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SEMIMICROSCOPIC DESCRIPTION OF THE GIANT QUADRUPOLE RESONANCES IN DEFORMED NUCLEI

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S U M M A R Y

The calculation results of the giant quadrupole isoscalar and isovector resonances performed within the random phase approximation are represented. The strength functions for E2-transitions are calculated for doubly even deformed nuclei in the regions $150 \leqslant A < 190$ and $228 \leqslant A < 248$ in the energy interval (0-40) MeV. The following integral characteristics of giant quadrupole resonances are obtained: the position, widths, the contribution to the energy weighted sum rule and the contribution to the total cross section of photoabsorption. The calculations have shown that giant quadrupole resonances are common for all the considered nuclei. The calculated characteristics of the isoscalar giant quadrupole resonance agree with the available experimental data. The calculations also show that the semimicroscopic theory can be successfully applied for the description of giant multipole resonances.

1. The existence of giant quadrupole resonances (GQR) was predicted in ref.[1] before their experimental detection. Recent experiments[2-4] have confirmed the presence of GQR in deformed nuclei. The progress in the experimental study of GQR stimulated theoretical works, beyond the framework of the phenomenological description. When calculating the characteristics of GQR, one should consider the isoscalar and isovector quadrupole-quadrupole interactions simultaneously. The preliminary results of such calculations performed for ^{154}Sm and ^{238}U are given in ref.[8].

The aim of the present paper is a systematic study of the isoscalar and isovector GQR in doubly even deformed nuclei.

2. The characteristics of giant quadrupole resonances are calculated on the basis of a simple generalization of the mathematical apparatus developed for the description of low-lying one-phonon states[9].

The nuclear Hamiltonian can be written as:

$$H = H_{av} + H_{pair} + H_Q , \qquad (1)$$

where H_{av} is the average field of the neutron and proton systems, H_{pair} is the interaction resulting in the pairing correlations of superconducting type, H_Q is the quadrupole-quadrupole interaction.

To describe the giant resonances we introduce the phonons with isotopic spin T=0 and T=1 with $T_z = 0$. In complex nuclei the collective vibrations with T=0 and T=1 are not independent because of $N > Z$, and we consider them simultaneously. The constants of the quadrupole-quadrupole interaction can be expressed by the isoscalar $\varkappa_0^{(2)}$ and isovector $\varkappa_1^{(2)}$ constants in the following way: $\varkappa_{nn}^{(2)}=\varkappa_{pp}^{(2)}=\varkappa_0^{(2)}+\varkappa_1^{(2)};\ \varkappa_{np}^{(2)}=\varkappa_0^{(2)}-\varkappa_1^{(2)}$.

Using the random phase approximation given in ref.[9], we find the secular equation for the descritpion of the energies of one-phonon states as:

$$(\varkappa_0^{(2)}+\varkappa_1^{(2)})(X_n^i+X_p^i)-4\varkappa_0^{(2)}\varkappa_1^{(2)}X_n^iX_p^i=1,$$

where

$$X_n^i=2\sum_{ss'}\frac{f(s,s')\tilde{f}(s,s')u_{ss'}^2(\mathcal{E}(s)+\mathcal{E}(s'))}{(\mathcal{E}(s)+\mathcal{E}(s'))^2-\omega_i^2},\ \tilde{f}(s,s')=f(s,s')-\frac{\Gamma_n^i(s)}{\gamma_n^i}\delta_{ss'}, \qquad (2)$$

Here ω_i is the one-phonon energy, $f(s,s')$ is the matrix element of the quadrupole operator, the addition to $f(s,s')$ concerns the $K^\pi=0^+$ states, γ_n^i, $\Gamma_n^i(s)$ are defined in ref.[9], $\mathcal{E}(s)$ is the quasiparticle energy, $u_{ss'}=u_s v_{s'}+u_{s'}v_s$, u_s, v_s are the Bogolubov canonical transformation coefficients. Our calculation of characteristics of giant resonances in the harmonic approximation is a first step in the programme of their study taking account of the quasiparticle-phonon interaction.

