequence Λ_n is monotonic and tends to R^2 by inclusion.

Let μ be any probabilistic regular (or completely regular) measure on the space $S'(R^2)$. By $\overset{\mu}{\underset{\eta}{\forall}}$ we shall understand the sentence: "for μ-almost η".

Estimate (1) [4]

Let $\Lambda_n \subset R^2$ and let $\{Y_n\}$ be another sequence as above and such that $\forall_n : Y_n \subset \Lambda_n$ and $\lim_{n\to\infty} \operatorname{dist}(Y_n, \partial\Lambda_n) = \infty$. Then, there exists a subsequence $(n')\subset(n)$ such that

$$\overset{\mu}{\underset{\eta}{\forall}}\ \forall_{\alpha<m_0}\ \lim_{n'\to\infty}\Big(\sup_{x\in Y_{n'}} \big(|\Psi^{\partial\Lambda_{n'}}_\eta(x)|\, e^{\frac{\alpha}{2}\operatorname{dist}(Y_{n'},\partial\Lambda_{n'})}\big)\Big) = 0. \qquad (A.9)$$

Estimate (2) [4]

Let C_0 be any fixed C^1-piecewise curve and let C_n be any sequence of piecewise C^1 curves in R^2 which tends to infinity in the sense that $\operatorname{dist}(0, C_n)\to\infty$ as $n\to\infty$. Then for any sequence $\{\Lambda_n\}$ of bounded sets in R^2 such that $\Lambda_n \subset R^2 - (C_0 \cup C_n)$ and dist $(\Lambda_n, C_n)\to\infty$ $n\to\infty$ there exists a subsequence $(n')\subset(n)$ such that

$$\overset{\mu}{\underset{\eta}{\forall}}\ \forall_{\alpha<m_0}\ \lim_{n'\to\infty}\Big(e^{\frac{\alpha}{2}\operatorname{dist}(\Lambda_{n'},C_{n'})}\Big)\sup_{x\in\Lambda_{n'}} |\Psi^{C_0\cup C_n}_\eta(x) - \Psi^{C_0}_\eta(x)| = 0. \qquad (A.10)$$

Estimate (3)

Let μ be completely regular measure on $\{S'(R^2),\Sigma\}$. Then for any bounded $\Lambda\subset R^2$ and a unit cube $\Delta\subset R^2$ there exist finite for μ-almost η constants $C_n(\eta,\Lambda)$ such that for all $\beta<[n]+1/2$ the following estimate holds:

$$\int_\Delta |\Psi^{\partial\Lambda}_\eta(x)|^n dx \leq C_n(\eta,\Delta)\Big\{\int_\Delta K^{\partial\Lambda}(x,x)\,dx\Big\}^\beta, \qquad (A.11)$$

where

$$K^{\partial\Lambda}(x,x) = (-\Delta+m_0^2)^{-1}(x,x) - (-\Delta^{\partial\Lambda}+m_0^2)^{-1}(x,x) \qquad \text{for} \quad x\notin\partial\Lambda.$$

If μ is regular measure then the estimate (A.11) holds with $n=2$.

In the case when μ is regular and Δ is such that $\epsilon = \operatorname{dist}(\Delta,\partial\Lambda)>0$, there exists a function $C(\eta,\Lambda,\epsilon)$ finite for almost every η and such that

$$\int_\Delta |\nabla\Psi^{\partial\Lambda}_\eta(x)|^2(x)\,dx \leq C(\eta,\Delta,\epsilon)\Big[\int_\Delta \Delta K^{\partial\Lambda}(x,x)\,dx + \Big(\int_\Delta K^{\partial\Lambda}(x,x)\,dx\Big)^\beta\Big] \qquad (A.12)$$

for any $\beta<1$.

