объединенный
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исследований
дубна

E5-83-255

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NOTE
ON ONE TYPE II COUNTER PROBLEM

Submitted to Aplikace Matematiky

1983

1. INTRODUCTION

The mathematical theory of particle counters is concerned with the formulation and study of stochastic processes associated with the registration of particles due to radioactive substances by a counting device designed to detect and record them and placed within the range of a radioactive material. The general problem can be described as follows.

We first consider a sequence of random events consisting of the arrival of the emitted particles. This sequence is called the primary sequence of events or primary process. We suppose that any arriving particle generates an impulse of a random length (may be constant one, too). Due to the inertia of the counting device, it is possible that all particles will not be counted. The time during which the device is unable to record is called the dead time. The sequence of registered particles forms a secondary process which is selected from the primary sequence according to a used type of a counter.

The basic problem in the counter theory is to determine the distribution function of the distance between two successive registered particles if the distribution function of the primary process, distribution of impulses and the counter type are known.

Our main aim in this note is to determine the joint Laplace transform of the above-mentioned distribution, and the generating function of the number of particles arriving to the counting device during the dead time for the so-called Type II counter, and to make some remarks on the registrations of m types of particles ($m \geq 1$).

2. NOTATIONS AND KNOWN RESULTS

The mathematical and physical literature on the counter theory deals mainly with two types of models. A Type I counter (counter with nonprolonging dead time) is one in which dead time is produced only after impulses of particles have been registered. A Type II counter (counter with prolonging dead time) is one in which dead time is produced after registration of all impulses of emitted particles. Examples of Type I and Type II counters are the Geiger-Müller counters and electron multipliers, respectively. An extensive bibliography of the counter theory is in Takács[24] and Smith[21]. From the physical literature dealing with this object see, for example, the monography[11].

Let us suppose that particles arrive at a counter at the instances $0 = \tau_0 < \tau_1 < \tau_2 < \ldots < \infty$, where the interarrival times $\tau_n - \tau_{n-1}$ $(n = 1,2,\ldots)$ are identically distributed, independent, positive random variables with the distribution function

$$F(x) = P(\tau_n - \tau_{n-1} < x).$$

Denote by χ_n the duration of impulse starting at τ_n, $(n=0,1,2,\ldots)$. It is supposed that $\{\chi_n\}$ is a sequence of identically distributed, independent, positive random variables with the distribution function

$$H(x) = P(\chi_n < x),$$

and independent of $\{\tau_n\}$.

At any instant t there are two mutually exclusive states in which counter may be: state E_0 when no impulse covers the instant, and state E_1 otherwise. The interval when the counter is in a state E_1 corresponds to the dead time, and the interval when it is in E_0 state corresponds to an idle time. The particles are registered only if the counter is idle, and let us suppose that the registration process starts from $\tau_0 = 0$.

In the Type II counter the n-th particle $(n = 1,2,\ldots)$ is registered iff $\tau_m + \chi_m < \tau_n$ for any $m = 0,1,\ldots,n-1$. If we define accordingly to Pyke[15] $n_0 = 0$,

$$n_j = \min\{k : k > n_{j-1},\ \tau_k > \tau_r + \chi_r,\ r = n_{j-1},\ldots,k-1\}$$

for $j = 1,2,\ldots$, then $\{n_j\}_{j=0}^{\infty}$ is a sequence of indices of registered particles. Since the primary process is recurrent one, the secondary process

$$Z_j = \tau_{n_j} - \tau_{n_{j-1}}, \quad j = 1,2,\ldots,$$

is recurrent, too.

The main problem is to determine the distribution function

$$G(x) = P(Z_j < x),$$

or, equivalently, its Laplace transform

$$\gamma(s) = M(e^{-sZ_j}), \quad s \geq 0.$$

This very interesting problem has been studied by several authors. The partial case of the Poisson primary process is

discussed by Takács[22-24,26,28], Pollaczek[14], Smith[21], Sankaranarayanan[17-19], Albert and Nelson[4], Afanaseva and Michajlova[1,2].