In the study of GQR the reduced probability of E2-transition from the ground state, is calculated as a rule for each state i then the $B(E2;0^+0\to2^+K_i)$-quantities are summed in a certain energy interval. Instead of calculating for each state i the quantities $B(E2;\omega_i)=(002\mu|2K)^2M^2(\omega_i)$, where $M(\omega_i)$ is the matrix element of E2-transition from the ground state to the one-phonon state i, we use the method of the direct calculation of averaged characteristics. We introduce the strength function:

$$b(E2,\omega)=(00\lambda\mu|IK)^2\sum_i M^2(\omega_i)\rho(\omega-\omega_i), \qquad (3)$$

$$\int_\omega^{\omega+\Delta\omega} b(E2,\omega')d\omega'\simeq\sum_i{}_{\Delta\omega}B(E2;\omega_i), \qquad (4)$$

where

$$\rho(\omega-\omega_i)=\frac{1}{2\pi}\frac{\Delta}{(\omega-\omega_i)^2+(\Delta/2)^2}. \qquad (5)$$

The energy interval of the averaging Δ is a free parameter. Following refs.[10,11] we represent the function (3) by the contour integral with the roots of eq.(2) as poles, and then pass to the integrals over contours around the poles $Z=\omega\pm i\Delta/2$.

As a result we get

$$b(E2,\omega)=(00\lambda\mu|IK)^2\frac{2-\delta_{K0}}{\pi}e^2\times$$
$$\times\, Im\left\{\left[\frac{(1+e_p^{(2)})^2X_p+(e_n^{(2)})^2X_n-X_nX_p(\varkappa_0^{(2)}(1+e_p^{(2)}-e_n^{(2)})^2+\varkappa_1^{(2)}(1+e_p^{(2)}+e_n^{(2)})^2)}{1-(\varkappa_0^{(2)}+\varkappa_1^{(2)})(X_n+X_p)+4\varkappa_0^{(2)}\varkappa_1^{(2)}X_nX_p}\right]_{\omega=\omega+i\Delta/2}\right\},$$

where $e_n^{(2)}$, $e_p^{(2)}$ are the effective charges.

3. Now we present some details of numerical calculations. We use the energies and wave functions of the Saxon-Woods axially symmetric potential which parameters are fixed in ref.[12] while studying the low-lying states. To calculate the strength functions $b(E2,\omega)$ up to the excitation energies (35-40) MeV a great number of single-particle states has been taken into account. So, in the region of transuranium elements the number of single-particle states exceeds 300, and the number of matrix elements of a a given multipolarity reaches 3000. Figure 1 represents the density of two-quasiparticle states with $K^\pi=2^+$ in ^{238}U. It also shows