Estimate (4) [5]

Let μ be regular measure on $\{S'(R^2),\Sigma\}$ and $\{\Lambda_n\}$ a sequence as above. Then for any $\epsilon>0$ there exists a subsequence $(n')\subset(n)$ and functions $D(\eta,\sigma)$ and $E(\eta,\epsilon,\delta)$ finite for μ-almost every η and such that:

$$\int_{\partial_1 \Lambda_n} |\Psi_\eta^{\partial\Lambda_{n'}}(x)|^2 dx < D(\eta,\epsilon)\,|\partial\Lambda_{n'}|^{1+\epsilon} \quad , \qquad (A.13)$$

where:

$$\partial_1\Lambda = \{x \in \mathrm{Int}\Lambda \,|\, \mathrm{dist}(x,\partial\Lambda) \le 1\},$$

$$\int_{\partial_\delta^1 \Lambda_{n'}} |\nabla \Psi_\eta^{\partial\Lambda_{n'}}(x)|^2 dx \le E(\eta,\epsilon,\delta)\,|\partial\Lambda_{n'}|^{1+\epsilon} \quad , \qquad (A.14)$$

where

$$\partial_\delta^1\Lambda_{n'} = \{x \in \mathrm{Int}\Lambda \,|\quad 0 < \delta \le \mathrm{dist}(x, \partial\Lambda_{n'}) \le 1\}.$$

All these estimates completed with the reverse martingale theorem are one of the fundamental mathematical tools used in the following.

Let us recall also what the problem of the global Markov property is. It is well known [6] that in (almost) all scalar two-dimensional models the following property holds. Let Λ be any bounded with C^1-piecewise boundary $\partial\Lambda$ subset of R^2. Then for two observables $F(\phi)$, $G(\phi)$ localized, respectively, in Λ and Λ^c we have

$$E_\mu\{F(\phi)G(\phi)|\Sigma(\partial\Lambda)\} = E_\mu\{F(\phi)|\Sigma(\partial\Lambda)\}E_\mu\{G(\phi)|\Sigma(\partial\Lambda)\} \qquad (A.15)$$

for any $\mu \in \mathcal{G}_r^t(\{U_\Lambda\})$. This is the so-called local Markov property of the field μ. Whenever (A.15) holds for one-connected and unbounded Λ then the corresponding field μ is called globally Markov. For the importance of verification of these properties in the field theory we refer to [7,10].

Conventional strategy of verifying the global Markov property is the following one. Let Γ be an arbitrary C^1-piecewise curve $\Gamma \subset R^2$ which divides R^2 exactly into two sets Ω_+ and Ω_-. Then the global Markov property for μ will be verified if we prove that for almost every η the measure

$$\mu^{\Gamma,\eta}(d\phi) = \lim_{\Lambda \uparrow R^2} (Z_\Lambda^{\Gamma,\eta})^{-1} \exp(U_\Lambda(\phi + \Psi_\eta^\Gamma))\mu_0^\Gamma(d\phi) \qquad (A.16)$$

is a pure Gibbs measure corresponding to the interaction

$$U_\Lambda^{\Gamma,\eta}(\phi) = U_\Lambda(\phi + \Psi_\eta^{\partial\Gamma}).$$

A.1. EXPONENTIAL INTERACTIONS IN d=2

These models correspond to the choice

$$U_\Lambda(\phi) = -z\int d\lambda(\alpha)\int_\Lambda :e^{\alpha\phi}:(x)\,dx\,, \qquad (A.17)$$

where $z \ge 0$, and $d\lambda$ is some positive finite and bounded measure supported inside interval $(-2\sqrt{\pi}, 2\sqrt{\pi})$. Fundamental existence theorems have been established in papers [8]. According to the conventional wisdom it is expected that this class of models does not exhibit any kind of phase transitions. This suggestion has founded the following mathematical formulation.

Theorem A.1. [9]

For any $z > 0$, $d\lambda$ as above there exists exactly one completely regular Gibbs measure μ_{exp} ($\equiv$ infinite volume half-Dirichlet state) corresponding to the interactions (A.17). This unique completely regular Gibbs measure has global Markov property.

Sketch of the proof:

Let Γ be C^1-piecewise curve in R^2 as in the introduction. It is not hard to prove that the conditioned measure

$$\mu_\Lambda^{\Gamma,\eta}(d\phi) = (Z_\Lambda^{\Gamma,\eta})^{-1}\exp(-z\int d\lambda(\alpha)\int_\Lambda :e^{\alpha\phi}:(x)\,e^{\alpha\Psi_\eta^{\partial\Lambda\cup\Gamma}}:(x)\,dx)\,\mu_0^{\Gamma\cup\partial\Lambda}(d\phi) \qquad (A.18)$$

fullfills the F-K-G correlation inequalities. This with the estimate:

$$\mu_\Lambda^{\Gamma,\eta}(\phi(x);\phi(y)) \le \mu^\circ(\phi(x);\phi(y)) \qquad (A.19)$$

which follows easily by the application of the Ginibre duplicate argument then yields the uniform in Λ and η exponential decay of the measure $\mu_\Lambda^{\Gamma,\eta}$.

