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EUCLIDEAN FIELDS: LATTICE MODELS

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1. Transversal lattice fields

We will reformulate the theory of the free Euclidean transversal vector field of mass $m \geq 0$, on a cubic space-time lattice $X^{(c)} \subset R^{d+1}$, for $c > 0$ and $d \geq 2$. For simplicity we fix the lattice constant to $c=1$ and the dimension of space $d=3$, except stated otherwise.

We keep most of the notations for the continuum theory which we have discussed in parts I and II of the paper (JINR E5-12780,12779).

In the Cartesian basis $\{n^\mu : (n^\mu)^\nu = \delta^{\mu\nu};\ \mu,\nu = 0,1,2,3\}$ in R^4 we write

(1) $\quad x = (x^0, \underline{x}) = x^\mu n^\mu \in X, \ x^\mu$ integer,

for a lattice point and $|x-y| = \sum_{\{\mu\}} |x^\mu - y^\mu|$ for the distance of $x, y \in X$. By the Fourier transformation T

(2) $\quad T:\ f \to \tilde{f}(p) = (2\pi)^{-2} \sum_X e^{ip.x} f_x, \quad p \in I = \times_{\{\mu\}} [-\pi, +\pi],$

we identify the complex Hilbert spaces $l^2(X)$ and $L^2(I,dp)$. We define the Nabla operator ∇^+ on the lattice by the action on a scalar function $f = (f_x)_{x \in X}$

(3) $\quad (\nabla^+ f)^\mu_x = f_{x+n^\mu} - f_x.$

Then

(4) $$(-\Delta + m^2)_{xy} = \begin{cases} 2(d+1)+m^2 & x=y \\ -1 & |x-y|=1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\Delta = \nabla^+ . \nabla^- = \nabla^- . \nabla^+$ is the lattice Laplacean, is a standard expression. The operator ∇^- is defined likewise by

(5) $\quad \nabla^- = T_a (\nabla^+) T_{-a}, \quad a = n^\mu ,$

where $(T_a f)_x = f_{x-a}$, $a \in R^4$, or by $\nabla^- = -(\nabla^+)^*$ in $l^2(X)$.

(6) $\quad T(-\Delta)T^{-1} = |w(p)|^2 = 2 \sum_{\{\mu\}} (1 - \cos p^\mu),$

where $w^\mu(p) = T(\nabla^+)^\mu T^{-1}$.

Let G^m be the Green's function for $-\Delta + m^2$ with free boundary conditions.

The one-particle Hilbert space for the free Euclidean scalar field of mass $m \geq 0$ on the lattice X is

(7) $\quad L = \{u = (u_x)_{x \in X}:\ \sum_X G^m_{xy} \bar{u}_x u_y < \infty\},$

in the momentum representation $L = L^2(I, \frac{dp}{|w|^2 + m^2})$.
For $m=0$, the map $u \to \nabla^+ u$ defines an isometric embedding of $l^2(X)$ in $N' = \sum_{\{\mu\}}^{\oplus} L$. Using the orthonormal basis $\{\delta_y : y \in X\}$ in $l^2(X)$, we can define a projection E in N' as follows:

(8) $\quad Ef = f - \sum_X \langle s_y, f \rangle_{N'} s_y ,$

(9) $$(s_y)^\mu_x = (\nabla^+ \delta_y)^\mu_x = \begin{cases} +1 & x = y - n^\mu \\ -1 & x=y \\ 0 & \text{otherwise} \end{cases}$$

are the unit sources on X. For $m \geq 0$, E is defined by

(10) $\quad N = \operatorname{ran} E = \{h \in N' : \nabla^- . h = 0\},$

in particular, for the model in Euclidean space-time R^3 E is given by the identity $\Delta E f = \nabla^+ \times (\nabla^- \times f)$, $f \in N'$. In the lemma 1 we change the notation and index the lattice model by "c", in order to distinguish it from the continuum one.