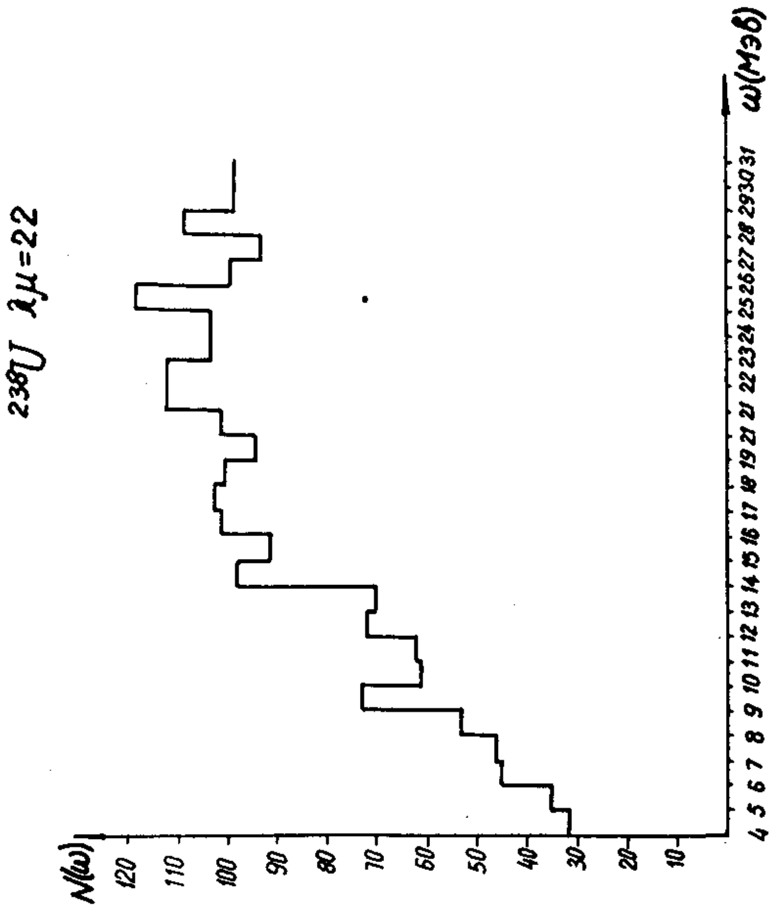
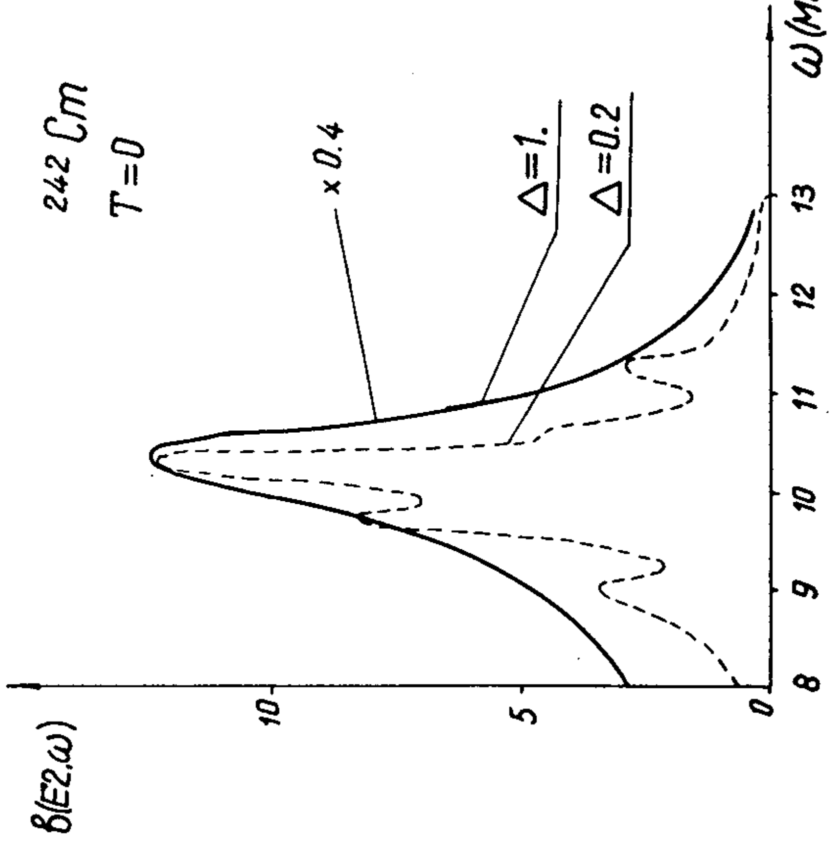


Fig. 1.

that the configurational space is sufficiently large, so, at the energy of 30 MeV the density does not yet decrease. The pairing constants G_N and G_Z are determined from the pairing energies.

