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ON A MODIFIED COUNTER WITH PROLONGING DEAD TIME

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The mathematical theory of particle counters is concerned with the formulation and study of stochastic processes associated with the registration of particles due to radioactive substances by a counting device designed to detect and record them and placed within the range of a radioactive material

We suppose that any arriving particle generates an impulse of a random length. Due to the inertia of the counting device, it is possible that all particles will not be counted. The time during that the device is unable to record is called th dead time. The physical and mathematical literature on counter theory deals mainly with two types of models. A counter with a nonprolonging dead time (type I counter, too) is one in which the dead time is produced only after impulses of particles have been registered. A counter with prolonging dead time is one in which the dead time is produced after registration of all impulses of emitted particles (type II counter, too). Examples of these counters with the nonprolonging and prolonging dead times are the Geiger-mulier counters and electron multipliers, respectively.

Usually it is assumed that the counter before registration process is idle and the particles arrive at a counter at the instances $0=r_{1}<r_{2}<\ldots$, where the interarrival times, $T_{n}=$ $=t_{\mathrm{n}+1}-\tau_{\mathrm{n}}, \mathrm{n} \geq 1$, are independent random variables with the distribution functions $F_{n}(x)=P\left(T_{n}<x\right), \quad n \geq 1$, and $F_{n}(x)=$ $=F(x)$ for any $n$. The duration of impulses of particles starting at $\tau_{n}, X_{n}$, with distribution functions $H_{n}(x)=P\left(\chi_{n}<x\right)$, $\mathrm{n} \geq 1$, are usually assumed independent identically distributed (i.i.d.) positive random variables independent of $\left\{T_{n}\right\}_{n=1}^{\infty}$.

For a modified counter with prolonging dead time we assume that $\left\{T_{n}\right\}_{n=1}^{\infty}$ and $\left\{x_{n}\right\}_{n=1}^{\infty}$ are not identically distributed, in general, and any dead time is repeatedly produced by interarrival times and lengths of impulses which are all independent and distributed by $\left\{\left.F_{n}\right|_{n=1} ^{\infty}\right.$ and $\{H\}_{n=1}^{\infty}$. Hence if $\left\{_{j}\right\}_{j=1}^{\infty} \quad$ is a sequence of indices of registered particles, $\mathrm{Z}_{\mathrm{j}}=r_{\mathrm{m}_{\mathrm{i}+1}}-r_{\mathrm{n}_{\mathrm{j}}}, \mathrm{j} \geq 1$, are i.i.d. random variables.
Our main aims are to determine the probability laws of the numbers of particles, $\nu$, arriving at the modified counters during the dead times, and the Laplace transform of the cycle $Z_{1}$.

These problems are important not only for the counter theory. The same problems, from the mathematical point of view, arise in the cases of the film or filmless measurements of the particle track ionization in the so-called bubble and streamer chambers (for details see, e.g. / 1,2 ). The description of the queuing systems with infinitely many servers leads to similar problems (see, e.g., /3/).

The known results on the mentioned random variables in question are available only in the particular case when $\mathbf{F}=\mathbf{F}_{\mathbf{1}}=\mathbf{F}_{\mathbf{2}}=\ldots$, and $H=H_{1}=H_{2}=\ldots$. Pyke/4/ derived the distribution of $\nu$, and some limit properties have been investigated by Afanas" eva and Mikhailova/5/ (in GI/GI/m terminology), and by Dvurečenskij and Ososkov $/ 6 /$.

The cycle has been studied only for the above particular case by Pollaczek/7/ in the form of complicated contour integrals. Pyke $/ 4 /$ and Takáce $/ 8 /$ have obtain only some integral equations. The Laplace transform in the explicit form is given by Dvureでenskij and Ososkov ${ }^{16 /}$.
2. NUMBER OF PARTICLES ARRIVING DURING DEAD TIME

Let us put
$A_{n}=X_{X_{1}}<T_{1}+\ldots+T_{n} \cdot X_{2}<T_{2}+\ldots+T_{n} \ldots . X_{n}<\left.T_{n}\right|_{n} \geq 1$.
Then $P_{n}=P(\nu=n)=P\left(\bar{A}_{1} \ldots \bar{A}_{n-1} \mathbf{A}_{n}\right), \quad n \geq 1$. It is clear that if the input process is recurrent and the lengths of impulses are i.i.d., then $\left|A_{n}\right|^{\infty}=1$, is a sequence of recurrent events in the sense of Feller $/ 9 /$ that is, for any $1 \leq i_{1}<\ldots<i_{n}, n \geq 2$, $P\left(A_{i_{2}} \ldots A_{i_{n}} \mid A_{i_{1}}\right)=P\left(A_{i_{2}-i_{1}} \ldots A_{i_{1}-i_{1}}\right)$, consequently $P$ may be easily found.

