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G.A.Emelyanenko, Š.Šujan

# A NEW LOOK UPON INFORMATION PROCESSING IN EXPERIMENTAL PHYSICS.

# LORENTZ EQUATION AND WALKING ON A UNIT SPHERE

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## 1. INTRODUCTION

In Ref.I we formulated several problems related to the development of a new approach to information processing for a certain class of physical experiments.

A basic aim of the present paper is to carry over a part of the geometrical program of investigation, namely, to construct a geometrical representation for the classical Lorentz equation. We focus our attention to problems connected with the influence of mean energy losses and of heterogeneity of the magnetic field upon the particle's motion. As will become clear, our method of investigation will differ from those ones adopted earlier /1-8/. The problems connected with incorporating processes of scattering and of radiation losses will be studied in a subsequent paper.

## 2. A GEOMETRICAL REPRESENTATION FOR LORENTZ EQUATION

The basic equation of motion of a particle with charge $e$ in an environment under the action of the magnetic field $\vec{H}$ is the (vector) Lorentz equation, which describes the time evolution of the impulse-vector $\vec{p}$:

$$\frac{d\vec{p}}{dt} = \frac{e}{c}[\vec{v}(t) \times \vec{H}(t)], \qquad (2.1)$$

where $c$ is the light-velocity in vacuum, $\vec{v}(t)$ designates the velocity vector at time instant $t$, $\vec{H}(t)$ stands for the vector of magnetic field at $t$ (this should be understood as the change of $\vec{H}$ in the working space of a detector along the particle's orbit), and the symbol $\times$ designates vector multiplication, respectively. For any vector $\vec{r}$ we denote by $r = |\vec{r}|$ its modulus. We shall assume that the function $t \to \vec{H}(t)$ satisfies the conditions that ensure there exists a solution to the Cauchy problem for (2.1). Let

$$\vec{n}(t) = \vec{p}(t)/p(t). \qquad (2.2)$$

Then

$$\vec{n}(t) = \vec{v}(t)/v(t), \qquad (2.3)$$



for at any time instant $t$ the vectors $\vec{p}(t)$ and $\vec{v}(t)$ are directed identically. Let

$$\vec{q}(t) = \vec{H}(t)/H(t), \qquad (2.4)$$

i.e., $\vec{q}(t)$ is the vector which corresponds to the direction of the magnetic field at moment $t$. The symbol $m_{eff}(t)$ will denote the effective mass at the time instant $t$. It is related to the rest mass $m$ via the relation

$$m_{eff}(t) = m[1 - (v(t)^2/c^2)]^{-1/2} \qquad (2.5)$$

and, furthermore, to the modulus of impulse by the relation

$$p(t) = m_{eff}(t)\, v(t). \qquad (2.6)$$

The expression inside the square brackets in (2.5) is a relativistic factor. Substituting (2.2) through (2.4) into (2.1) yields

$$\frac{d\vec{n}}{dt} = \frac{e}{c}\frac{v(t)}{p(t)}H(t)[\vec{n}(t) \times \vec{q}(t)] - \frac{1}{p(t)}\frac{dp}{dt}\vec{n}(t). \qquad (2.7)$$

Of course, (2.7) is valid when $p(t) \neq 0$ (and this corresponds physically to a particle which does not stop moving inside the time interval $[0, T]$ within which we shall study (2.1)). We return to the case of stopped particles later. Using (2.6) we may rewrite (2.7) in the form

$$\frac{d\vec{n}}{dt} = Q_1(t)[\vec{n}(t) \times \vec{q}(t)] + S(t)\vec{n}(t), \qquad (2.8)$$

where

$$Q_1(t) = \frac{e}{c}\frac{H(t)}{m_{eff}(t)}, \quad S(t) = -\frac{1}{p(t)}\frac{dp}{dt}. \qquad (2.9)$$