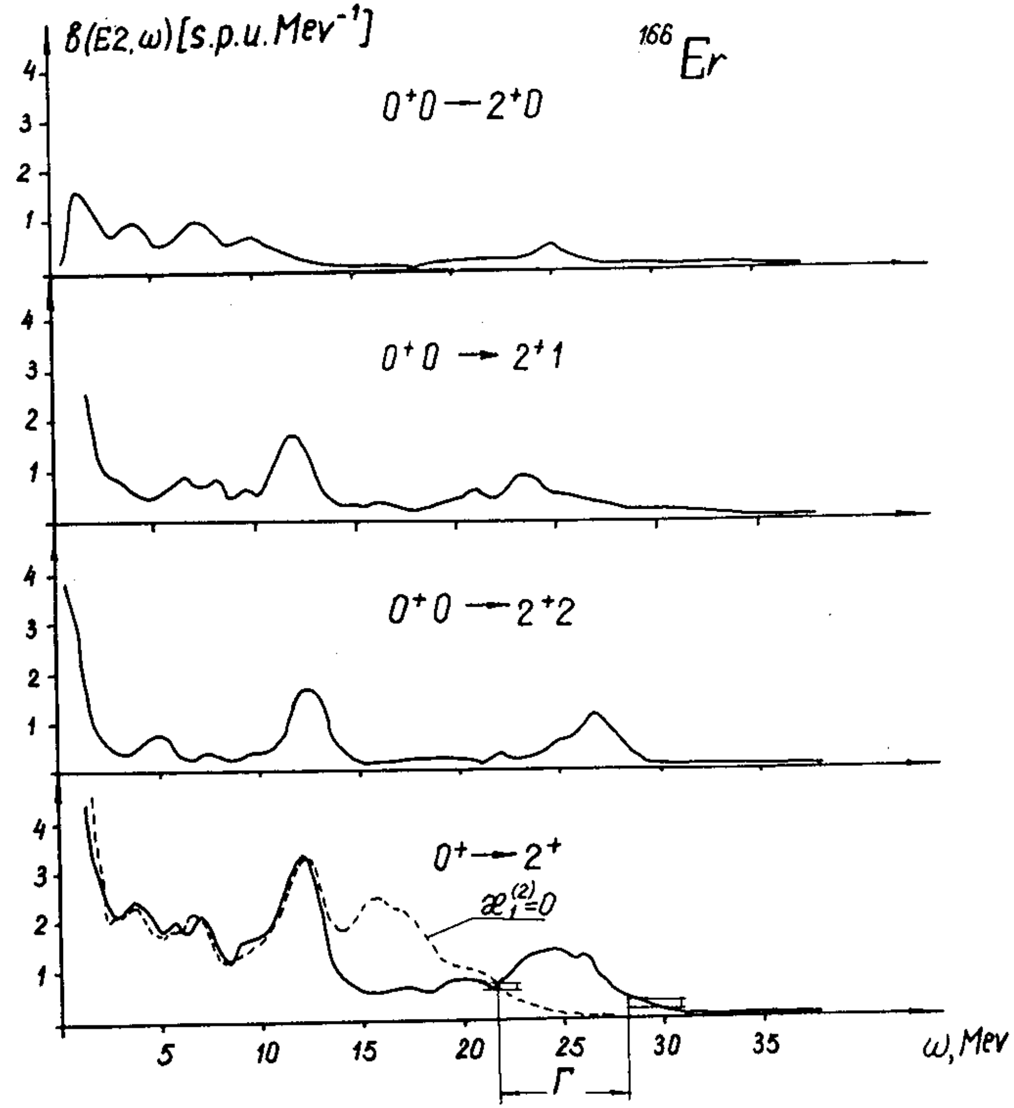
The constants $\varkappa_0^{(2)}$ are chosen so, as to obtain the correct description of the energies of the first $K^{\pi}=2^+$ states. The constants $\varkappa_0^{(2)}$ smoothly decrease with increasing A in both the regions of the deformed nuclei $150 \leqslant A < 190$ and $228 \leqslant A \leqslant 248$. The constant $\varkappa_1^{(2)}$ is a free parameter, and its determination will be discussed later. The effective charges $e_n^{(2)}$ and $e_p^{(2)}$ are taken to be zero. The quantity Δ is not, in fact, a free parameter, since the choice of the fixed value of Δ determines the form of representation of the calculation results. From (5) it is seen, that if the quantity Δ is taken to be very small, then we obtain a curve enveloping each value of $B(E2,\omega_i)$. With increasing Δ, the curves are smoothed. Figure 2 represents the strength functions $b(E2,\omega)$ for ^{242}Cm calculated with Δ =1.0 MeV and Δ =0.2 MeV. It is seen from the figure that for the correct description of the fine structure, we should take a small value of Δ. In further calculations in the harmonic approximation, one should not expect correct description of the fine structure, therefore it is chosen that Δ =1,0 MeV.

4. Now we discuss the results of our calculations which are given in figs.3 and 4 and in Tables 1,2 and 3. The strength functions b (E2, ω) for E2-transitions from the ground state to the excited states with $I^{\pi}=2^+$ and K =0,1,2 in ^{166}Er are given in fig.3. In deformed nuclei GQR comprise the mixture of components with K =0,1,2. The resonances for each value of K have



242 Cm
T=0
x 0.4
Δ=1.
Δ=0.2
ω (Mev)
B(E2,ω)

Fig. 2.



B(E2,ω)[s.p.u.Mev-1]
166 Er
0+0 — 2+0
0+0 → 2+1
0+0 — 2+2
0+ → 2+
æ(2)1=0
ω, Mev
Γ

Fig. 3.

