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ON THE REDUCTION OF YANG-MILLS POTENTIALS IN GAUGE FIELD THEORIES WITH SPONTANEOUSLY BROKEN SYMMETRY

1980

Николова Л., Ризов В.А. **E5-80-485**

О редукции потенциалов Янга-Миллса в калибровочных теориях со спонтанным нарушением симметрии

Рассматривается спонтанное нарушение симметрии в калибровочной теории поля вследствие присутствия в теории нелинейно преобразующихся полей. Нелинейно преобразующиеся поля играют роль "редуцирующих полей" согласно теореме о редукции структурных групп главных расслоенных пространств. На основе последней делается пересмотр некоторых свойств редукции калибровочной теории поля с калибровочной группой G до теории, в которой калибровочная группа является подгруппой группы G. Обсуждается прежде всего редукция формы связности на главном расслоенном пространстве (потенциала Янга-Миллса). В рамках редукции естественным образом возникает форма связности принимающая значения в алгебре Ли группы, содержащей калибровочную группу как замкнутую подгруппу. Изучается более детально спонтанное нарушение симметрии в случае, когда теория допускает разложение формы связности в виде суммы сводимой формы связности и горизонтальной 1-формы.

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Nikolova L., Rizov B.A. **E5-80-485**

On the Reduction of Yang-Mills Potentials in Gauge Field Theories with Spontaneously Broken Symmetry

The spontaneous breaking of the symmetry in a gauge field theory due
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We consider a gauge field theory with spontaneous breaking of the symmetry due to the presence of nonlinearly transforming fields. Treating the gauge group G as a structure group of a principal fibre bundle and the fields as cross-sections in associated fibre bundles, the nonlinearly transforming fields play the role of "reducing fields" according to the reduction theorem for structure groups of principal fibre bundles. Based on the latter we review some features of the reduction of the gauge field theory with gauge group G to a gauge field theory, which is gauge invariant with respect to a subgroup of G. We put an emphasis on the reduction of the Yang-Mills potential (a connection form on the principal fibre bundle). Within the frames of that reduction there arises in a natural way a Yang-Mills potential, which takes values in the Lie algebra of a group, containing the gauge group as a closed subgroup (Eq.(3)). We elaborate in more detail the spontaneous symmetry breaking in case the Lie algebra of the gauge group satisfies Eq.(4). Then every "reducing field" relates to the Yang-Mills potential in the theory another one, which is "correlated" with the "reducing field" (Eqs.(5),(9),(11)). The two Yang-Mills potentials are connected by a 1-form field (Eq.(10)). Finally, we write the relation between the latter and the "reducing field" (Eq.(8)).

We consider a gauge field theory as being defined by (a Yang-Mills potential) a connection form on a principal trivial fibre bundle $P(X,G)$ with base X and structure Lie group G and a set of (fields) cross-sections of fibre bundles associated to P (see e.g. /1,2,3/ ,) without fixing the details of their dynamics.

1. Let ω be a connection form on P, i.e., $\omega_p : \tau_p(P) \to \mathcal{G}$, for every $p \in P$, where $\tau_p(P)$ is the tangent space to P at the point p* and $\mathcal{G}$ denotes the Lie algebra of G. The pull back of ω by a cross-section $q : X \to P$, $\Gamma_q = q^*\omega$ is the Yang-Mills potential (corresponding to ω) in the gauge q. In terms of another cross-section q', $q'(x) = q(x)\,g(x)$,

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*We denote by $\tau(M)$ the tangent bundle of the manifold M and by $\tau_m(M)$ the tangent space at $m \in M$.

$g : X \to G$, one has

$$\Gamma_{q'(x)} = (q'^{*}\omega)_x = \mathrm{ad}(g^{-1}(x))\,\Gamma_{q(x)} + g^{-1}(x)\,dg(x) .$$

2. Every field $\hat{\phi}$ is considered as a cross section of a fibre bundle $E = (P\times\Phi)/G$ associated to P with standard fibre, the manifold Φ (the action of G on Φ, $T(g) : \Phi \to \Phi$, $g \in G$ is given). Relative to the cross-section (the gauge) $q : X \to P$ every $\hat{\phi} : X \to E$ is represented by a (smooth) mapping $\phi : X \to \Phi$. By means of the cross-section q', $q'(x) = q(x)\,g(x)$, $g : X \to G$, one has $\phi' : X \to \Phi$, $\phi'(x) = T(g^{-1}(x))\,\phi(x)$.