We see that the function $t \to p(t)$ enters (2.8) as a parameter. It is well-known that this function satisfies a certain differential equation (the explicit form of which is not important for us at present)

$$\frac{dp}{dt} = \phi(t, X), \qquad (2.10)$$

where $X$ stands for the parameter of environment (in fact, we usually have not (2.10) itself but its solution - the loss function - at our disposal). In order the Cauchy problem be fully determined we must add initial conditions for (2.8) and (2.10):

$$p(t) = p_0, \quad \vec{n}(t) = \vec{n}_0 \quad \text{for } t = 0. \qquad (2.11)$$

In order to simplify forthcoming considerations we need several notations more. If $\vec{\alpha}$ and $\vec{\beta}$ are two vectors then $\pi[\vec{\alpha}, \vec{\beta}]$ designates the plane containing those vectors and $\sphericalangle(\vec{\alpha}, \vec{\beta})$ the spatial angle between them (if defined). Let $\theta(t) = \sphericalangle[\vec{n}(t), \vec{q}(t)]$. Then

$$\vec{n}(t) \times \vec{q}(t) = |\vec{n}(t)|\ |\vec{q}(t)|\ \sin\theta(t)\, \vec{e}(t) = \sin\theta(t) \cdot \vec{e}(t), \tag{2.12}$$

where $\vec{e}(t)$ is the unit vector orthogonal to the plane $\pi[\vec{n}(t), \vec{q}(t)]$; in symbols

$$\vec{e}(t) \perp \pi[\vec{n}(t), \vec{q}(t)]; \quad e(t) \equiv 1. \tag{2.13}$$

Before proceeding to the study of the system (2.8) through (2.11), several remarks seem worth to make. In the original equation (2.1) the energy losses are not taken into account. Therefore it might appear more reasonable to start with a modified Lorentz equation

$$\frac{d\vec{p}}{dt} = \frac{e}{c}[\vec{v}(t) \times \vec{H}(t)] + \frac{\vec{p}(t)}{p(t)}\frac{dp}{dt}. \tag{2.14}$$

In fact, such a strategy is usually followed even at present (see Refs. /1,2/ ). Using the formula (3.2) of Ref.I for $\vec{\theta} = \vec{p}$ we get in place of (2.8)-(2.11) the following system of equations:

$$\left.\begin{aligned} &\frac{d\vec{n}}{dt} = \frac{e}{c}\frac{H(t)}{p(t)}[\vec{n}(t) \times \vec{q}(t)]; \\ &\frac{dp}{dt} = \phi(t, X); \\ &p(t) = p_0, \quad \vec{n}(t) = \vec{n}_0 \quad \text{for} \quad t = 0. \end{aligned}\right\} \tag{2.15}$$

From the geometrical point of view the system (2.15) is more evident than (2.8)-(2.11), for the vectors $d\vec{n}/dt$ is nothing but the normal vector to the trajectory at the point $t$. Since $\vec{n}(t)$ is a tangent vector at the same point, elementary geometrical arguments show that the first equation in (2.15) is geometrically "consistent" in the sense that the vectors on both sides are identically directed. Moreover, we have from elementary geometry that

$$\frac{d\vec{n}}{dt} = \kappa(t)\, \vec{\eta}(t), \tag{2.16}$$

where $\vec{\eta}(t)$ is the unit normal vector and $\kappa(t)$ stands for the



curvature at the point $t$ (2.15) and (2.12) entail

$$\kappa(t) = \frac{e}{c}\frac{H(t)}{p(t)}\sin\theta(t). \tag{2.17}$$

On the other hand, (2.15) does not correctly reflect the energy losses. Indeed, it follows from (2.17) that the energy losses merely change the curvature. However, physically it is evident that the process of energy losses works essentially in the direction tangent to the integral curve of (2.1). Consequently, a certain displacement of the centre of curvature should result. But this means there exists an additional term (in the direction of $\vec{n}(t)$ ) to the rotation tensor $\vec{n}(t) \times \vec{q}(t)$. Such an additional term is present in (2.8). Hence, the system (2.8)-(2.11) seems to be more reasonable, from the physical point of view, than (2.15) is.