Although in the physical practice we deal mainly with the Poisson primary process, due to the repeated handling of particles by several counters, the initial process inputting at the last counter will not be Poisson, but only recurrent one.

The recurrent primary process and constant length of impulses are studied in refs.[15,24,28] and the exponentially distributed impulse times are discussed in papers[14,15,25,27,28].

The general case has been taken into account by Pollaczek[14] who has given the solution in the form of complicated contour integrals. Takács[24] and Pyke[15] obtained only integral equations without their solutions. Similar results are obtained using the multiplicative processes by Smith[21]. In authors' paper[9] the explicit computable formulae for the discrete primary process and discrete lengths of impulses are given. Many other problems of the theory of Type II counters are investigated, for example, in refs.[6,20].

This problem is important not only for the counter theory. The same problems (from the mathematical point of view) are studied in the film or filmless measurements of the particle track ionization in the so-called bubble and streamer chambers (for details see, for example, refs.[7,8,12]). The description of the queueing systems with infinitely many servers leads to analogical problems, in general.

3. TYPE II COUNTER

As has been noticed by several authors[5,15,21] the determination of G or γ is an extremely difficult problem. However, there are the integral equations which formally, but not always in practice, determine G or γ, respectively. Takács obtained an integral equation in $M(t)$ – the expected number of registered particles in a time interval $(0,t)$ for all $t \geq 0$.

<u>Lemma 1</u>. (Takács[24]) For all $t \geq 0$

$$M(t) = \int_0^t H(y)\,dF(y) + H(t)\int_0^t M(t-y)\,dF(y) - \int_0^t\int_0^t M(z-y)\,dH(z)\,dF(y). \tag{1}$$

If we know $M(t)$, then $\gamma(s)$ may be determined by

$$\gamma(s) = \int_0^\infty e^{-st}\,dM(t) \Big/ \left(1 + \int_0^\infty e^{-st}\,dM(t)\right), \quad s \geq 0. \tag{2}$$

Barlow[5] generalized the equation (1) to the case of the semi-Markov primary process.
Pyke[15] obtained the integral equation for G.

Lemma 2. (Pyke[15]) For all $x \geq 0$

$$G(x) = \int_0^x \int_0^{x-y} (1 - G(x-y-t))H(y+t)dN(t)dF(y), \tag{3}$$

and for all $s \geq 0$

$$\gamma(s) = \lambda(s)(1+\lambda(s))^{-1}, \tag{4}$$

where

$$\lambda(s) = \int_0^\infty \int_0^\infty e^{-s(x+t)} H(x+t)dF(x)dN(t).$$

These two representations are equivalent in the sense that G and N are uniquely determined one by another. Below we give an explicit form of the Laplace transform γ of the distribution function G which determines the solutions of the equations (1) and (2), respectively.

Let us denote by q_n the number of the impulses present at the arrival of the n-th particle $(n = 0,1,2,\ldots)$. The sequence of events A_n defined by

$$A_n = \{q_n = 0\}, \quad n = 0,1,\ldots,$$

is a sequence of recurrent events in the sense of Feller[10], i.e.,

$$P(A_{i_n} / A_{i_1} \ldots A_{i_{n-1}}) = P(A_{i_n - i_{n-1}})$$

for any finite system of indexes

$$i_1 < i_2 < \ldots < i_n, \quad n = 2,3,\ldots,$$

Hence we have

$$P(A_n) = \int_0^\infty \ldots \int_0^\infty H(x_1)\ldots H(x_1 + \ldots + x_n)dF(x_1)\ldots dF(x_n), \tag{5}$$

$$n \geq 1,$$

$$P(A_0) = 1.$$

We suppose that $P(A_1) > 0$ (the case $P(A_1) = 0$ corresponds to the case when during the dead time there arrive infinitely many particles). If we define by ν the number of particles which arrive at the counter during the dead time, then $P(\nu = n) = P(\bar{A}_1 \ldots \bar{A}_{n-1} A_n)$ and for $P_n = P(\nu = n)$ we have

$$P_1 = P(A_1),$$

$$P_n = P(A_n) - \sum_{i=1}^{n-1} P(A_i)P_{n-i}, \quad n \geq 2. \tag{6}$$

It is clear that $P(A_n) \geq P(A_{n+1})$, and there exists $\lim_n P(A_n) = P_\infty$ and $M(\nu) = 1/P_\infty$. In ref.[3] there are the sufficient conditions to ensure $P_\infty > 0$.