For $1 \leq i \leq j$ define
$\left.A_{i, j}=\mid X_{i}<T_{i}+\ldots+T_{i}, x_{i+1}<T_{i+1}+\ldots+T_{j} \ldots, x_{i}<T_{j}\right\}$.
Then
$P\left(A_{i, j}\right)=\int_{0}^{\infty} \ldots \int_{0}^{\infty} H_{i}\left(t_{i}+\ldots+t_{j}\right) H_{i+1}\left(t_{i+1}+\ldots+t_{j}\right) \ldots H_{j}\left(t_{j}\right) d F_{i}\left(t_{i}\right) \ldots d F_{j}\left(t_{j}\right)$
and for $1 \leq i_{1}<\ldots<i_{n}, n \geq 2$, we have

$$
\begin{equation*}
P\left(A_{i_{2}} \ldots A_{1} \mid A_{i_{1}}\right)=P\left(A_{i_{1}}+1, i_{2} \ldots A_{i_{1}}+1, i_{1}\right) \tag{2.3}
\end{equation*}
$$

It is easy to verify that $\left|A_{n}\right|_{m=1}^{\infty}$ from (2.1) is a sequence of recurrent events iff $P\left(A_{j-1}\right)=P\left(A_{1+1, j}\right)$ for $1 \leq i \leq j$.

To determine $P_{n}, n \geq 1$ let us put for $k \geq 1 \quad A_{n}^{E}=A_{k, k+n-1}, \underline{n} \geq 1$.
The integer-valued variable $\nu_{k}, k \geq 1$, defined by $P_{n}^{k}=P\left(\nu_{k}=n\right)=$
$=P\left(\bar{A}_{1}^{\mathbf{k}} \ldots \bar{A}_{n-1}^{\mathbf{k}} A_{n}^{\mathbf{k}}\right), n \geq 1$, may be interpreted as a number of particles arriving at the $k$-th modified counter with prolonging dead time in that the dead time is produced according to distribution functions of interarrival times beginning from $F_{k}$, $\mathrm{F}_{\mathbf{k}+1} \ldots$, and the distribution functions of the lengths of impulses begin from $H_{k}, H_{k+1}, \ldots$. It is clear that $\nu_{1}=\nu$ and $P_{n}{ }^{1}=P_{n}$.

To determine $P_{n}$ we proceed as follows:

$$
\begin{aligned}
P_{n}= & P\left(\bar{A}_{1} \ldots \bar{A}_{n-1} A_{n}\right)= \\
& P\left(A_{n}\right)-\sum_{j=1}^{n-1} \sum_{1 \leq i_{1}<\ldots<i_{i} \leq n-1}(-1)^{j-1} \\
& P\left(A_{i_{1}} \ldots A_{i_{i}} A_{n}\right) .
\end{aligned}
$$

Using (2.3) we obtain in conclusion
$P_{1}=P\left(A_{1}\right), \quad P_{n}=P\left(A_{n}\right)-\sum_{j=1}^{n-1} P\left(A_{j}\right) P_{n-j}^{1+j}, \quad n \geq 2$.
Since the sequence $\left\{A_{n}^{k}\right\}_{n=1}^{\infty}, k \geq 1$, has the property analogical to (2.3) for $\left\{A_{n}\right\}_{n=1}^{\infty}$, that is, for any
$1 \leq i<\ldots<i, n \geq 2, \quad P\left(A_{i_{2}}^{k} \ldots A_{i_{n}}^{k} \mid A_{i_{1}}^{k}\right)=P\left(A_{i_{2}-i_{1}}^{k+i_{1}} \ldots A_{i_{n}^{-i} 1}^{k+i_{1}}\right)$,
we may obtain the next formula for the $k-$ th modified counter

where $P\left(A_{j}^{k}\right)$ may be evaluated by (2.2).
If $\left\{A_{n}\right\}_{n=1}^{\infty}$ is a sequence of recurrent events, then $P_{n}^{k}=P_{n}$ for every $k, n$, and from (2.4) we conclude the known formula $P_{n}=P\left(A_{n}\right)-\sum_{j=1}^{n-1} P\left(A_{j}\right) P_{n-j}, \quad n \geq 1$, derived by Pyke/4/.