For a region $\mathcal{O} \subset R^4$, we let $\mathcal{O}^{(c)} = \mathcal{O} \cap X^{(c)}$, where for $c > 0$ $X^{(c)} = \{x = x^\mu n^{(c)\mu} : x^\mu \text{ integer and } n^{(c)\mu} = c.n^\mu\}$. We denote by $\chi_{I^{(c)}}$ the characteristic function of the corresponding momentum box

(11) $\quad I^{(c)} = \{p \in R^4 : |p^\mu| \leq \frac{\pi}{c}\}.$

Lemma 1 Let m=0. Then

(12) $\tilde{J}^{(c)}: \tilde{u} \to x_{I(c)} \cdot |w^{(c)}| \cdot |p|^{-1} \tilde{u}$

defines a non-local isometric embedding of $L^{(c)}$ in L. Generalized to the vector case, $J^{(c)}$ is neither local nor transversal, i.e. for given $h \in N_{\mathcal{O}}(c)$

(13) supp $J^{(c)}h \not\subset \mathcal{O}$ and $\nabla . J^{(c)}h \neq 0$.

Proof: The multiplication operator $\tilde{J}^{(c)}$ has a non-local Fourier transform, and $J^{(c)}w(c) \neq ip$, except c=0. Note that lemma 1 generalizes to the massive case as well.

Moreover, using modified arguments given by Guerra, Rosen and Simon for the scalar model we can prove a convergence theorem $(c \downarrow 0)$. Q.E.D.

We call a region $\mathcal{O}$ regular if $x \in \mathcal{O}^{int}$ implies $x+n^\mu-n^\nu \in \mathcal{O}$; $\mu,\nu=0,1,2,3$. Of course, the interior of the half-space $\Lambda_+ = \{x: x^0 \geq 0\}$ is regular. Let

(14) $N_+ = \{h \in N: \ h_x = 0 \text{ for } x \notin \Lambda_+\}$,

and E_+ the corresponding projection in N. We introduce the reflection $\theta_0: x \to (-x^0, \underline{x})$ at the hyperplane S_0, and the unitary operator

(15) $(\theta_0 h)^\mu_x = g^{\mu\nu} h^\nu_{\theta_0 x}$, $g^{\mu\nu} = \text{diag}\,(-+++)$.

Lemma 2 For $h \in N_+$

(16) $\langle h, \theta_0 h \rangle_N = \sum g^{\mu\nu} G^m_{x/\theta_0 y} \bar{h}^\mu_x h^\nu_y \geq 0.$

In particular, for m=0 $E_+\theta_0 E_+ = E_0$ is the projection onto the subspace $N_{S_0} = \{h \in N: \ h_x = 0 \text{ for } x^0 \neq 0\}$.

Moreover, for $m \geq 0$ holds reflexivity with respect to the hyperplane S_0, i.e. $h^0_x = 0$ for elements $h \in N_{S_0}$.

Proof:

(17) $(\Delta E)^{\mu\nu}_{xy} = \text{const}\ \delta_{x/y+n^\mu-n^\nu} + \dots$,

where $+\dots$ contains all the contributions from coinciding points and next neighbours. Hence ΔE does not couple $\Lambda^{int}_\pm$ to each other which by standard arguments implies in the case m=0 the Markoff property with respect to S_0.

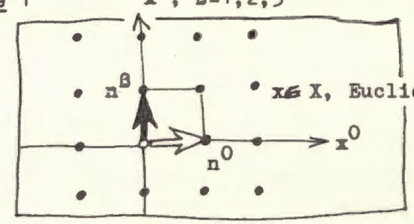
For $h \in N_{S_0}$ let us rewrite the transversality $\nabla^+ . h = 0$ in the form

(18) $h^0_y = h^0_x + \sum_{\beta=1,2,3} (h^\beta_{x+n^\beta} - h^\beta_x)$, $y = x+n^0$.

If $y \notin S_0$, $h^0_y = 0$ since supp $h \subset S_0$. If $y \in S_0$, $h^0_y = 0$ since $x, x+n^\beta \notin S_0$ and the r.h.s. vanishes.

The proof of the reflection positivity (16) is similar as for the continuum model. Q.E.D.

Figure 1



$x \in X$, Euclidean space-time lattice

Let be A the Gaussian random field indexed by transversal real functions of finite support in X, with mean zero and covariance given by the scalar product in N_m, $m \geq 0$.

For a suitable choice of the underlying probability space (Q, B^m, β^m) we can realize A as coordinate functions

(19) $A(h)(q) = \sum_X h_x \cdot q_x$, $q \in Q = \times_{\{x,\mu\}} R$.