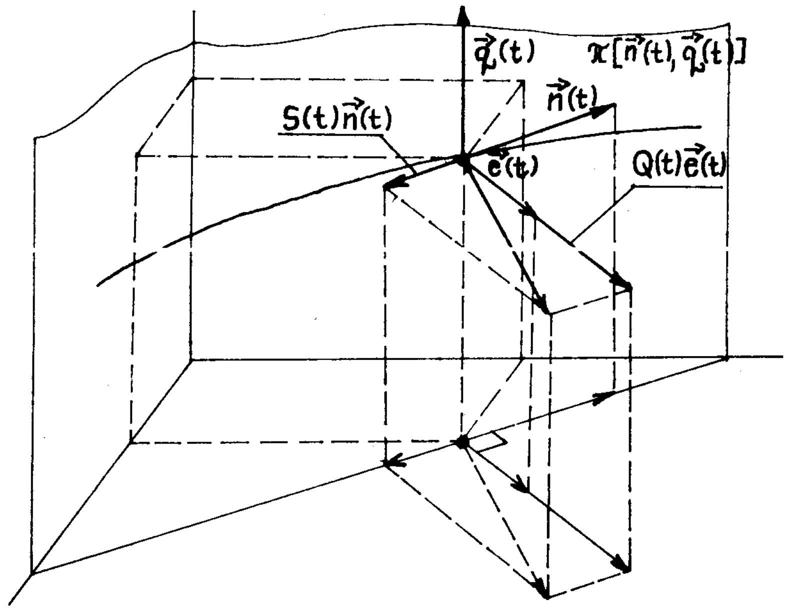


Fig.1

Now let us return to the former system. The corresponding geometrical picture is given on Fig.1. Introducing the quantity

$$Q(t) = Q_1(t)\ \sin\theta(t) = \frac{e}{c}\frac{H(t)\ \sin\theta(t)}{m_{eff}(t)} \tag{2.18}$$

the equation (2.8) will assume on the form

$$\frac{d\vec{n}}{dt} = Q(t)\,\vec{e}(t) + S(t)\,\vec{n}(t)\,. \tag{2.19}$$

It follows from (2.9) and (2.18) that $Q(t)$ and $S(t)$ are (in physically interesting situations) continuous functions. From (2.16) it follows that the vector $d\vec{n}/dt$ is a unit one only when $\vec{n}(t)$ is a vector tangent to the surface of a unit sphere $S_3$, i.e., when $\kappa(t) = 1$. In the general case we must renormalize $d\vec{n}/dt$. Since the vectors $\vec{e}(t)$ and $\vec{n}(t)$ are orthogonal, (2.19) shows that

$$\left|\frac{d\vec{n}}{dt}\right| = [Q(t)^2 + S(t)^2]^{1/2}\,. \tag{2.20}$$

The quantity (2.20) differs from zero unless the particle moves along a straight line (see below). Using (2.20) we finally come to a non-linear algebraic equation

$$\vec{\eta}(t) = A(t)\,\vec{e}(t) + B(t)\,\vec{n}(t)\,, \tag{2.21}$$

where

$$\left.\begin{aligned} A(t) &= Q(t)\,[Q(t)^2 + S(t)^2]^{-1/2}\;; \\ B(t) &= S(t)\,[Q(t)^2 + S(t)^2]^{-1/2}\,. \end{aligned}\right\} \tag{2.22}$$