Let $\phi_{\mathbf{k}}\left(z^{j}\right)=\sum_{n=1}^{\infty} P_{n}^{k} z^{n}$ be the generating function for $\nu_{k}, k \geq 1$.
Due to (2.5) we have
$\phi_{k}(z)=\sum_{n=1}^{\infty} P\left(A_{n}^{k}\right)\left(1-\phi_{k+n}(z)\right), \quad|z|<1$.
A very interesting case is obtained when $F=F_{1}, F=F_{2}=F_{3}=\ldots$, and $H=H_{1}, H=H_{2}=H_{3}=\ldots$. If we put $P\left(A_{n}^{*}\right)=P\left(A_{n}^{2}\right)$, then $P_{1}=P\left(A_{i}\right), P_{n}=P\left(A_{n}\right)-\sum_{j=1}^{n_{2}-1} P\left(A_{j}\right) P_{n-j}^{*}, \quad n>2$, where $P_{n}^{*}=P_{n}^{2}=P_{n}^{3}=\ldots$.

For its generating function we conclude $\phi(z)=\left(1-\phi^{*}(z)\right) U(z)$ $|z|<1$, where $\phi^{*}(z)$ is the generating function for $P_{n}, n \geq 1$, and $U(z)=\sum_{n=1}^{\infty} P\left(A_{n}\right) z^{n}$.

## 3. PROPERTIES OF $A_{n}^{k}$

Here we investigate the structure of the events $\left\{A_{n}^{\mathbf{k}}\right\}_{n=1}^{\infty}, \mathbf{k} \geq 1$, for arbitrary sequences $\left\{F_{n}\right\}_{n=1}^{\infty}$ and $\left\{H_{n}\right\}_{n=1}^{\infty}$.

Lemma 1. If $P\left(A_{1}^{k}\right)>1$ for any $k \geq 1$, then $P\left(A_{n}^{k}\right)>0$ for any $k, n \geq 1$.

Proof. Let there be two integers $k$ and $n$ such that $P\left(A_{n}^{k}\right)=0$. Denote by $n_{0}$ the minimal integer $n$ (for the given $k$ ) for which $P\left(A_{n}^{k}\right)=0$. If $n_{0}>1$, then $P\left(A_{n_{0}-1}^{k}\right)=P\left(A_{\mathbf{n}_{0}-1}^{k} \bar{A}_{n_{0}}^{k}\right)=$
$=P\left(A_{n_{0}-1}^{k}\right)\left(1-P\left(A_{1}^{k+n} 0^{-1}\right)\right)$. Hence $P\left(A_{1}^{k+n} 0^{-1}\right)=0$ which contradicts our assumption.
Q.E.D.

Let us define $U_{k}(z)=\sum_{n=1}^{\infty}-P\left(A_{n}^{k}\right) z^{n}, \quad|z|<1$, and $B_{n}^{k}=\left\{\chi_{k}<T_{k}+\ldots\right.$ $+T_{k+n-1}$ ]. Then the following holds:

Theorem 2. For any $k \geq 1$ we have
(i) $P\left(A_{n}^{k}\right) \leq P\left(A_{n-1}^{k+1}\right), \quad n \geq 2$,

$$
\begin{equation*}
\mathrm{U}_{\mathbf{k}}(z) \leq 1+\mathrm{U}_{\mathrm{k}+\mathrm{l}}(\mathrm{z}) . \tag{3.1}
\end{equation*}
$$