The rest of the proof then applies reverse martingale theorem to reduce the control of some thermodynamic limits by passing to the subsequences for which estimate [1] and estimate [2] hold. For more details we refer to the papers [9].

A.2. SINE-GORDON-LIKE MODELS [10]

These models corresponds to the choice

$$U_\Lambda^\epsilon(\phi) = z\int d\lambda(\alpha)\int_\Lambda :\cos\alpha\phi_\epsilon:(x)\,dx\,, \qquad (A.20)$$

where $z \in R^1$, $d\lambda$ is some bounded with bounded support positive measure of the real line, $\phi_\epsilon = \phi * f_\epsilon$, where $f_\epsilon \in C_0^\infty(R^d)$, $f_\epsilon \geq 0$ and supp f_ϵ is of the (small) size ϵ. These models do not find room strictly the situation described in the introduction because of nonlocality of the interactions. However, it is not a serious obstacle (see [11]). Basing on the analysis of the Kirkwood-Salsburg equations the following theorem has been proved in the paper [11].

Theorem A.2.1.[11]

Assume that $|z| \leq C_\epsilon^{-1} \exp(-2a_*^2 S_0^\epsilon(0)-1)$, where

$$C_\epsilon = \sup_a \int |e^{-a\beta S_0^\epsilon(x)} - 1| \, d\lambda(\beta)\, dx, \quad a_* = \sup\{|a| \mid a \in \operatorname{supp} d\lambda\}.$$

Then there exists a unique regular Gibbs measure μ_∞^ϵ corresponding to the interactions (A.20). This unique regular Gibbs measure has global (almost)-Markov property.

Proof of this theorem is based entirely on the analysis of the Kirkwood-Salsburg equations which can be easily written for the conditioned correlation functions

$$\rho_{\Lambda,\epsilon}^{\eta_\epsilon}((\hat{x})_n) = z^n \int \mu_\Lambda^{\eta_\epsilon}(d\phi) \prod_{i=1}^n : e^{ia_i \phi_\epsilon} : (x_i) \quad \text{(A.21)}$$

From the detailed analysis of the Kirkwood-Salsburg operator it follows that almost surely:

$$\lim_{\Lambda \uparrow R^d} \rho_{\Lambda,\epsilon}^{\eta_\epsilon}((\hat{x})_n) = \rho_{\infty,\epsilon}((\hat{x})_n),$$

where

$$\rho_{\infty,\epsilon}((\hat{x})_n) = z^n \int \mu_\infty^\epsilon(d\phi) \prod_{i=1}^n : e^{ia_i \phi_\epsilon} : (x_i) \quad \text{(A.22)}$$

and μ_∞^ϵ denotes the infinite volume limit of μ_Λ^ϵ.

In the two-dimensional situation it is possible to pass to the local limit $\epsilon = 0$. Basic construction of these models with restrictions supp $d\lambda \subset (-2\sqrt{\pi}, 2\sqrt{\pi})$ is contained in the papers [10]. The most simplifying feature of these models is a'priori bound on the effect of the conditioning in the interaction:

$$\sup_{\eta \in \operatorname{supp}\mu} |: \cos \epsilon \Psi_\eta^{\partial\Lambda} : (x)| \leq e^{\frac{\epsilon^2}{2} K^{\partial\Lambda}(x,x)} \quad \text{(A.23)}$$

This uniform bound is sufficient to prove uniform in the boundary data convergence of the high-temperature cluster expansion. This uniform convergence yields the following result.

Theorem A.2.2.

Let $d = 2$, $\epsilon = 0$.

$$d\lambda(a) = \frac{1}{2}\{\delta(a-\epsilon) + \delta(a+\epsilon)\} \quad |\epsilon| < 2\sqrt{\pi}. \qquad \text{(for simplicity)}$$

Then for sufficiently small $|z|$ there exists a unique regular Gibbs measure μ_∞ corresponding to the interaction (A.20). This unique regular Gibbs measure has global Markov property.