Let us put

$$a_n = P(A_n) - P(A_{n+1}), \quad n = 0,1,\ldots .$$

Hence for the generating function $f(z) = M(z^\nu)$ of ν we have

$$f(z) = z(1 - \sum_{n=0}^\infty a_n z^n)(1 - z\sum_{n=0}^\infty a_n z^n)^{-1}, \quad |z| \leq 1. \tag{7}$$

Define

$$a(s) = \int_0^\infty e^{-sx} dF(x), \quad s \geq 0,$$

$$\mu = \int_0^\infty x dF(x).$$

With the given recurrent primary process $\{r_n\}_{n=0}^\infty$ we define the new recurrent one $\{r_n^s\}_{n=0}^\infty$ for any $s \geq 0$ with the distribution function

$$F_s(x) = P(r_n^s - r_{n-1}^s < x) = a^{-1}(s) \int_0^x e^{-st} dF(t).$$

Let $f_s(z)$ be the generating function of the number ν_s of the particles arriving during the dead time according to the primary process $\{r_n^s\}_{n=0}$ and the lengths of impulses $\{\chi_n\}_{n=0}^\infty$. Then we obtain the following forms for $\Phi(s,z) = M(e^{-sZ_1} z^\nu)$ and $\gamma(s)$, respectively.

Theorem 1. For any $s \geq 0$, $|z| \leq 1$

$$\Phi(s,z) = f_s(a(s)z), \tag{8}$$

$$\gamma(s) = f_s(a(s)), \tag{9}$$

$$M(Z_1) = \mu M(\nu). \tag{10}$$

Proof. Since $Z_1 = r_\nu$ we have

$$\Phi(s,z) = \sum_{n=1}^{\infty} \int_{\{\nu=n\}} e^{-sr_\nu} z^\nu dP =$$

$$= \sum_{n=1}^{\infty} \int \cdots \int_{C_n} e^{-s(t_1+\dots+t_n)} z^n dF(t_1)\dots dF(t_n) dH(x_1)\dots dH(x_n),$$

where the integration area C_n is of the following form

$$(x_1 < t_1)^c, \begin{pmatrix} x_1 < t_1 + t_2 \\ x_2 < t_2 \end{pmatrix}^c, \dots, \begin{pmatrix} x_1 < t_1 + \dots + t_{n-1} \\ \vdots \\ x_{n-1} < t_{n-1} \end{pmatrix}^c,$$

$$\begin{pmatrix} x_1 < t_1 + \dots t_n \\ \vdots \\ x_n < t_n \end{pmatrix}$$

(here the sign "c" denotes the complement of the set mentioned in the parentheses).

Hence

$$\Phi(s,z) = \sum_{n=1}^{\infty} a^n(s) z^n P(\nu_s = n) = f_s(a(s)z).$$

Analogically we proceed for $\gamma(s)$. The mean value of Z_1 is obtained from the Wald identity.

Q.E.D.

4. THE PROPERTIES OF ν

The integer-valued random variable $\nu - 1$ determines the number of the nonregistered particles between two successive registrations. Although in the physical practice it is hardly observable value during the registration process by counters, it has an importance in other practical applications. For example, in the film handling of track information the ν means the number of the streamers with constant diameters in the blobs [7,12]. Some limiting properties of ν, when $P_\infty \to 0$, are investigated in ref. [3] and it was proved that $P(\nu > n) \approx e^{-P_\infty n}$.