(ii) $P\left(A_{n}^{k}\right) \geq P\left(B_{n}^{k}\right) P\left(A_{n-1}^{k+1}\right), \quad n \geq 2$.
(iii) If for some $n P\left(B_{n}^{k}\right)>0$, then
$\sum_{n=1}^{\infty} P\left(A_{n}^{k}\right)=\infty$ iff $\sum_{n=1}^{\infty} P\left(A_{n}^{k+1}\right)=\infty$.
(iv) If $\lim _{n \rightarrow \infty} P\left(B_{n}^{k}\right)=1$ and $\sum_{n=1}^{\infty} P\left(A_{n}^{k}\right)=\infty$, then
$\lim _{\mathrm{z} \rightarrow \mathrm{I}^{-}}(\mathrm{z}) / \mathrm{U}_{\mathrm{k}+\mathbf{l}^{\prime}}(\mathrm{z})=1$.
Proof. (3.1) and (3.2) are evident. In order to prove (3.3) we use the Chebychev inequality: If $\psi_{i}\left(x_{1}, \ldots, x_{n}\right), i=1,2$, are non-negative real-valued functions either all are nonincreasing or all nondecreasing, and if $G_{j}, j=1, \ldots, n$ are distribution functions, then
$\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \psi_{1}\left(x_{1}, \ldots, x_{n}\right) \psi_{2}\left(x_{1}, \ldots, x_{n}\right) d G_{1}\left(x_{1}\right) \ldots \mathrm{dG}_{n}\left(x_{n}\right) \geq$
$\geq \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \psi_{1}\left(x_{1}, \ldots, x_{n}\right) d G_{1}\left(x_{1}\right) \ldots d G_{n}\left(x_{n}\right) \times$
$\times \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \psi_{2}\left(x_{1}, \ldots, x_{n}\right) d G_{1}\left(z_{1}\right) \ldots d G_{n}\left(x_{n}\right)$.

For (iv) we have the following. Let $i \geq 3$. Then
$U_{k}(z) \geq \sum_{n=1}^{i-1} P\left(A_{n}^{k}\right) z^{n}+P\left(B_{i}^{k}\right) z \sum_{n=i}^{\infty} P\left(A_{n-1}^{k+1}\right) z^{n-1}=$
$=\sum_{n=1}^{i-1} P\left(A_{n}^{k}\right) z^{n}+P\left(B_{i}^{k}\right) z\left(U_{k+1}(z)-\sum_{n=1}^{i-2} P\left(A_{n}^{k+1}\right) z^{n}\right)$.
Using (3.2) we may obtain $1 \geq \lim _{z \rightarrow 1^{-}} U_{k}(z) / U_{k+1}(z) \geq P\left(B_{i}^{k}\right)$. Letting $i \rightarrow \infty$ we prove (3.4).
Q.E.D.

Theorem 3. If for some $n_{0} \geq 1$ we have $\phi_{n_{0}+1}=\phi_{n_{0}+2}=\ldots$ and $P\left(A_{1}^{i}\right)<1, i=1, \ldots, n_{0}$, then for any $n \quad \phi_{n}(1)=1$ ff $\sum_{i=1}^{\infty} P\left(A_{i}\right)=\infty$.

Proof. If $\mathrm{n}_{0}=1$, then (2.6) implies $\phi_{1}(z)=\mathrm{U}_{1}(\mathrm{z}) /\left(1+\mathrm{U}_{2}(\mathrm{z})\right)$,
$\phi_{2}(\mathrm{z})=\mathrm{U}_{2}(\mathrm{z}) /\left(1+\mathrm{U}_{2}(\mathrm{z})\right)$. assertion.

If $n_{0} \geq 2$, then $\phi_{n_{0}+1}(z)=U_{n_{0}+1}(z) /\left(1+U_{n_{0}+1}(z)\right), \quad \phi_{n_{0}}(z)=$
$=U_{n_{0}}(z) /\left(1+U_{n_{0}+1}(z)\right)$. For $1 \leq i<n_{0}$ we have
$\phi_{n_{0}-i}(z)=U_{n_{0}-i}(z)\left(1-\phi_{n_{0}+1}(z)\right)+\sum_{j=1}^{i} P\left(A_{j}^{n_{0}-i}\right) \times$
$\times\left(\phi_{n_{0}+1}(z)-\phi_{n_{0}}{ }^{-j+j}(z)\right)=U_{n_{0}-1}(z) / U_{n_{0}}(z) \phi_{n_{0}}(z)+\sum_{j=1}^{\vdots} P\left(A_{j}^{u_{0}^{-i}}\right) \times$
$\times\left(\phi_{\mathbf{n}_{0}+1}(z)-\phi_{n_{0}-i+j}(z)\right)$.
Using repeatedly (3.4) we prove the general case.
Q.E.D.

Observe that assumption $P\left(A_{1}^{i}\right)<1, i=1, \ldots, n_{0}$, is not superfluous. Indeed, let $T_{n}=1, n \geq 1$, and $H_{1}(1)=1, H_{2}=H_{3}=\ldots$, $H_{2}(1)=0$. Then $P\left(A_{1}^{1}\right)=1$ and $U_{1}(z)=z, U_{2}(z)=0$.