Recent investigations of the author significantly improved the uniqueness statements of these two theorems.

Theorem A.2.3.

Assume that $z_0 \geq 0$ is a regular point of the infinite volume free energy corresponding to the models (A.20). Then the set of regular Gibbs measures corresponding to the interactions (A.20) which have translationally invariant first moment consists of exactly one element.

Main steps of the proof.

The following correlation inequalities of the Ginibre-type are valid in the models (A.20).

Let us denote by $\langle \ \rangle_\Lambda^{(0,\eta_\epsilon)}$ expectation value with respect to the tensor product measure $\mu_{\Lambda,\epsilon}(d\phi) \otimes \mu_{\Lambda,\epsilon}^{\eta_\epsilon}(d\phi')$.

Take $\mu \in \mathcal{G}_r^t(\{U_\Lambda^\epsilon\})$ and $\eta \in \operatorname{supp}\mu$ arbitrary. Then for any $z \geq 0$; $n \in N$ the following inequality holds:

$$0 \leq \Big\langle \prod_{i=1}^n (:\cos a_i \phi_\epsilon : (x_i) - \prod_{i=1}^n :\cos a_i \phi'_\epsilon : (x_i)) \times \exp(\pm \delta \int g(x) :\cos a\phi_\epsilon : (x) :\cos a\phi'_\epsilon : (x)\, dx) \Big\rangle_\Lambda^{(0;\eta_\epsilon)}, \quad \text{(A.24)}$$

where $\delta \in R$ and $0 \leq g \in S(R^2)$.

The second useful correlation inequality is the following one. For any choice of the numbers $\theta_i \in [0, 2\pi)$, $i = 1, 2, \dots, n$ any $\mu \in \mathcal{G}_r^t(\{U_\Lambda^\epsilon\})$ and $\eta \in \operatorname{supp}\mu$ we have:

$$\Big\langle \prod_{i=1}^n :\cos(a_i \phi(x_i) + \theta_i): \Big\rangle_{\Lambda,\epsilon}^\eta \leq \Big\langle \prod_{i=1}^n :\cos a_i \phi(x_i): \Big\rangle_{\Lambda,\epsilon}^{\eta=0}, \quad \text{(A.25)}$$

Correlation inequality (A.24) leads to the following inductive statement. Take $\mu \in \mathcal{G}_r^t(\{U_\Lambda^\epsilon\})$ and $\eta \in \operatorname{supp}\mu$. If

$$0 < \int \mu_\infty^\epsilon(d\phi) : \cos a\phi_\epsilon : (x) = \lim_{\Lambda \uparrow R^d} \int \mu_{\Lambda,\epsilon}^{\eta_\epsilon}(d\phi) : \cos a\phi_\epsilon : (x) \quad \text{(A.26)}$$

then for any $n \geq 1$:

$$\lim_{\Lambda \uparrow R^d} \int \mu_{\Lambda \epsilon}^{\eta_\epsilon}(d\phi) \prod_{i=1}^{n} :\cos a_i \phi_\epsilon :(x) = \qquad (A.27)$$
$$= \lim_{\Lambda \uparrow R^d} \int \mu_{\Lambda,\epsilon}(d\phi) \prod^{n} :\cos a_i \phi_\epsilon :(x_i).$$

Equalities (A.27) combined with the correlation inequality (A.25) yields, then

$$\lim_{\Lambda \uparrow R^d} \int \mu_{\Lambda,\epsilon}(d\phi) \prod_{i=1}^{n} :e^{i a_i \phi_\epsilon} :(x_i) = \lim_{\Lambda \uparrow R^d} \int \mu_{\Lambda,\epsilon}^{\eta_\epsilon}(d\phi) \prod_{i=1}^{n} :e^{i a_i \phi_\epsilon} :(x_i) \qquad (A.28)$$

assuming that (A.26) holds. Thus, the proof has been reduced to statement (A.26).

Lemma A.2.