For $m=0$

$$(20)\qquad d\beta \sim e^{\frac{1}{2}q.\Delta q}\, dq = \prod_{\{x,y,\mu\}} e^{+\frac{1}{2}q_x^\mu q_y^\mu} \prod_{\{x,\mu\}} d\lambda(q_x^\mu),$$

where the first product is over all next neighbours, $/x-y/=1$, and $d\lambda/ds = \exp(-4s^2)$. β is a ferromagnetic measure of Ising type. We let B the smallest σ-algebra of subsets in Q so that all $A(h)$, $h\in N$ real, are β-measurable.

Hence for any source s_x, $x\in X$, $A(s_x)$ is not a measurable Q-space function with respect to B. Passing to plaquette variables we will find a simpler expression for $d\beta\restriction_B$.

First we introduce the sets of lattice bonds and plaquettes with positive orientation

$$(21)\qquad b^+(X)=\{b=(x/x+n^\mu)\},$$

$$p^+(X)=\{p=(n^\mu x n^\nu/x),\ \mu<\nu\}.$$

Then $rb=(x+n^\mu/x)\in b^-(X)$ and $rp=(n^\nu x n^\mu/x)\in p^-(X)$. For the notations below we refer to the text book of Singer and Thorpe:

$$(22)\qquad \partial b=\{x,(x+n^\mu)\},$$

$$\partial p=\{(x/x+n^\mu),(x+n^\mu/x+n^\mu+n^\nu),(x+n^\mu+n^\nu/x+n^\nu),(x+n^\nu/x)\},$$

$$\delta x=\{(x/x\pm n^\mu)\}$$

Now we identify a given vector field $h=(h_x^\mu)_{x\in X}$ on X with the function on $b(X)=b^+(X)\cup b^-(X)$

$$(23)\qquad h_b=\begin{cases} h_x^\mu & b=(x/x+n^\mu)\in b^+(X)\\ -h_x^\mu & b=(x+n^\mu/x)\in b^-(X)\ ,\end{cases}$$

i.e. $h_b+h_{rb}=0$. In a similar manner, we identify a given tensor field $f=(f_x^{\mu\nu})_{x\in X}$ on X with the function

$$(24)\qquad f_p=f_x^{\mu\nu}\ ,\ p=(n^\mu x n^\nu/x)\in p(X).$$

If f is antisymmetric then $f_p+f_{rp}=0$. Hence a vector field h on the lattice X is a sequence $h=(h_b)_{b\in b(X)}$, $h_b+h_{rb}=0$ for all b, and an antisymmetric tensor field is a sequence $f=(f_p)_{p\in p(X)}$, $f_p+f_{rp}=0$.

With the above identifications, for any scalar field u on X holds $(\nabla^+u)_b=u_{\partial b}$, in particular

$$(25)\qquad (\nabla^+\delta_y)_b=(s_y)_b=\begin{cases}-1 & b=(y/y\pm n^\mu)\\ 0 & \text{otherwise}\ .\end{cases}$$

In Euclidean space-time R^3, using the Hodge duality map for any vector field h on $X\subset R^3$ hold the identities

$$(26)\qquad (\nabla^-.h)_x=h_{\delta x}\quad\text{and}\quad *(\nabla^+xh)_p=h_{\partial p}.$$

In particular, for $p=(n^\alpha x n^\beta/y)\in p^\pm(X)$ let $(1_p)_x^\mu=\epsilon^{\mu\alpha\beta}\delta_{xy}$. Then

$$(27)\qquad (c_p)_b=(\nabla^-x1_p)_b=\begin{cases}\pm 1 & b\in\partial p\\ 0 & \text{otherwise}\end{cases}$$

is a unit curl on X. Of course $(c_p)_{\partial p}=\pm 4$.

In section 2 of part II (JINR E5-12779) we have discussed the map $\overset{e}{I}$ which allows to identify in a unitary manner the Euclidean one-photon states in terms of $f^{\mu\nu}$ with those in the transversal gauge.