Now we examine the behaviour of $P(\nu = n)$ in dependence on n. Therefore we need the next notion.

A distribution function F concentrated on $<0,\infty)$ is Cramer one if

$$\sup\{\lambda \geq 0 : \int_0^\infty e^{\lambda x} dF(x) < \infty\} = \infty.$$

It is clear that the set C of all the Cramer distribution functions is convex. Moreover, if $F(x_0) = 1$ for some $x_0 > 0$, then $F \in C$, and if $dF(x) = a\exp(-bx^c), x \geq 0$, for some $a \geq 0, b \geq 0, c \geq 2$, then $F \in C$.

Theorem 2. Let the primary recurrent process have the Cramer distribution function F and $P_\infty > 0$. Then

$$P_n = (\beta - 1)\beta_1 \beta^{-n-1} + r_n, \tag{11}$$

where

$$\beta = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^{k-1}}{dz^{k-1}} [\psi^k(z)]|_{z=1}, \tag{12}$$

$$\beta = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{d^k}{dz^k} [\psi^k(z)]|_{z=1}, \tag{13}$$

and

$$|r_n| \leq CR^{-n}$$

(the constant C does not depend on n, and $R > 1$). Here

$$\psi(z) = P(A_1)z + \sum_{n=2}^{\infty} (P(A_n) - P(A_{n-1}))z^n.$$

Proof. According to the Cauchy formula [13] we have

$$P_n = \frac{1}{2\pi i} \oint_{|z|=1} \frac{f(z)dz}{z^{n+1}} = \frac{1}{2\pi i} \oint_{|z|=1} \frac{\psi(z)dz}{(1 - z + \psi(z))z^{n+1}}.$$

From the conditions of the theorem we have

$$0 \leq a_n = P(A_n) - P(A_{n+1}) = P(\chi_1 < r_1, \dots, \chi_n < r_n, \chi_{n+1} \geq$$
$$\geq r_{n+1}) \leq P(\chi_{n+1} \geq r_{n+1}).$$

$$\gamma(s) = f_s(a(s)), \tag{9}$$

$$M(Z_1) = \mu M(\nu). \tag{10}$$

Proof. Since $Z_1 = \tau_\nu$ we have

$$\Phi(s,z) = \sum_{n=1}^{\infty} \int_{\{\nu=n\}} e^{-s\tau_\nu} z^\nu dP =$$

$$= \sum_{n=1}^{\infty} \int \cdots \int_{C_n} e^{-s(t_1+\dots+t_n)} z^n dF(t_1)\dots dF(t_n) dH(x_1)\dots dH(x_n),$$

where the integration area C_n is of the following form

$$(x_1 < t_1)^c, \quad \begin{pmatrix} x_1 < t_1 + t_2 \\ x_2 < t_2 \end{pmatrix}^c, \dots, \begin{pmatrix} x_1 < t_1 + \dots + t_{n-1} \\ \vdots \\ x_{n-1} < t_{n-1} \end{pmatrix}^c,$$

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From the conditions of the theorem we have

$$0 \le a_n = P(A_n) - P(A_{n+1}) = P(\chi_1 < \tau_1, \dots, \chi_n < \tau_n, \chi_{n+1} \ge$$
$$\ge \tau_{n+1}) \le P(\chi_{n+1} \ge \tau_{n+1}).$$

Since $F \subset C$ we obtain

$$P(\chi_{n+1} \geq r_{n+1}) \leq M(e^{\lambda \chi_1}) M(e^{-\lambda r_1})^{n+1}$$

for any $\lambda \geq 0$. Hence the series $\sum_{n=0}^{\infty} a_n z^n$ has the convergence radius

$$R \geq 1/M(e^{-\lambda r_1}) > 1;$$

when $\lambda \to \infty$, then $R = \infty$. Therefore the equation $1 - z + \psi(z) = 0$ has a unique (simple) positive root $z = \beta > 1$ with the minimal module. This follows from the following

$$1 - z + \psi(z) = 1 - z \sum_{n=0}^{\infty} a_n z^n, \quad \text{and } a_n \geq 0.$$