3. We suppose that among the fields there are cross-sections of the fibre bundle $K = (P\times G/H)/G$ associated to P with standard fibre the homogeneous space G/H of G, where H is a closed subgroup of G and the action of G on G/H, $F(g) : G/H \to G/H$, $g \in G$, is the natural one. The cross-sections $\hat{z} : X \to K$ are those, which "break the symmetry spontaneously". In the context of a Lagrangian field theory they arise usually from the values of the Higgs field, which minimize the "potential energy" or from boundary conditions, which ensure finite energy (e.g., monopole type) solutions of the field equations (see [5,6]). By means of the cross-section (the gauge) $q : X \to P$ every such field $\hat{z} : X \to K$ is described by a mapping $z : X \to G/H$.

The covariant derivative (with respect to ω) $\nabla\hat{z}$ of the field $\hat{z} : X \to K$ is a 1-form field on X with values in the associated bundle $K^{\tau} = (P\times\tau(G/H))/G$ (cf. [4,3]). In terms of the gauge q it is given by a 1-form ∇z on X with values in $\tau(G/H)$:

$$\nabla z_x(t) = dz_x(t) + F(z(x))_{*}(\Gamma_{q(x)}(t)) , \tag{1}$$

where $x \in X$, $t \in \tau_x(X)$, and the second term in the right-hand side of (1) is the value at $z(x) \in G/H$ of the fundamental vector field on G/H corresponding to $\Gamma_{q(x)}(t) \in \mathcal{G}$.

According to the reduction theorem [8, Ch.1], to every $\hat{z} : X \to K$ there corresponds a reduction of the principal fibre bunde $P(X, G)$ to a principal fibre bundle $Q(X, H)$ with a structure group H. We call $\hat{z} : X \to K$ reducing field. Q appears as a subbundle of P with projection $\pi_Q : Q \to X$ which is the restriction of $\pi : P \to X$ to Q. In general Q is not a trivial bundle, although P is.

A collection of local trivializations $\{q_\alpha\}$ of Q is obtained in the following way (cf. [3]). Let $\{V_\sigma : \sigma \in I\}$ be an

open covering of X such that for every $\sigma \in I$ there exists a differentiable map $g_\sigma : V_\sigma \to G$ which satisfies

$$z|_{V_\sigma}(x) = F(g_\sigma(x))(eH) , \tag{2}$$

where e is the unit element of G. Then a set of local trivializations (local gauges) $q_\sigma : V_\sigma \to Q$ is given by $q_\sigma(x) = q(x)\,g_\sigma(x)$ for every $\sigma \in I$ and

$$\pi_Q^{-1}(V_\sigma) = \{ q_\sigma(x) h : x \in V_\sigma ,\ h \in H \}$$

4. The restriction

$$\omega^{\#} = \omega|_Q \tag{3}$$

of the connection form ω to $Q \subset P$ determined by

$$\Gamma_{q_\sigma(x)} : \tau_x(X) \to \mathcal{G}$$

$$\Gamma_{q_\sigma(x)} = \mathrm{ad}(g_\sigma^{-1}(x))\,\Gamma_{q(x)} + g_\sigma^{-1}(x)\,dg_\sigma(x)$$

is a $\mathcal{G}$ - valued connection form on the reduced H-subbundle Q *.

Let $\mathcal{G}$ admit the decomposition (as a vector space):

$$\mathcal{G} = \mathcal{H} \oplus \mathcal{K} , \quad \mathrm{ad}(h)\mathcal{K} \subset \mathcal{K} , \qquad \text{for every} \quad h \in H , \tag{4}$$

where $\mathcal{H}$ is the Lie algebra of H, and $\mathcal{K}$ is identified with $\tau_{eH}(G/H)$.

Then an $\mathcal{H}$-valued connection form $\alpha^{\#}$ on Q is determined [8, Ch.11] by the projection

$$A_{q_\sigma(x)} = (\Gamma_{q_\sigma(x)})_{\mathcal{H}} \tag{5}$$

of $\Gamma_{q_\sigma(x)}$ onto $\mathcal{H}$ ($A_{q_\sigma(x)} = q_\sigma^{*}\alpha^{\#}$ is the Yang-Mills potential corresponding to $\alpha^{\#}$ in the local gauge q_σ).

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*Since Q is a subbundle of P, $\tau_p(Q)$ for every $p \in Q$ is a subspace of $\tau_p(P)$ and can be decomposed into horizontal and vertical parts (with respect to ω): $\tau_p(Q) = H_p(Q) + V_p(Q)$. In general, of course $V_p(Q) \not\subset \tau_p(Q)$. The form $\omega^{\#}$ annihilates the vectors from $H_p(Q)$ and only them. These vectors we call horizontal in Q with respect to $\omega^{\#}$.