For $\tilde{f}\in L^2(I,P_{ph}/w/^{-2}dp)$, where $P_{ph}^{\alpha\beta/\mu\nu}(p)=\delta^{\alpha\mu}\frac{\beta}{w}\frac{\nu}{w}+\dots$ is antisymmetric, we define

$$(28)\qquad \tilde{\overset{e}{I}}:\ \tilde{f}\to\tilde{h}^\mu(p)=w^\nu(\tilde{f}^{\mu\nu}-\tilde{f}^{\nu\mu}),\ p\in I.$$

$\{c_p:\ p\in p(X)\}$ is a basis in N. Using (II.18) we find the formula $c_p=\overset{e}{I}(*1_p)$. Then, for $h=\sum m_p c_p\in N$ real, we can express the random variable $A(h)$ in terms of the Euclidean $F^{\mu\nu}$'s on the lattice X, $F_p = F(\star 1_p)$, by the formula $A(h) = \sum_{p(X)} m_p F_p$. Their covariances are given by

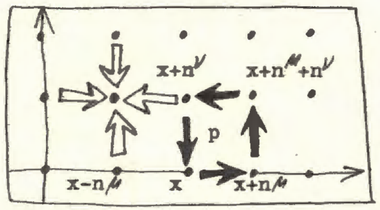
$$(29) \qquad \langle F_p F_q \rangle_{\beta} = \int_I dp \, \frac{P_{ph}^{\alpha\beta/\mu\nu}(p)}{|w(p)|^2} e^{ip.(x-y)} \, ,$$

for $p=(n^{\alpha} x n^{\beta}/x)$ and $q=(n^{\mu} x n^{\nu}/y)$. Let $\varsigma_x = (q)\delta_x = (\nabla^- .q)_x$, $x \in X$, which is formally equal to $A(s_x)(q)$. Since any Q-space function measurable with respect to B is constant in the variables ς_x, the measure β factorizes into $d\beta = d\beta(F) \otimes d\beta(\varsigma)$, where

$$(30) \qquad d\beta(F) \sim e^{-\frac{1}{2}F.VF} \, dF.$$

V denotes the inverse of the covariance matrix (29).

Figure 2



plaquette $p=(n^{\mu} x n^{\nu}/x)$

The above discussion of the Euclidean photon field on the lattice shows that it is a non-local theory in the sense that we have a map from plaquettes $p \in p(X)$ to random fields F_p, rather than from lattice points.

Equivalently, for any loop $\mathcal{C}$ the random variable

$$(31) \qquad A(\mathcal{C}) = \sum_{b \in \mathcal{C}} A_b$$

is β-measurable since $\mathcal{C}$ is just the boundary of the union of some plaquettes. To avoid confusion in the next section we return to our notations $\overset{e}{A}$, respectively $\overset{e}{F}{}^{\mu\nu}$ from part II.

2. Steady currents and magnetic dipoles

The Euclidean photon field $\overset{e}{A}$ is the Gaussian random field indexed by elements $j \in S_r^T$, i.e. real $j \in \sum^{\oplus} S(R^{d+1})$ satisfying $\nabla.j=0$, with mean zero and the covariance

$$(32) \qquad \langle \overset{e}{A}(j)\overset{e}{A}(j') \rangle = \iint \frac{dx\,dy}{c_d |x-y|^{d-1}} \, j(x).j'(y),$$

where c_d denotes the area of the unit sphere in Euclidean space-time R^{d+1}, $d \geq 2$.

In the lattice model which we have discussed in section 1 the basic objects are the random variables $\overset{o}{A}(\mathcal{C})$, where $\mathcal{C}$ is a loop on the lattice $X \subset R^{d+1}$. Similar in the continuum formulation: The test functions j can be identified with steady currents. This idea is due to Gross. We will study the localization, Markoff property, reflexivity, etc., for Euclidean one-photon states in the language of classical $d+1$-dimensional magnetostatics.

Given a steady current j we let $U = G^0 j$ the vector potential and

$$(33) \qquad W = -\int_{R^{d+1}} U.j \, dx$$

the self-energy. In Euclidean space-time R^3 the kernel of $G^0 = (-\Delta + m^2)^{-1}$, $m=0$, is given by the Coulomb potential $\frac{1}{4\pi|x-y|}$. Hence $W = -\|j\|_N^2$, where

$$(34) \qquad N = \left\{ \tilde{j} \in \sum^{\oplus} L^2(R^{d+1}, |p|^{-2} dp) : p.\tilde{j}(p) = 0 \text{ a.e.} \right\} .$$

Note that $\nabla.U=0$ in the sense of a vector-valued tempered distribution.