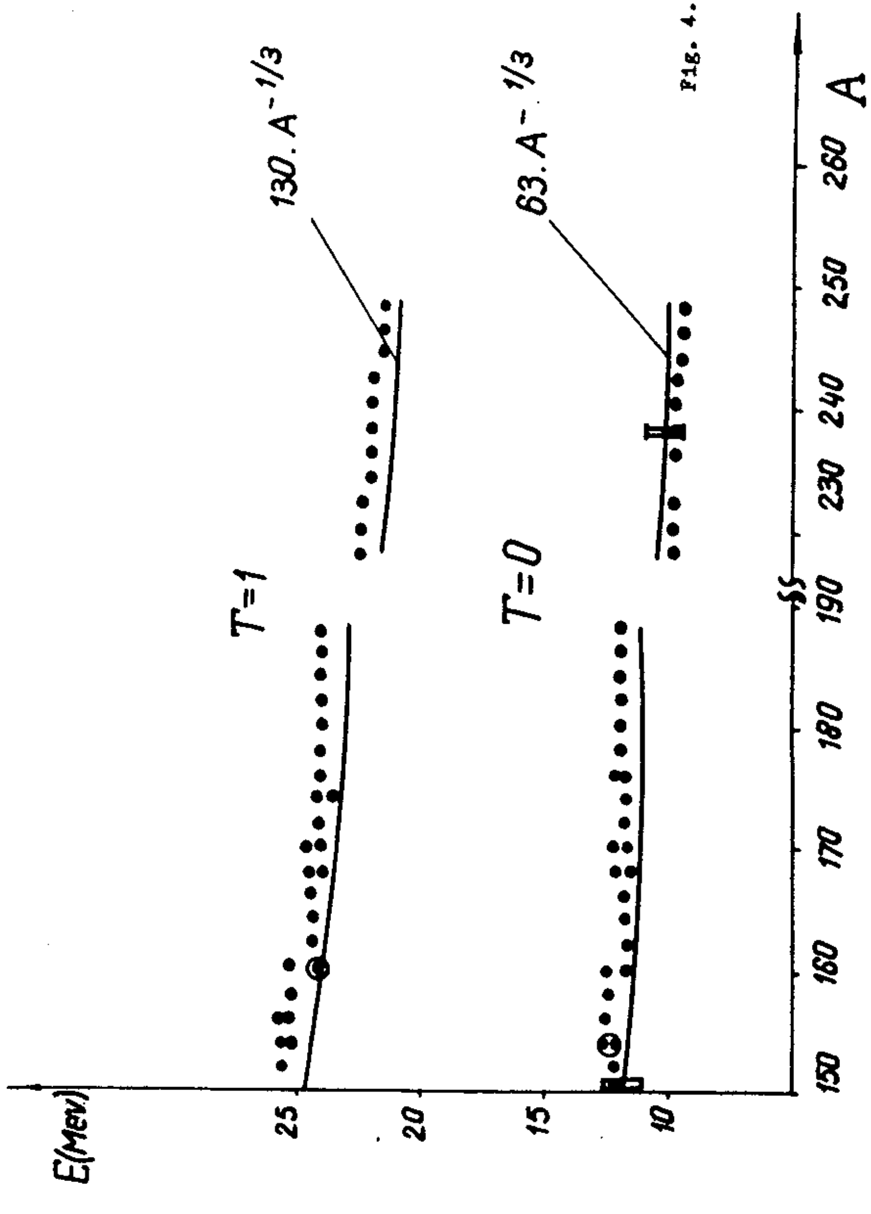


Fig. 4.

a very complicated form which is smoothed due to a great value of the parameter Δ = 1 MeV. The positions of maxima of isoscalar and isovector resonances with K =0,1,2 do not coincide. This results in the broadening of isoscalar and isovector resonances in deformed nuclei as compared to spherical nuclei. This is consistent with experimental data [3,4]. Note, that there is a broadening of giant quadrupole resonances but not their splitting. The calculated positions of the isoscalar maxima with K =0,1,2 essentially differ from the calculation results in ref. [4] for ^{154}Sm and qualitatively agree with the calculations in ref. [7] for ^{160}Dy. Note, that the calculated sequence of peaks with different values of K is conserved in all the nuclei.

The total strength function $\mathcal{B}$ (E2, ω) for E2-transitions to the states with $I^{\pi} = 2^+$ with all possible values of K is represented in the bottom of fig.3. The dotted line represents the values of $\mathcal{B}$ (E2, ω) calculated with $\varkappa_1^{(2)}=0$. It is seen from the figure that when $\varkappa_1^{(2)}=0$ the width of the quadrupole isoscalar resonance is highly overestimated. The position of the isovector resonance is defined by the quantity $\varkappa_1^{(2)}$.