Let $\mu \in \mathcal{G}_r^t(\{U_\Lambda^\epsilon\})$ and take $\eta \in \operatorname{supp}\mu$
Define:

$$p_\Lambda^{\eta_\epsilon}(z) = \frac{1}{|\Lambda|} \int e^{U_\Lambda(\phi + \Psi_{\eta_\epsilon}^{\partial\Lambda})} \mu_0^{\partial\Lambda}(d\phi). \qquad (A.29)$$

Assume that $\Lambda \uparrow R^d$ in the sense of van Hove and that $\partial\Lambda$ are C^1 - piecewise. Then $\lim_{\Lambda \uparrow R^d} p_\Lambda^{\eta_\epsilon}(z)$ exists and is equal to $\lim_{\Lambda \uparrow R^d} p_\Lambda^{(\eta_\epsilon = 0)}(z)$, i.e., half-Dirichlet infinite volume pressure.

Proof of the Lemma A.2.1 is based on the estimates like E3) and E4) from the introduction. The proof of Theorem A.2.3 is then completed using some well known properties of the convex functions.

In the two-dimensional case we have used the so-called $:\cos a\phi:$ bound in the form of [106]. This yields (technical) restriction on the size of a: $|a| < \dfrac{2}{1 - \frac{1}{2\pi}}$.

A.3. IS THERE A FIRST ORDER PHASE TRANSITION IN THE TWO-DIMENSIONAL SCALAR YUKAWA THEORY?

It is well known that the two-dimensional pseudoscalar Yukawa model exhibits phase transition of the first order /12/.

Indecisive suggesstions were adressed in the literature towards the case of scalar Yukawa model /13/.

Let us denote by $\mathcal{L}^Y(\Psi,\phi)$ Lagrangian of the two-dimensional Yukawa models. To the Lagrangian $\mathcal{L}^Y(\Psi,\phi)$ let us add the Schwinger term $\mathcal{L}^S(\phi)$ and the Thirring term $g\mathcal{L}^T(\Psi)$. Then the chargeless sector of the theory described by the total lagrangian $\mathcal{L}^{YTS}(\Psi,\phi) = \mathcal{L}^Y(\phi,\Psi) + \mathcal{L}_c^S(\phi) + g\mathcal{L}^T(\psi)$ can be described purely in the boson language. The chargeless sector of the Y-T-S theory can be described fully in the Euclidean region by the following formal measure on $\{S'(R^2), \Sigma\} \otimes 2$

$$\mu_\infty(d\Phi, d\phi) = \exp(\lambda \int :\cos(\epsilon\Phi + \theta):(x)\, \phi(x)\, dx) \times$$
$$\times \exp(\mu \int :\cos \epsilon\Phi:(x)\, dx)\, \mu_0(d\Phi) \otimes \mu_0(d\phi), \qquad (A.30)$$

where λ is the Yukawa coupling constant, μ is the bare fermionic mass, $\epsilon^2 = 4\pi \dfrac{1}{1 + g/\pi}$, where g is the Thirring coupling constant, $\mu_0(d\Phi)$ is the free field Gaussian measure with the mass $m_0^f = e^2/(1+g/\pi))$ where e^2 is the Schwinger coupling constant.

The case $\theta = 0$ corresponds to the scalar Yukawa interaction and the case $\theta = 0$ to the pseudoscalar interaction.

Various constructions of the measure $\mu_\infty(d\Phi, d\phi)$ were presented in /14/. From the Coleman-(Fröhlich-Seiler) theorem and analyticity for small λ and μ proven in /14/ it follows that $\mu_\infty(d\Phi, d\phi)$ really describes chargeless sector of Y-T-S theory whenever $g > 0$.

Let us denote by $\mathcal{G}_r^t(\lambda,\mu)$ the set of refular Gibbs measures on $[S'(R^3)] \otimes 2$ corresponding to the interactions contained in the definition (A.30). The following result about coexistence of phase(s) has been proved in /14/.

Theorema A.3. /14/

Let $\theta = 0$, $e^2 > 0$, $g > 0$.