The Euclidean transverse potentials $\overset{e}{A}$ live in the Fock space $\Gamma(N_r)$. Using the map J_o we can identify the real one-photon states in Coulomb gauge

(35) $$A^C(0,\tilde{\underline{i}})\Omega \quad ; \underline{i} \in S^C_r(R^d),$$

with the steady currents $J_o\underline{i} = (0, \delta_o \otimes \underline{i}) \in N_r$ localized on the hyperplane S_o. On the other hand (35) can be identified with the restriction $\tilde{j}^B(p)$, $p=(+/\underline{p}/,\underline{p}) \in V^0_+$, $B= 1,2,..,d$. Then (32) gives for $d=3$

(36) $$\iint_{R^3 x R^3} \frac{d^3x\, d^3y}{/\underline{x}-\underline{y}/^2} \underline{i}(\underline{x}).\underline{i}(\underline{y}) \;\tilde{=}\; \int_{V^0_+} \frac{d^3p}{2p^0} \tilde{\underline{i}}^* . \tilde{\underline{i}} ,$$

where $(\tilde{j}^*)^\mu(\underline{p}) = g^{\mu\nu}\tilde{j}^\nu(-\underline{p})$.

The reflexivity condition in N says that $\Theta_o j = j$, for any $j \in N_{S_o}$. Steady currents like

(37) a) $(-\underline{\Delta}, \underline{\nabla}\nabla^0)\delta_o \otimes u$

b) $(0, \nabla^0\delta_o \otimes \underline{\nabla} x \underline{f})$,

with $\Theta_o j = -j$ are excluded because of their infinite self-energy. Note that for $d=2$ we have the identity

(38) $$(-\underline{\Delta}, \underline{\nabla}\nabla^0)u = \nabla x(\nabla x\, n^0 u), \quad \nabla = (\nabla^0, \underline{\nabla}).$$

We now come to the Markoff property. Given a steady current $j \in N_\Lambda$, Λ compact, the interaction of j with any test current i localized in the complement Λ^c is projected onto the hypersurface $\overline{\Lambda^c} \cap \Lambda = S$.

(39) $$\langle j,i \rangle_N = \langle E_S j,i \rangle_N , \quad j \in N_\Lambda ,$$

is equivalent to $U(j-E_S j)\restriction_{\overline{\Lambda^c}} = 0$ in the sense of a vector-valued tempered distribution.

In other words: j induces on S magnetic moments which are equivalent to the surface distribution $E_S j$, and $E_S j$ produces in Λ^c the same magnetic field as j itself.

Replacing the Coulomb by the Yukawa potential with mass $m>0$, in particular for $j \in N_{\Lambda_+} \cap \ker C$ we find instead of (39)

(40) $$\langle j,i \rangle_{N_m} = \langle E_> j,i \rangle_{N_m} ,$$

$$E_> \tilde{j} = (1, -\underline{p}p^0/k^2) \frac{i(w+k)}{p^0+ik} \check{j}^0, \quad w = +\sqrt{\underline{p}^2+m^2} .$$

We remark that $E_> j$ has support in Λ_+ but not on the hyperplane S_0 (see section 3 of part I -(JINR E5-12780)).

Contrary to the situation in electrostatics - where we have reflection principles for Dirichlet and Neumann boundary conditions - in magnetostatics the problem is more complicated.

Lemma 3 Given $j \in N_{\Lambda_+}$, let $\Theta_o j$ the mirror image with respect to S_o. Then the vector potential of the total distribution $U_+ = G^0(j+\Theta_o j)$ satisfies

(41) a) $-\Delta U_+ = j$ in Λ^{int}_+ and $\nabla . U_+ = 0$,

b) $U^0_+\restriction_{S_o} = 0$ (Dirichlet for normal component)

$(n_{S_o}.\nabla)U^B_+\restriction_{S_o} = 0$ (Neumann for tangential comp. U^B_+, $B=1,2,3$).