5. The difference $\tilde{\nu}^{\#} = \omega^{\#} - \alpha^{\#}$ is a horizontal $\mathcal{K}$-valued 1-form on Q. It corresponds to a 1-form field $\hat{\nu}^{\#}$ on the base X with values in the associated bundle $K_Q = (Q \times K)/H$. In terms of the local trivializations $\{q_\sigma\}$ we give $\hat{\nu}^{\#}$ through the set of 1-forms

$$\nu_\sigma = q_\sigma^* \tilde{\nu}^{\#} = (\Gamma_{q_\sigma})_{\mathcal{K}} , \tag{6}$$

where $(\Gamma_{q_\sigma})_{\mathcal{K}}$ is the projection of Γ_{q_σ} onto $\mathcal{K}$. The local forms ν_σ transform according to the representation $h \to \mathrm{ad}(h^{-1})$ of H.

6. The reduction of $P(X,G)$ to $Q(X,H)$ puts in correspondence to every field $\hat{\phi}$ a reduced field $\hat{\phi}^{\#}: X \to E_Q = (Q \times \Phi)/H$. The latter is defined by a mapping $\phi_\sigma: V_\sigma \to \Phi$, $\phi_\sigma(x) = T(g_\sigma^{-1}(x))\phi(x)$ in every local gauge q_σ, $\sigma \in I$.

7. The set of reduced fields $\{\hat{\phi}^{\#}\}$ obtained from the fields $\{\hat{\phi}\}$ of the initial theory with gauge group G, the $\mathcal{H}$-valued connection form $\alpha^{\#}$ in Q and the 1-form field $\hat{\nu}^{\#}$ constitute the reduced field theory gauge invariant with respect to H. The field $\hat{z}^{\#}: X \to K_Q = (Q \times G/H)/H$, corresponding to the reducing field $\hat{z}$, in every local gauge q_σ is given by the constant mapping (cf.Eq.(2)):

$$z_\sigma : V_\sigma \to \Phi, \quad z_\sigma(x) = eH . \tag{7}$$

Its covariant derivative with respect to $\omega^{\#}$ equals the 1-form field $\hat{\nu}^{\#}$ (cf.Eqs.(1),(6)):

$$\nabla \hat{z}^{\#} = \hat{\nu}^{\#} \tag{8}$$

($\mathcal{K}$ is identified with $\tau_{eH}(G/H)$).

This relation and Eq.(6) enable us to find a parallel between the massive vector fields in the gauge theories with spontaneously broken symmetry via a Higgs-Kibble mechanism and the covariant derivative of the nonlinearly transforming field which appears in field theories based on nonlinear Lagrangians (see, e.g., /7,9/).

8. Since $Q \subset P$ is a subbundle of P it is possible to extend $\alpha^{\#}$ and $\tilde{\nu}^{\#}$ to a $\mathcal{G}$-valued connection form α on P and to a horizontal $\mathcal{G}$-valued 1-form $\tilde{\nu}$ on P, respectively (so that $\alpha|_Q = \alpha^{\#}$, $\tilde{\nu}|_Q = \tilde{\nu}^{\#}$). To this end we gauge transform $A_{q_\sigma(x)}$ and $\nu_{\sigma(x)}$ by $g_\sigma^{-1}(x)$:

$$A_{q_\sigma(x)} \to A_{q(x)} = \mathrm{ad}(g_\sigma(x)) A_{q_\sigma(x)} + g_\sigma(x)\, dg_\sigma^{-1}(x),$$
$$\nu_{q_\sigma(x)} \to \nu_x = \mathrm{ad}(g_\sigma(x))\, \nu_{\sigma x}, \quad x \in V_\sigma . \tag{9}$$

This provides us in the gauge $q: X \to P$ both with a Yang-Mills potential $A_q = q^*\alpha$ and a $\mathcal{G}$-valued 1-form $\nu = q^*\tilde{\nu}$ on X. The latter transforms according to the representation $g \to \mathrm{ad}(g^{-1})$ of G. We have, of course,

$$\Gamma_q = A_q + \nu . \tag{10}$$

Therefore, when the structure group G of P reduces to a subgroup H and its Lie algebra $\mathcal{G}$ satisfies Eq.(4), with every connection form ω on P, we can associate, firstly, a $\mathcal{G}$-valued connection form α on P reducible to an $\mathcal{H}$-valued connection form on Q, and, secondly, a horizontal $\mathcal{G}$-valued 1-form $\tilde{\nu}$ (so that $\omega = \alpha + \tilde{\nu}$). The horizontal 1-form $\tilde{\nu}$ corresponds to a 1-form field $\hat{\nu}(\tilde{\nu}|_Q = \hat{\nu}^{\#})$ on the base X with values in the associated fibre bundle $(P \times \mathcal{G})/G$.

Finally let us point out the fact that the reducing field which gives rise to the reduction is covariantly constant with respect to the connection form α on P. Denoting the covariant differentiation with respect to α by ∇' we have (cf.Eqs.(1),(5) and (7)): $\nabla' z = 0$ in every gauge. That property has been used in /10/ to consider the so-called inverse Higgs effect.

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