Let $R > 1$ be a radius of a circle in which the function $1 - z + \psi(z)$ has a unique zero $z = \beta$. Then

$$r_n = \frac{1}{2\pi i} \oint_{|z|=R} \frac{f(z)\,dz}{z^{n+1}} = \frac{\psi(\beta)}{(\psi'(\beta) - 1)\beta^{n+1}} + P_n. \tag{14}$$

The integral in the left-hand side of (14) may be estimated by the maximum module $|r_n| < CR^{-n}$. Putting $\beta_1 = 1/(1 - \psi'(\beta))$ we obtain the formula (11).

To obtain the explicit expressions for β and β_1, respectively, we consider a function $w = z - \psi(z)$ which in a conform way transforms some neighbourhood of the point $w = 1$ to other one. Therefore $w = w(z)$ has its inverse function $z = z(w)$. It is clear that $\beta = z(1)$ and $\beta_1 = z'(1)$. Using the Lagrange expansion formula we obtain the (11) and (13).

Q.E.D.

5. TYPE III COUNTER

G.E.Albert and L.Nelson[4] used more general form of the counter model which contains both the mentioned Type I and Type II counters as special cases.

They supposed that if the particle arrives at the counter, then the impulse (of the length χ_n) starts with probability p if at time r_n an impulse is in course, and with probability 1 otherwise. If $p = 0$, then we obtain the Type I counter; and if $p = 1$, the discussed above Type II counter.

This model has been studied by several authors [4,24,26-28,15,17-19]. The distribution function related with the secondary process for $p > 0$ may be easily deduced from the one of the Type II counter as it was mentioned by Takács[28].

Indeed, let us suppose that $0 < p < 1$. We define the primary process with the distribution function

$$\hat{F}(x) = p \sum_{n=1}^{\infty} q^{n-1} F_n(x),$$

where $q = 1 - p$, and $F_n(x)$ denotes the n-th iterated convolution of the distribution function with itself. It is easy to see that the only difference between the secondary process of the Type III counter determined by $F(x)$, $H(x)$ and p, and of the Type II determined by $\hat{F}(x)$, $H(x)$, is that the latter contains an additional interval spent in the state E_0 immediately before every transformation $E_0 \to E_1$. The lengths of these intervals are identically distributed, independent random variables with the distribution function

$$Q(x) = p \sum_{n=0}^{\infty} q^n F_n(x),$$

and these random variables are independent of any other random variables involved.

If $\gamma(s)$ and $\hat{\gamma}(s)$ denote the Laplace transforms of the distances between successive registrations by the Type III counter and by the Type II counter mentioned above, then we have

$$\hat{\gamma}(s) = \gamma(s) p (1 - q a(s))^{-1},$$

where $a(s)$ is the Laplace transform of the distribution function $F(x)$. Hence we have

$$\gamma(s) = \hat{\gamma}(s)(1 - q a(s)) p^{-1}. \tag{15}$$

Here $\hat{\gamma}(s)$ is determined by (9) in which we change $F(x)$ by $\hat{F}(x)$.

6. MARKOV RENEWAL PRIMARY PROCESS OF ZERO ORDER

In this section the problem of registration of one type of particles is generalized to the case of m types of particles $(1 \leq m < \infty)$ which arrive at the Type II counter.

Let us suppose that there be m types of radioactive materials which emit m types of particles according to the Markov renewal primary process of zero order. These processes have been introduced and studied by Pyke[16].