Figure 4 and Table 1 represent the energies of giant isoscalar and isovector quadrupole resonances.The energies of isoscalar resonances are approximately equal to 12 MeV for the rare-earth nuclei and 10 MeV for actinides. The position of giant isoscalar quadrupole resonances and the behaviour of the strength functions below them do not practically depend on the quantity $\varkappa_1^{(2)}$. The calculation results of the energies of isoscalar resonances are consistent with the available experimental data [2,4].

The position of the isovector GQR is defined by the constant $\varkappa_1^{(2)}$. If, following ref. [5], we put $\varkappa_1^{(2)} = -3,5\varkappa_0^{(2)}$ then the

Table 1.

Characteristics of the giant quadrupole resonances

Nucl.	T ≈ 0			T ≈ 1			EWSR	
	E, MeV	Γ, MeV	EWSR	E, Mev	Γ, MeV	EWSR	tot.	mod/ind.
^{I50}Nd	I2.I	5.	II2(22)	25.5	5.5	I38(27)	449	88
^{I52}Sm	I2.2	5.	I24(24)	25.5	5.5	I5I(29)	462	88
^{I54}Sm	I2.3	4.5	I04(20)	25.5	5.	I45(29)	454	88
^{I54}Gd	I2.3	4.8	II0(2I)	25.2	5.5	I57(29)	474	89
^{I56}Gd	I2.5	4.	96(I8)	25.3	5.7	I63(3I)	446	84
^{I58}Gd	I2.5	4.	94(I8)	25.I	6.	I6I(3I)	457	87
^{I60}Gd	I2.7	4.	93(I8)	25.I	5.8	I58(30)	449	86
^{I56}Dy	I2.5	4.7	I03(I9)	25.5	5.5	I6I(29)	483	88
^{I58}Dy	I2.5	3.8	90(I7)	25.5	5.5	I63(30)	475	88
^{I60}Dy	I2.7	3.5	86(I6)	25.5	7.	I7I(32)	467	87
^{I62}Dy	II.8	3.5	92(I7)	24.3	6.5	I77(33)	509	95
^{I64}Dy	II.9	3.5	93(I8)	24.3	6.2	I62(3I)	50I	95
^{I62}Er	II.8	3.7	93(I7)	24.3	5.9	I76(32)	527	96
^{I64}Er	II.9	3.	92(I7)	24.3	6.	I77(32)	5I8	95
^{I66}Er	I2.	3.I	97(I8)	24.4	6.	I70(3I)	5I0	94
^{I68}Er	I2.I	3.I	96(I8)	24.4	6.	I67(3I)	502	94
^{I70}Er	I2.I	3.I	9I(I7)	24.5	6.	I64(3I)	494	93
^{I70}Yb	I2.2	3.	93(I7)	24.0	6.5	I79(33)	5I6	94
^{I72}Yb	I2.	3.	96(I8)	24.	6.5	I80(33)	524	96
^{I74}Yb	II.8	3.5	II3(2I)	23.5	6.9	I84(34)	5I5	96
^{I76}Yb	II.9	3.	95(I8)	24.	6.8	I76(33)	508	95
^{I72}Hf	II.8	3.	I00(I8)	24.	6.8	I92(34)	550	98
^{I74}Hf	II.8	3.2	I05(I9)	24.	6.7	I79(32)	54I	98
^{I76}Hf	II.8	3.	99(I8)	24.	6.9	I87(34)	533	97
^{228}Th	9.8	3.I	I05(I8)	22.4	6.	2I5(37)	572	99
^{230}Th	9.9	3.I	I04(I8)	22.4	5.8	204(35)	566	98
^{232}Th	9.9	3.3	I07(I9)	22.3	5.9	207(36)	559	98
^{232}U	9.9	3.	I00(I7)	22.3	5.4	202(35)	566	97
^{234}U	9.7	2.9	I09(I9)	22.	6.	2I4(37)	575	99
^{236}U	9.8	3.	II9(2I)	22.	6.	2I6(37)	569	98
^{238}U	9.8	2.9	II0(I9)	22.	6.5	2I2(37)	562	98
^{238}Pu	9.8	2.8	94(I6)	22.	5.7	I97(34)	574	98
^{240}Pu	9.9	2.8	I05(I8)	22.	6.2	208(36)	568	97
^{242}Pu	9.9	2.7	93(I6)	22.	6.7	2II(36)	56I	97
^{242}Cm	9.8	2.9	II4(I9)	22.	6.5	2I0(35)	574	98
^{244}Cm	9.5	2.9	I00(I7)	2I.5	7.3	232(39)	574	97
^{246}Cm	9.5	2.9	I07(I8)	2I.5	7.5	233(40)	568	97

Table 2.