Proof: The boundary conditions (41) are found for a current j in front of a thin paramagnetic hyperplane S_o. The Poisson equation and the transversality for U_+ follow from the facts that U_+ is linear in the currents j and $\Theta_o j$ and $\text{supp}\, \Theta_o j \cap \Lambda^{int}_+ = \emptyset$. Moreover,

(42) $$j+\Theta_o j = \begin{cases} j^0(x) - j^0(\Theta_o x) \\ j^B(x) + j^B(\Theta_o x), \quad B=1,2,3. \end{cases} \quad \text{Q.E.D.}$$

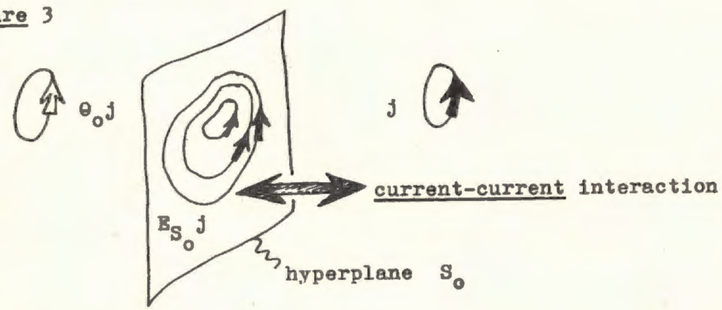
Let $W_+ = -\int U_+ . j \, dx$ the corresponding energy with the mixed boundary conditions (41). Then reflection positivity in N implies

$$(43) \qquad W - W_+ = -\|j\|_N^2 + \langle (1+\theta_0)j, j \rangle_N \geq 0.$$

Hence j is attracted by S_0. Conversely, j induces on S_0 magnetic moments with corresponding distribution of steady currents $E_{S_0}j$ which attract j. Therefore $W - W_+$ should be positive.

The above interpretation of reflection positivity generalizes an idea of Uhlmann for a similar situation in electrostatics.

Figure 3



Let us decompose j into $j = t \oplus l$, where $t = Cj$. Using $E_{S_0} = Ce_{S_0}$, there is no magnetization of S_0 resulting from l.

In other words: There is no contribution of l to the amount of work (43) we have to do in order to remove j to infinity.

Remark Lemma 3 generalizes to the situation of a current $j \in N$ with support in the ball $\Lambda_R = \{x: |x| \leq R\}$ and its inversion image $\theta_R j$ with respect to the sphere S_R as follows. Rewriting the vector potential $U_R = G^0(1+\theta_R)j$ in spherical coordinates as $U_R = (U_R^r, U_R^w)$, it satisfies the mixed boundary conditions on $S_R = \{x: |x| = R\}$.

$$(44) \qquad U_R^r \restriction_{S_R} = 0 \quad \text{and} \quad (n_{S_R}.\nabla)U_R^w \restriction_{S_R} = 0.$$

For the model in Euclidean space-time R^3 the steady current $j = \sum_i \nabla x(m_i \delta_{a_i})$, where $m_i \in R^3$ and supp $j = \{a_i\}$ is finite, represents a generalized one-photon state with total magnetic dipole moment

$$(45) \qquad \mathfrak{M}(j) = 1/2 \int_{R^3} r x j \, d^3r = \sum_i m_i .$$

To avoid confusion with the cross product "x" we pass to the notation $r \in R^3$ instead of x. From the relation $\langle \nabla x m^\mu \delta_a, \nabla x n^\nu \delta_b \rangle_N = E^{\mu\nu}(a,b)$ we conclude that these currents build a total set.

Hence for the proof of reflection positivity in N_r, m=0, it is sufficient to study the expression

$$(46) \qquad \langle \overset{e}{F}(f)\overset{e}{F}(\theta_0 f) \rangle_S = \sum_{i,j} \langle \nabla x m_i \delta_{a_i}, \nabla x \underline{\theta} m_j \delta_{\theta_0 a_j} \rangle_N ,$$

where f is related to j by the map $\overset{e}{I}$. Explicitly holds $f = * \sum m_i \delta_{a_i}$, where $*$ denotes the Hodge duality map. We have used the pseudo-vector character of m_i, i.e. $\theta_0 j = \sum_i \nabla x(\underline{\theta} m_i \delta_{\theta_0 a_i})$, $\underline{\theta}: r \to (r^0, -\underline{r})$.