Thus, we suppose that the relationships between different types of particles and their impulses are as follows. Let the n-th particle $(n \geq 0)$ arriving at the counter be of the type J_n and let the type of the particle do not depend on the previous types of particles, and

$$P(J_n = k) = p_k,\ k = 1,\dots,m,\ \sum_{k=1}^{m} p_k = 1.$$

Let $F_i(x)$ and $H_i(x), i = 1,\dots,m$, be distribution functions with $F_i(0) = H_i(0) = 0$ for each i. Then we define the process of interarrival times $\{T_n\}_{n=1}^{\infty}$, where T_n is the time between the arrival of the (n-1)th and n-th particles, specifying $T_0 = 0$ and

$$P(T_n < x/T_0,\dots,T_{n-1}, J_0,\dots,J_n) = F_{J_n}(x) \quad \text{a.s.} \tag{16}$$

for all $x \geq 0$ and $n > 0$.

We see that the primary process $\tau_n = \sum_{i=0}^{n} T_i, n \geq 0$, is recurrent one determined with the function

$$F(x) = \sum_{j=1}^{m} p_j F_j(x).$$

The lengths of impulses $\{\chi_n\}_{n=0}^{\infty}$ of particles arriving at the moments τ_n, $n \geq 0$, are determined as follows

$$P(\chi_0 < x/J_0, T_0) = H_{J_0}(x) \quad \text{a.s. for all } x \geq 0,$$

$$P(\chi_n < x/J_0,\dots,J_n, \chi_0,\dots,\chi_{n-1}, T_0,\dots,T_{n-1}) = H_{J_n}(x) \quad \text{a.s.} \tag{17}$$

for all $x \geq 0$ and $n > 1$.

It results from (16) and (17) that the lengths of impulses are independent, positive random variables with the common distribution function

$$H(x) = \sum_{j=1}^{m} p_j H_j(x), \quad x \geq 0,$$

and they do not depend on $\{\tau_n\}$. Then the above interesting characteristics of the resulting secondary process may be determined by the methods developed in the third section.

7. EXAMPLES

Example 1. Let $F(x) = 1 - e^{-\lambda x}$, $x \geq 0$, and $H(x)$ be an arbitrary distribution function, then

$$P(A_n) = \lambda^{n+1}/n! \int_0^{\infty} \{ \int_0^{t} H(x)dx \}^n e^{-\lambda t}\, dt, \quad n = 0,1,\dots,$$

$$f(z) = 1 - (\lambda \int_0^{\infty} \exp(-\lambda \int_0^{t} (1 - zH(x))dx)dt)^{-1}, \quad |z| \leq 1,$$

$$\Phi(s, z) = 1 - ((\lambda + s) \int_0^{\infty} \exp(-st - \lambda \int_0^{t} (1 - zH(x))dx)dt)^{-1},$$

$$s \geq 0, \ |z| \leq 1,$$

$$\gamma(s) = 1 - ((\lambda + s) \int_0^{\infty} \exp(-st - \lambda \int_0^{t} (1 - H(x))dx)dt)^{-1},$$

$$s \geq 0.$$

If $h = \int_0^{\infty} x dH(x) < \infty$, then by /29/ $\lim_n P(A_n) = e^{-\lambda h}$. Hence

$$M(\nu) = e^{\lambda h},$$

$$M(Z_1) = e^{\lambda h}/\lambda.$$

The expression for γ agrees with one in ref./24/. By an analogical way we may obtain the formula for the γ for the Type III counter since $F(x) = 1 - e^{-\lambda p x}$, $x \geq 0$.

Example 2. Let $F(x) = 1$ if $x > a > 0$ for some a, elsewhere 0 and H be an arbitrary distribution function with $H(a) \neq 0$. Then

$$P(A_n) = \prod_{i=1}^{n} H(ia), \quad n = 0,1,\dots,$$

Here the empty product is equaled to 1. Hence, if $H(na) \neq 1$ for any $n = 1,2,\dots$, then $\lim_n P(A_n) = 0$ and therefore $M(\nu) = \infty$.