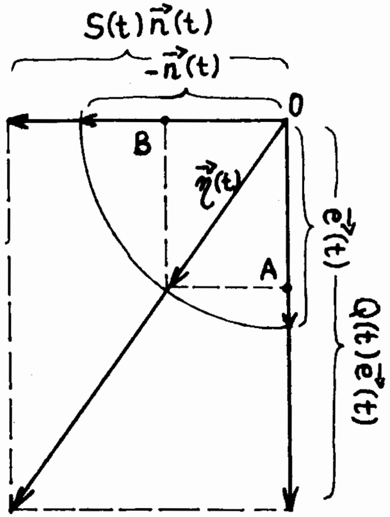

Fig.2

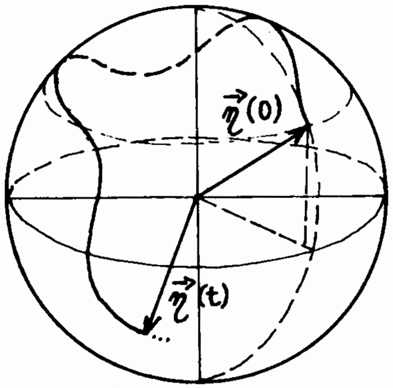

Fig.3



On Fig.2 we have $|OA| = A(t)$, $|OB| = B(t)$, and the plane of that figure is orthogonal to $\pi[\vec{n}(t), \vec{q}(t)]$. This means that the rotation from $\vec{e}(t)$ to $\vec{\eta}(t)$ acts in a plane orthogonal to $\pi[\vec{n}(t), \vec{q}(t)]$ and intersecting the latter in a straight line containing the vector $\vec{n}(t)$.

Equations (2.21), (2.10), and (2.11) completely determine the function $t \to \vec{\eta}(t)$. If the change of the unit vector $\vec{\eta}$ is represented by the motion of its end point, then we finally get a representation of the system (2.8)-(2.11) in the form of a (deterministic) walk on the surface of the sphere $S_3$ (Fig.3).

Next let us clarify the meaning of objects entering the geometrical representation obtained so far:

$$\left.\begin{aligned}
&\text{(a)}\quad \vec{\eta}(t) = A(t)\,\vec{e}(t) + B(t)\,\vec{n}(t)\,; \\
&\text{(b)}\quad A(t) = Q(t)\,[Q(t)^2 + S(t)^2]^{-1/2}\;; \\
&\text{(c)}\quad B(t) = S(t)\,[Q(t)^2 + S(t)^2]^{-1/2}\;; \\
&\text{(d)}\quad Q(t) = eH(t)\,\sin\theta(t)\,[c\,m_{eff}(t)]^{-1}; \\
&\text{(e)}\quad S(t) = -p(t)^{-1}\,dp/dt\,; \\
&\text{(f)}\quad dp/dt = \phi(t, X)\,; \\
&\text{(g)}\quad p(t) = p_0,\quad \vec{n}(t) = \vec{n}_0 \quad \text{for } t = 0\,.
\end{aligned}\right\} \tag{2.23}$$

Let

$$\left.\begin{aligned} \varphi(t) &= \measuredangle[\vec{e}(t), \vec{\eta}(t)]\,, \\ \psi(t) &= \measuredangle[\vec{n}(t), \vec{\eta}(t)]\,. \end{aligned}\right\} \tag{2.24}$$

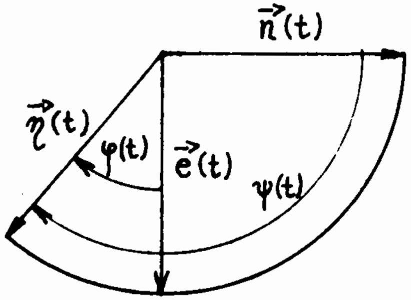

Fig.4.