Lemma 4 (46) is equal to

$$(47) \qquad \langle g, \theta_0 g \rangle_L \geq 0,$$

where $g = -\sum (m_i.\nabla)\delta_{a_i}$ is the related charge distribution of the electric dipoles m_i localized in a_i.

Proof: The interaction energy of individual magnetic dipoles with associated steady currents $j = \nabla x(m\,\delta_a)$ and $i = \nabla x(n\delta_b)$, $a \neq b$, is given by

$$(48) \qquad \langle j, i \rangle_N = m.K(a-b)n - (m.n)\,\mathrm{Tr}\,K(a-b),$$

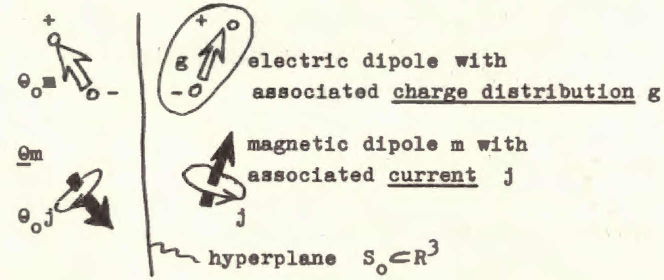
where $K(r) = (2P_r^{\perp} - P_r)./r/^{-3}$ with the projection P_r onto $r \in R^3$ denotes the coupling matrix. Using the expression $B(n) = K(a-b)n$ for the magnetic field produced by the dipole n in the point $a \in R^3$, and $\mathrm{Tr}\, K=0$, we get the well known formula

$$(49) \qquad \langle j,i \rangle_N = + m.B(n).$$

We now reinterprete m, n as <u>electric dipoles</u> with the associated charge distributions $-m.\nabla\delta_a$ and $-n.\nabla\delta_b$. Their Coulomb interaction energy is given by a similar expression to (49), i.e. $-m.E(n)$, where $E(n)=B(n)$. Finally, using $\theta_o(-n.\nabla\delta_b) = -(\theta_o n).\nabla\delta_{\theta_o b}$ and $\theta_o \underline{\Theta} n = -n$, we find for the individual dipole moments the identity

$$(50) \qquad \langle \nabla x m\delta_a, \theta_o(\nabla x m\delta_b)\rangle_N = \langle m.\nabla\delta_a, \theta_o(n.\nabla\delta_b)\rangle_L . \qquad \text{Q.E.D.}$$

<u>Figure</u> 4



The inequality (47) remains valid if we replace the Coulomb by the Yukawa potential

$$(51) \qquad \frac{e^{-m/x-y/}}{4\pi/x-y/}, \quad m>0.$$

Then the reflection positivity for the Euclidean free transversal <u>vector field of mass $m>0$</u> is also a consequence of lemma 4. We expect that our proof for the models in Euclidean space-time R^3 extends to the general case.

3. Conclusion

We have studied the local structure of the Euclidean models for the free vector fields of mass $m \geq 0$ in the transverse gauge.

Reflection positivity and reflexivity are the most important features and they allow various interpretations, in particular for the Euclidean photon field.

First of all we mention the consequence of positive-definite metric in the Hilbert space of <u>quantum one-photon states</u> in the Coulomb gauge and the fact that the time-zero quantum fields act irreducibly in the corresponding Fock space.

For the model in space-time R^3 we have introduced generalized <u>Euclidean one-photon states</u> with definite magnetic dipole moments. Then reflection positivity is related to the very plausible assertion - and we proved it - that the interaction of any finite system of such dipoles with its mirror image has negative energy. A collection of proofs for reflection positivity - even in the massive case and for the lattice models - will be published in a separate paper.

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References (III)

/SiTh/ I.M.Singer, J.A.Thorpe: Lecture notes on elementary topology and geometry, ed. Scott, Foresman and Co. (1967),

/GuRoSi2/ F.Guerra, L.Rosen, B.Simon: Boundary conditions for the $P(\phi)_2$ Euclidean field theory, Ann. Inst. Poincaré, Vol. <u>XXV</u>, No 3 (1976),

/Uh/ A.Uhlmann: Remarks on reflection positivity, Czechoslovak Journal of Physics <u>B 29</u> (1979).

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