Let ${}^+\mathcal{G}_r^{t,T}(\lambda,\mu_0)$ be a sebset of $\mathcal{G}_r^t(\lambda,\mu_0)$ obtained as limits of $\mu_\Lambda^{(\eta_1 \eta_2)}(d\Phi, d\phi)$ with $\mu \geq 0$, $\eta_2 \geq 0$ which have translationally invariant first moment. Assume that the half-Dirichlet infinite volume pressure $p_\infty^{H.O.}(\lambda,\mu)$ is differentiable at $\mu = \mu_0$.

Then for any $\mu \in {}^+\mathcal{G}_r^{t,T}(\lambda,\mu_0)$ and $f_i \in S(R^2)$ we have

$$\mu(e^{i\Phi(f_1)}\, e^{i\phi(f_2)}) = \mu_\infty^{H.O.}(e^{i\Phi(f_1)}\, e^{i\phi(f_2)}), \qquad (A.31)$$

where $\mu_\infty^{H.O.}$ is the infinite volume half-Dirichlet state corresponding to (A.30).

The proof of this theorem is similar to that of Theorem A.2.3. With the assumptions made on μ and η_2 the correlation inequalities of type (A.14), (A.15) hold. These correlation inequalities reduce the proof to the statement about independence of the infinite-volume pressure of the typical boundary conditions $\eta = \eta_1 \otimes \eta_2$.

A.4. SOME REMARKS ABOUT $\mathcal{P}(\phi)_2$ MODEL

The choice of $U_\Lambda(\phi) = -\lambda \int_\Lambda :\mathcal{P}(\phi):(x)\, d^2x > \lambda \geq 0$ and where $\mathcal{P}$ is a polynomial bounded from below corresponds to the so-colled $\mathcal{P}(\phi)_2$ theories. In contrast with the models considered above the phase diagram of the $\mathcal{P}(\phi)_2$ theories might have a fairly complicated structure /15/. Adaptation of the elements of the Pirogov-Sinai theory to extract some information about phase-diagrams of the $\mathcal{P}(\phi)_2$ theories is possible.

However, from the point of view of the program sketched in the Introduction the results obtained are more than poor. We note here the following applications of the Estimates 3 and 4 from the Introduction.

Theorem A.4.1./18/

Let μ be a completely regular $\mathcal{P}(\phi)_2$ measure. Then the infinite volume pressure p^η_∞ does not depend on the typical boundary condition η. This theorem in a sense generalized the results of /17/ concerning the independence of the so-called classical boundary conditions. But from the point of view of the Gibbs approach this class of boundary conditions is presumably of measure zero.

It is expected that in the region of the convergence of the high-temperature cluster expansion, the obtained Gibbs measure is pure. However, we have not seen a proof of this. Some particular result follows from the estimates 3 and 4 from the Introduction.

Theorem A.4.2.

Let μ be a completely regular $\mathcal{P}(\phi)_2$ measure. Assume that Θ is a subset of suppμ such that:

$$\sup_{\eta\in\Theta} C_n(\eta,\Lambda) < \infty, \quad \sup_{\eta\in\Theta} D(\eta,\epsilon) < \infty; \quad \sup_{\eta\in\Theta} E(\eta,\cdot) < \infty,$$

where C_n, D and E are the functions from E(3), E(4) and E(4), respectively.

Then, the cluster expansion for the conditioned measure converges uniformly in the set Θ.

Corollary A.4

Let μ, η and Θ be as in theorem A.

Then $\lim_{\Lambda\uparrow R^2} \mu^\eta_\Lambda(d\phi)$ exist and do not depend on $\eta \in \Theta$.

However, the set Θ seems to be not of a measure 1.

More prespectively seems an application of the Theorem A.4.1. to the $:\phi^4:$ -like theories where the Lee-Yang theorem works.