In the rest of this part we will assume $H_{1}(x) \geq H_{2}(x) \geq \ldots$, $H_{n} \Rightarrow H$, and $F=F_{1}=F_{2}=\ldots$.

## Theorem 4. We have

(i) $P\left(A_{1}^{k}\right) \geq P\left(A_{2}^{k}\right) \geq \ldots$
(ii) $p_{k}=\lim _{\mathbf{n}} P\left(A_{\mathbf{n}}^{k}\right)=\lim _{\mathbf{n}} P\left(A_{\mathbf{n}}^{k+1}\right)=p_{k+1}$.
(iii) If $P\left(A_{n}^{\infty}\right)=\lim _{k} P\left(A_{n}^{k}\right)$, then $p_{\infty}=\lim _{n} P\left(A_{n}^{\infty}\right)=p_{k}$.

Proof. It suffices to prove (iii). Let us put $p_{k}=p$. Clearly $\mathrm{p} \geq \mathrm{p}_{\infty}$. Now let k be fixed. Then
$P\left(A_{n}^{k}\right) \geq P\left(A_{n-1}^{k+1}\right) P\left(B_{n}^{k}\right) \geq P_{k+1} P\left(B_{n}^{k}\right)=p P\left(B_{n}^{k}\right)$.

$$
\begin{equation*}
\infty \tag{1}
\end{equation*}
$$

Hence $P\left(A_{n}^{\infty}\right) \geq p \int_{0}^{\infty} \cdots \int_{0}^{\infty} H\left(t_{1}+\ldots+t_{n}\right) d F\left(t_{1}\right) \ldots d F\left(t_{n}\right)$ and, cone-
quently $p_{\infty} \geq p$.

Theorem 5. (i) If
$\lim _{k} P\left(A_{1}^{k}\right)>0, \quad \lim \int_{0}^{\infty} x d H_{k}(x)<\infty$,
then $p>0$. (ii) If $0<\int_{0}^{\infty} x \mathrm{dF}(\mathrm{x})<\infty$, then (3.5) is the necessary and sufficient condition for $p>0$.

Proof. From (3.3) and (3.4) we have
$p=p_{k}=p_{\infty}=\lim _{n} P\left(A_{n}^{\infty}\right)=\lim _{n} \int_{0}^{\infty} \ldots \int_{0}^{\infty} H\left(t_{1}\right) \ldots H\left(t_{1}+\ldots+t_{n}\right)$
$d F\left(t_{1}\right) \ldots d F\left(t_{n}\right) \geq \prod_{n=1}^{\infty} P\left(\left\{X_{n}^{\infty}<T_{1}+\ldots+T_{n}\right\}\right)$,
where $\chi_{n}^{\infty}, n \geq 1$, are i.i.d. random variables with the distri-
 ${ }_{n=1}^{\infty} P\left(\left(T_{1}+\ldots+T_{n}>\chi_{n}^{\infty}\right\}\right)>0, \quad$ then the first part of the therem will be proved. For this it is sufficient to prove that
$\sum_{n=1}^{\infty} P\left(\left\{X_{n}^{\infty}>T_{1}+\ldots+T_{n}\right\}\right)<\infty$.
Let $T_{n}^{K}=\min \left(T_{n}, K\right)$, where $K>0$ is a real number such that $M\left(T_{n}^{K}\right)>0$. Put $S_{n}^{K}=T_{1}^{K}+\ldots+T_{n}^{K}$ and $S_{n}=T_{1}+\ldots+T_{n}$. Then
$P\left(\left\{S_{n} \leq \chi_{n}^{\infty}\right\}\right) \leq P\left(\left\{S_{n}^{K} \leq \chi_{n}^{\infty}\right\}\right) \leq P\left(\left\{S_{n}^{K} \leq \chi_{n}^{\infty}, \mid S_{n}^{K} / n-\right.\right.$
$\left.\left.-M\left(T_{1}^{K}\right) \mid \leq \epsilon\right\}\right)+P\left(\left\{S_{n}^{K} \leq \chi_{n}^{\infty},\left|S_{n}^{K} / n-M\left(T_{1}^{K}\right)\right|>c \mid\right) \leq 1-\right.$
$\left.-\mathrm{H}\left(\mathrm{nM}\left(\mathrm{T}_{1}^{K}\right)-\epsilon\right)\right)+\mathrm{A} / \mathrm{n}^{2}$.
where $A$ is a constant independent of $n$. We choose $\epsilon$ such that $M\left(T^{K}\right)>_{\epsilon}$. Hence (3.6) converges and therefore $p>0$.