Now let $n_0 + 1$ be the minimal integer n such that $H(na) = 1$, then

$$M(\nu) = 1/\prod_{i=1}^{n_0} H(ia),$$

and for the generating function of ν we have

$$f(z) = z(1 - \sum_{n=0}^{n_0-1} a_n z^n)(1 - \sum_{n=0}^{n_0-1} a_n z^n)^{-1},$$

where

$$a_n = (1 - H((n+1)a)) \prod_{i=1}^{n} H(ia).$$

Hence we obtain

$$\Phi(s, z) = f(ze^{-as}),\quad s \geq 0,\quad |z| \leq 1,$$

$$\gamma(s) = f(e^{-as}),\quad s \geq 0.$$

Example 3. Let F be an arbitrary distribution function, and $H(x) = 1$ if $x > b > 0$ for some b, and zero elsewhere. Then

$$P(A_n) = I,\quad n = 1,2,\ldots,$$

$$P_n = I(1 - I)^{n-1},\quad n = 1,2,\ldots,$$

where

$$I = 1 - F(b - 0)$$

(we assume that $F(b-0) \neq 1$). Moreover

$$M(\nu) = 1/I,$$

$$f(z) = Iz(1 - (1 - I)z)^{-1},\quad |z| \leq 1,$$

$$\Phi(s, z) = z \int_b^{\infty} e^{-sx} dF(x)/(1 - z\int_0^b e^{-sx} dF(x)),\quad s \geq 0,\ |z| \leq 1,$$

$$\gamma(s) = \int_b^{\infty} e^{-sx} dF(x)/(1 - \int_0^{\infty} e^{-sx} dF(x)),\quad s \geq 0.$$

The expression for $\gamma(s)$ is the same as one in refs./15,28/.

Let us put

$$\bar{F}(x) = P(r_1 < x / r_1 < b),$$

$$\bar{\bar{F}}(x) = P(r_1 < x / r_1 \geq b),$$

and let $\bar{\mu}_r$ and $\bar{\bar{\mu}}_r$ be the r-th moments with respect to the distribution functions $\bar{F}(x)$ and $\bar{\bar{F}}(x)$, respectively. Then for the dead time B we obtain

$$M(B) = b + (1 + I)\bar{\mu}_1/I,$$

$$M(B^r) = b^r + (1 - I)I^{-1} \sum_{j=0}^{r-1} M(B^j)\bar{\mu}_{r-j},\quad r = 2,3,\ldots,$$

and for the moments of Z_1 we have

$$M(Z_1) = \mu/I,$$

$$M(Z_1^r) = \sum_{\substack{r_1 = r_2 + r_3 = r \\ r_i \geq 0}} M(B^{r_1})(-b)^{r_2}\,\bar{\bar{\mu}}_{r_3},\quad r = 2,3,\ldots,$$

when

$$\mu = \int_0^{\infty} dx H(x) < \infty.$$

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Received by Publishing Department
on April 19, 1983.

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Двуреченский А., Ососков Г.А. E5-83-255

К одной проблеме счетчиков продлевающего (II) типа

В работе исследуется явный вид совместного преобразования Лапласа расстояний между двумя соседними моментами регистрации частиц счетчиком типа II /счетчик с мертвым временем продлевающего типа/ в общем случае и числа частиц, пришедших за период мертвого времени. Этим найдено явное решение сложных интегральных уравнений, выведенных другими авторами. Кроме того, исследуется геометрическое поведение распределения числа частиц. Получено решение для счетчика типа III и изучается регистрация m типов частиц. Эти задачи применимы также к проблемам определения распределения длин блобов в стримерных камерах в физике высоких энергий.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Dvurečenskij A., Ososkov G.A. E5-83-255

Note on One Type II Counter Problem

An explicit form of the joint Laplace transform of the distances between two neighbouring moments of particle registration by Type II counter (the counter with prolonged dead time) in the general case, and the number of particles arriving during the dead time is investigated. An explicit solution is given to complicated integral equations determined by other authors. Moreover, the geometric behaviour is studied of the distribution of the number of particles. The Type III counter problem is mentioned, and the case of registration of m types of particles is discussed. These problems are applicable to ones of the distribution determination of blob length in the streamer chambers in the high energies physics.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983