See Fig.4 and observe that $\vec{e}(t)$, $\vec{n}(t)$ and $\vec{\eta}(t)$ each are lying in the same plane. By scalar multiplication of (2.23a) by $\vec{\eta}(t)$, $\vec{e}(t)$, and $\vec{n}(t)$, respectively, and by evaluating the corresponding scalar products we obtain

$$\left.\begin{aligned} 1 &= A(t)\,\cos\phi(t) + B(t)\,\cos\psi(t)\,; \\ A(t) &= \cos\phi(t)\,; \\ B(t) &= \cos\psi(t)\,. \end{aligned}\right\} \tag{2.25}$$

Since $\cos\psi(t) = \cos[\phi(t) + \pi/2] = -\sin\phi(t)$, the first equation in (2.25) follows from the second and the third ones, and reduces to $A(t)^2 + B(t)^2 = 1$.

We now describe some particular cases throwing light upon the physical meaning of objects in (2.23). Suppose the field is constant in each direction and modulus (i.e., $\vec{q}(t) \equiv \vec{q}(0)$,

$H(t) \equiv H(0)$) and that there are no losses of energy (i.e., $p(t) \equiv p_0$). In this case $S(t) \equiv 0$. Consequently, $A(t) \equiv 1$ and $B(t) \equiv 0$. Hence $\phi(t) \equiv 0$ and the vectors $\vec{\eta}(t)$ and $\vec{e}(t)$ coincide at each time instant $t$. This is natural for in the absence of energy losses also the displacement of the curvature centre must be absent. The equation (2.23a) reduces to

$$\vec{\eta}(t) = \vec{e}(t) = \frac{1}{\sin\theta(t)}[\vec{n}(t) \times \vec{q}(0)]\,. \qquad (2.26)$$

In case when $\measuredangle[\vec{n}_0, \vec{q}(0)] = \pi/2$, equation (2.26) transforms into the tautology $\vec{\eta}(t) = \vec{e}(t)$, and this corresponds to the motion along a unit circle placed in the plane orthogonal to $\kappa[\vec{n}_0, \vec{q}(0)]$.

In order we get also a physical information about the motion, return to (2.7), which, in our particular case, assumes on the form

$$\frac{d\vec{n}}{dt} = \frac{e}{c}\,\frac{v(0)}{p(0)}H(0)\,\vec{e}(t)\,. \qquad (2.27)$$

We already know that the scalar multiplicative constant in (2.27) has the meaning of curvature. Hence, we see from (2.27) that the additional information about a homogeneous field merely entails the change of the unit curvature to some constant one, $\kappa$, where

$$\kappa = \kappa(0) = \frac{e}{c}\,\frac{v(0)}{p(0)}H(0) \qquad (2.28)$$

(cf. Fig.5).

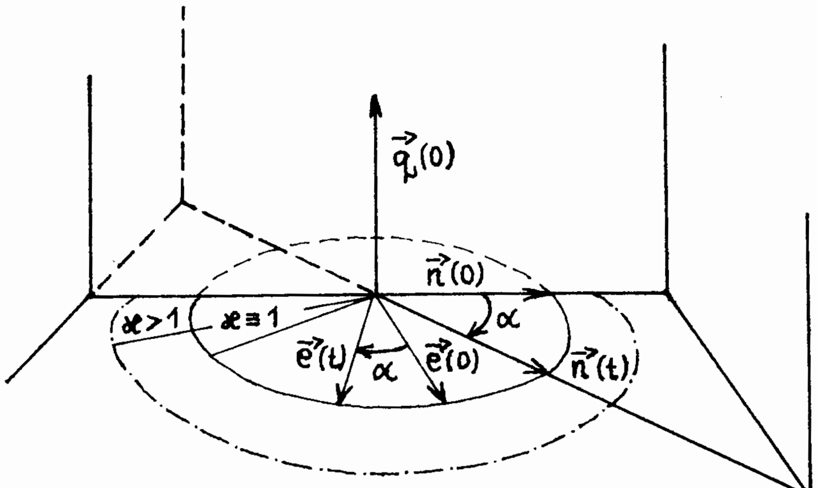


Fig.5



All reasoning remains true also for a field $\vec{H}(t)$ for which $\vec{q}(t) \equiv \vec{q}(0)$ and in $H(t)$ only the component along $\vec{q}(t)$ changes in time. The only difference consists of the fact that the curvature will no longer be constant:

$$\kappa(t) = \frac{e}{c}\,\frac{v(0)}{p(0)}H(t)\,. \qquad (2.29)$$

Now let us consider the case when $H(t) \equiv H(0)$, $\vec{q}(t) \equiv \vec{q}(0)$, and $\measuredangle[\vec{n}_0, \vec{q}(0)] \neq \pi/2$. The scalar term $(\sin\theta(t))^{-1}$ in (2.26) is a periodic function which changes the curvature, however, the motion itself again lives on the plane orthogonal to $\pi[\vec{n}_0, \vec{q}(0)]$.

When the field is absent (i.e., $H(t) \equiv 0$ so that $Q(t) \equiv 0$) we get a singularity in (2.23) to the effect that the end point of the vector $\vec{\eta}(t)$ becomes a fixed point. This corresponds to the motion along a straight line. Indeed, in this case we get from (2.7) the equation

$$p(t)\frac{d\vec{n}}{dt} = -\frac{dp}{dt}\vec{n}(t)\,. \qquad (2.30)$$

It follows from these considerations that all general results (to be obtained in subsequent papers) will work equally well also for straight trajectories.

We conclude this section with the following remark concerning the relation between $\theta(t)$ and $\phi(t)$. Using (2.23), (2.24), and (2.25) we get

$$\frac{B(t)}{A(t)} = \frac{\sin\phi(t)}{\cos\phi(t)} = \mathrm{tg}\,\phi(t) = -\frac{(1/p(t))\ (dp/dt)}{(e/c)\ (H(t)\sin\theta(t)\ /\ m_{eff}\ (t))}\,. \qquad (2.31)$$

## 3. CONCLUSION

A thorough investigation on the geometrical representation of equations of motion made it possible to recognize clearly the relations between geometrical and physical properties of braking processes and of the magnetic field, respectively (these are processes $P_1$ and $P_5$ in the notations of Ref.I). The general system (2.23) shows that, even in absence of random factors, the processes $P_1$ and $P_5$ are not independent.

In principle, all considerations of the present paper remain valid even in the presence of random factors. However, it turns out that it is more convenient to pass to an operator representation of separate physical processes obeying random character. This will be the aim of the next paper.

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REFERENCES

I. Emelyanenko G.A., Sujan S. JINR, E5-83-693, Dubna, 1983.
1. Soop K. Bubble Chamber. Measurements and Data Processing. (Russian). "Nauka", M., 1970.
2. Emelyanenko G.A. JINR, 11-6076, Dubna, 1971.
3. Zhigunov V.P., Sokolov S.N. Sov.J.Part.Nucl., 1982, vyp.5, p.1024-1069.

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Емельяненко Г.А., Шуян Ш. E5-83-694
Новый взгляд на обработку информации
в экспериментальной физике.
Уравнение Лоренца и блуждание на единичной сфере

В настоящей работе выполнена первая часть программы, сформулированной в [1]. Приводится геометрическое представление для классического уравнения Лоренца в виде блуждания по поверхности единичной сферы в трехмерном пространстве. Такое представление позволяет наглядно описать вклад влияния магнитного поля и потерь энергии в полное временное изменение геометрических и физических переменных.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Emelyanenko G.A., Šujan Š. E5-83-694
A New Look upon Information Processing
in Experimental Physics.
Lorentz Equation and Walking on a Unit Sphere

In this paper the first part of the program formulated in Ref.I is carried over. A geometrical representation of classical Lorentz equation is given in the form of a walk on the surface of the three-dimensional unit sphere. Such a representation makes possible to describe in an evident manner the contributions of magnetic field and energy losses to the full temporal change in geometrical and physical variables.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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