Now let us prove (ii). The sufficient condition has been
proved. Let $p>0$. The strong law of large numbers implies that for an arbitrary $\epsilon>0$ we have $P\left(\bigcap_{j=n_{0}}^{\infty}\left\{S_{j}<2 j M\left(T_{1}\right)\right\}\right) \geq 1-\epsilon$,
where $n_{0}$ is a suitable integer such that $H\left(2 n_{0} M\left(T_{1}\right)\right)>0$. If $p>0$, then there is $\epsilon>0$ such that $p>\epsilon$. Then

and, consequentiy

$$
\begin{aligned}
0 & <P\left(\bigcap_{n=1}^{\infty}\left\{\chi_{n}^{\infty}<S_{n}\right\} \cap \bigcap_{j=n_{0}}^{\infty}\left\{S_{j}<2 j M\left(T_{1}\right)\right\}\right) \leq \\
& \leq P\left(\prod_{n=n_{0}}^{\infty}\left\{\chi_{n}^{\infty}<2 n M\left(T_{1}\right)\right\}\right)=\prod_{n=n_{0}}^{\infty} H\left(2 n M\left(T_{1}\right)\right) .
\end{aligned}
$$

Hence $\sum_{n=1}^{\infty}\left(1-\mathrm{H}\left(2 \mathrm{nM}\left(\mathrm{T}_{1}\right)\right)\right)<\infty \quad$ which implies $\int_{0}^{\infty} \mathrm{xdH}(\mathrm{x})<\infty$. Condition $\lim _{\mathbf{k}} \mathrm{P}\left(\mathrm{A}_{1}^{\mathbf{k}}\right)>0$ is evident.

Let us note that $\operatorname{limP}\left(A_{1}\right)>0$ is not superfluous condition. Indeed, let $H=H_{1}=H_{2}^{k}=\ldots, H(1)=0, T_{n}=1, n \geq 1$. Then $P\left(A_{n}\right)=0$ for any $n$ and $p=0$ although $M(T)=1$.

## 4. THE CYCLE

The cycles of a counter are defined as interarrival times between the moments of registered particles. As has been noticed by several authors (Barlow $/ 10 /$, Pyke $/ 4 /$, Smith/11/) the determination of the distribution function of the cycle, $G$, or its Laplace transform, $\gamma$, respectively, is an extremely difficult problem even for the recurrent input process and i.i.d. lengths of impulses. However, there are some integral equations Takács $/ 8$ / and Pyke/4/ which formally, but not always in practice, determine $G$ or $y$.

Here we determine $\Phi(s, z)=M\left(e^{-s Z_{2}}{ }_{z}{ }^{\prime}\right)$ for the modified counter with prolonging dead time in the case of recurrent input of particles.

Let $F=F_{1}=F_{2}=\ldots$. Define $a(s)=\int_{0}^{\infty} e^{-s} \mathrm{dF}(\mathrm{x}), \mathrm{s}>0, \mu=\int_{0}^{\infty} \mathrm{xdF}(\mathrm{x})$, and let $0<\mu<\infty$. With the given recurrent process $\left\{\tau_{\mathbf{n}}\right\}_{\mathbf{n}=1}^{\mathbf{0}}$ we define a new recurrent one $\left\{\tau_{\mathbf{n}}^{\mathbf{s}}\right\}_{\mathbf{n}=1}^{\infty}$ for any $\mathbf{s} \geq 0$ with the distribution function $F_{s}(x)=P\left(\tau_{n+1}^{s}-\tau_{n}^{s}<x\right)=a(s)^{-1} \int_{e^{x}}^{\mathbf{s}} \mathbf{e}^{-s t} d F(t)$.

Let $\phi_{s}(z)$ be the generating function of the number of particles $\nu_{s}$ arriving at the modified counter during its dead time according to the imput process $\left\{\tau_{n}^{s}\right\}_{n=1}^{\infty}$ and the lengths of impulses $\left\{\chi_{n}\right\}_{n=1}^{\infty}$.