REFERENCES

1. Simon B. The $\mathcal{P}(\phi)_2$ (Euclidean Quantum) Field Theory. - Princeton: Princeton University Press, 1974; Glimm J., Jaffe A. "Quantum Physics. A Functional Integral Point of View" - Springer-Verlag, New-York, Heidelberg, Berlin, 1982; Preston G. "Random Field", Lecture Notes in Mathematics. - 534, Springer Verlag: Berlin, Heidelberg, New-York, 1976.
2. Winkler G. "Inverse Limits of Simplices and Application in Stochastics". - Hab-PHD-University of Münich, 1983 and References Therein.
3. Dobrushin R.L., Minles R.A. In: "Problems of Mechanics and Mathematical Physics". - Moscow, 1976 (in Russian); Benfatto G., Gallavotti G., Nicollo F. - Journ. of Funct. Anal., 1980, 36, p. 343-400.
4. Albeverio S., Moegh-Kröhn R. Comm.Math.Phys., 1979, 68, p.95-128.
5. Gielerak R. On the D-L-R-Equations for the Two-Dimensional Sine- Gordon-Model. - Submm. to Ann.Inst. H.Poincare.
6. Newman Ch. Journ. of Funct. Anal., 1973, 14, p. 44-61.
7. Albeverio S., Hoegh-Kröhn R. Comm.Math.Phys., 1979, 68, p. 95-128.
8. Albeverio S., Hoegh-Kröhn R. Journ of Funct.Anal., 1974, 16, 39; Fröhlich J., Park Y.M. Helv.Phys.Acta, 1977, 50, p. 315-329.
9. Gielerak R. Journ.Math.Phys., 1983, 24, 2, p.347-355; Gielerak R. In: "Critical Phenomena Theoretical Aspects", Birkäser, 1984; Zegarlinski B. Wroclaw University Ph.D. Thesis 1984 and Paper Submm. to Comm. Math. Phys.
10. Albeverio S., Hoegh-Kröhn R. Comm.Math.Phys., 1973, 30, p. 171-200; Fröhlich J. Comm.Math.Phys., 1976, 47, p.233-269. Fröhlich J., Seiler E. Helv.Phys.Acta, 1977, 49, p.889-924. Fröhlich J., Park Y.M. Comm.Math.Phys., 1978, 59, p.235-266.
11. Gielerak R., Zegarlinski B. Fortschr.der Physik/Progress on Physics, 1984, 36, p.1-24.
12. Fröhlich J. Acta Phys.Austriaca Suppl. XV, 1976, p.133-269. Bataban T., Gawedzki K. Ann.Inst. d.H.Poincare, vol.XXXVI, (4), 1982, p. 271-400.

13. Glimm J., Jaffe A. Critical Problems in Q.Field Theory, 1975. Talk presented at the International Colloquium on Math. Methods of Quantum Field Theory, Marseille, 1975; Glimm J., Jaffe A. in /1b/, p. 388.
14. Gielerak R. "On the Correlation Inequalities in Some Models Involving Fermions", 1984, Submm. to Fortschr. d.Physik.
15. Glimm J., Jaffe A., Spencer T. Comm.Math.Phys., 1975, 45, p. 203-216; Glimm J., Jaffe A., Spencer T. Parts I, II Ann. Phys., 1976, 101, p. 610-630, 631-669; Gawedzki K. Comm. Math.Phys., 1978, 59, p. 117-142; Gidas B. Ann Phys., 1979, 118m p. 18-83; Summers S. Annl'Inst. Henri Poincare, 1981, 34, p. 173-229; Imbrie J. Comm.Math.Phys., 1981, 82, p.261-343; Fröhlich J. Acta Phys.Austriaca, Supp.XY, 1976, p. 133-269; Fröhlich J., Simon B., Spencer T. Comm.Math.Phys., 1980, 50, p. 79-85.
16. Gielerak R. Vacuum Energy Density in the $P(\phi)_2$ Theories. Independence of (Noclassical) Boundary Conditions. - Submm. to Lett. on Math.Phys.
17. Guerra F., Rosen L., Simon B. Ann.1'Inst. Henri Poincare, 1975, 15, p. 111-259.

Received by Publishing Department
on February 15, 1985.

Гелерак Р. E5-85-108

Уравнения ДЛР в евклидовой квантовой теории поля и статистической механике. Евклидова теория поля

Дан обзор недавних результатов, полученных в гиббсовском подходе в двумерной евклидовой квантовой теории поля. Рассматривается применение корреляционных неравенств к анализу уравнений ДЛР.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Gielerak R. E5-85-108

On the D-L-R Equations in the Euclidean Field Theory and Statistical Mechanics. Euclidean Fied Theory

Present status of the Gibbsian approach to the Euclidean (scalar) field theory is outlined. Main effort is made at the application of the correlation inequalities in the analysis of the D-L-R equations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985