Theorem 6. For any $s \geq 0,|z|<1, \Phi(s, z)=\phi_{s}(a(s) z), \gamma(s)=\phi_{s}(a(s))$ $\mathrm{M}\left(\mathrm{Z}_{1}\right)=\mu \bar{M}(\nu)$.

$$
\text { Proof. Since } Z_{1}=\tau_{\nu+1} \text { we have }
$$

$\Phi(s, z)=\sum_{n=1}^{\infty} \mid\left\{{ }_{\{\nu=n} e^{-s \tau_{n+1}} z^{n} d P=\right.$

$$
=\sum_{n=1}^{\infty} \Gamma_{C_{n}} \ldots \int e^{-s\left(t_{1}+\ldots+t_{n}\right)} z^{n} d F\left(t_{1}\right) \ldots d F\left(t_{n}\right) d H_{1}\left(x_{1}\right) \ldots d H_{n}\left(x_{n}\right),
$$

where the integration area $\mathrm{C}_{\mathrm{n}}$ has the following form
$\left(x_{1}<t_{1}\right)^{c},\binom{x_{1}<t_{1}+t_{2}}{x_{2}<t_{2}}^{c}, \ldots,\left(\begin{array}{l}x_{1}<t_{1}+\ldots+t_{n-1} \\ \vdots \\ x_{n-1}<t_{n-1}\end{array}\right)^{c},\left(\begin{array}{l}x_{1}<t_{1}+\ldots+t_{n} \\ \vdots \\ x_{n}<t_{n}\end{array}\right)$
(here the sign "c"denotes the complement of the set mentioned in the parentheses).

Hence $\Phi(s, z)=\sum_{n=1}^{\infty} a(s)^{n} z^{n} P\left(\nu_{s}=n\right)=\phi_{s}(a(s) z)$.
The mean value of $Z_{l}$ is obtained using the Wald identity.
Q.E.D.

## 5. EXAMPLES

Example 1. Let $0<D_{1} \leq D_{2} \leq \ldots$, and let $H_{n}(x)=1$ if $x>D_{n}$ and O otherwise, and let $\left\{F_{n}\right\}_{n=1}^{\infty}$ be an arbitrary sequence with $F_{n}\left(D_{n}\right) \neq 0$. If we put $p_{i}=F_{i}^{n}\left(D_{i}\right), i \geq 1$, then

$$
\begin{align*}
& P\left(A_{n}^{k}\right)=1-p_{k+n-1} \\
& P_{1}^{k}=1-p_{k}  \tag{5.1}\\
& P_{n}^{k}=P_{k} \ldots p_{k+n-2}\left(1-p_{k+n-1}\right), \quad n \geq 2 .
\end{align*}
$$

Example 2. Let $F_{n}(x)=1$ if $x>a$, for some $a>0$, and 0 otherwise, $n>1$, and let $0<H_{n}(a)<1, H_{n}(2 a)=1, n \geq 1$. Putting $p_{i}=$ $=1-H_{i}(a), i>1$, we may obtain the formula (5.1).

In the above two examples $\sum_{n=1}^{\infty} P_{n}^{\mathbf{k}}=1$ iff $\sum_{i=1}^{\infty}\left(1-p_{i}\right)=\infty, k \geq 1$.

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Двуреченский А., Ососков Г.А.
E5-83-882 о модифицировакном счетчике с мертвым временем продлеваюмегося типа

Исследуется способ определения числа частиц, припедших на счетчик с мертвым временем продлеваюмегося типа за период мертвого времени. При этом предлолагается, что характеристики приходов частиц и их импульсов за период мертвого времени изменяются. Определено в явном виде прєобразованне Лапласа длины цикла - интервала между зарегистрированными частицами. Эти результаты применимы также к проблеме о фильмовом и бесфильмоном измерениях трековои информации в стримерных камерах в экспериментах по физике высоких энергй̆.

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Drurečenskij A., Ososkov G.A. E5-83-882

On a Modified Counter with Prolonging Dead Time
The way of determination of the number of particles, arriving at the counter with prolonging dead time during its dead time, is investigated. It is assumed that the characteristics of inputs and lengths of impulses of particles during the dead time are variable. The Laplace transform of the cycle (interval between registered particles) is determined in the explicit form. These results are applicable to some problems of the film and filmless track measurements in the streamer chambers of the high energy physics experiments.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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