Calculated values EWSR in ^{160}Dy and ^{232}Th

Nucleus	$\int_0^{40} \omega B(E2,\omega) d\omega$ (s.p.u. MeV)				
	K = 0	K = I	K = 2	все K	Model / mod.indep. (%)
^{I60}Dy	97	205	I7I	473	87
^{232}Th	II4	243	207	564	98

Table 3.

Calculated values EWSR in ^{160}Dy and ^{232}Th

$\int \omega B(E2,\omega) d\omega$ (s.p.u. MeV)					
^{160}Dy			^{232}Th		
Energy interval	Value	Model / mod.indep. %	Energy interval	Value	Model / mod.indep. %
0-3 MeV	44	8	0-3 MeV	56	I0
3-9 MeV	5I	9	3-8 MeV	72	I3
9-I4 MeV	I09	20	8-I2 MeV	III	I9
I4-22 MeV	80	I5	I2-I9 MeV	73	I3
22-28 MeV	I59	30	I9-25 MeV	202	35
28-40 MeV	34	6	25-40 MeV	45	8

isovector GQR is located at the energy 31-33 for the nuclei from $150 \leqslant A < 190$ mass region and at the energy 29-31 MeV for actinides. If we put $\varkappa_1^{(2)} = -1.5\, \varkappa_0^{(2)}$, then the calculated energies of the isovector resonance follow the dependence $130 \cdot A^{-1/3}$ and for ^{160}Gd the calculated energy is close to the expected measured value[13]. Figs.3 and 4 and Tables 1,2 and 3 represent the data calculated with $\varkappa_1^{(2)} = -1.5\, \varkappa_0^{(2)}$.

The widths of the isoscalar and isovector GQR in deformed nuclei is to a great extent determined by some incoincidence of resonances with different values of K. It is seen from the figure that the resonance form strongly differs from the Gauss curve. Therefore, one should define the energy interval of the localization of the resonance(total width Γ)and the energy of its center of mass (E), but not the halfwidth and position of a peak, as for the dipole resonances which are calculated for the deformed nuclei in ref.[14]. For the sake of definiteness we fix the edges of this interval on the level of 20% of the total hight of the corresponding peak counted off "phon". Figure 3 represents these values by the example of the isovector resonance. The results of thus determined energies and widths are given in fig.4 and Table 1. Note, that the calculated total widths Γ do not practically depend on Δ, since $\Gamma > \Delta$. One should also bear in mind that when calculating the energies E and widths Γ, we did not take into account the contribution of higher configurations. The consideration of quasiparticle-phonon interaction allows the investigation of the role of higher configurations.

It is seen from Table 1 that the position of giant quadrupole resonances is weakly changed inside the rare-earth and actinides regions. This change is even smaller than that of the energies of the first $K^{\pi} = 2^+$ states. The change of energies and $B(E\lambda)$ -quantities for the first one-phonon states is due to the change of the chemical potential when passing from one nucleus to another, and, thus, due to the change of matrix elements corresponding to the first poles. The position of giant resonances is determined by the matrix elements with $\Delta N = \lambda$ and the corresponding poles, the role of which does not change when passing from one nucleus to another. This fact is connected with a slight change of the energies of GQR with increasing A. An analogous situation is in the calculation of giant octupole resonances[15].

It should be mentioned that the calculation results remain stable at a small change of the parameters of the Saxon-Woods potential. The characteristics of GQR were calculated for a number of isotopes Yb and Hf with a slight change of the parameter of the quadrupole equilibrium deformation β_{20} and strong change of the parameter of the hexadecapole deformation β_{40}. The results have changed slightly. In ref.[16] there was developed the mathematical apparatus for the calculation of B (E2,ω)-quantities in odd-A-deformed nuclei and performed the calculations for the giant isoscalar quadrupole resonance. For ^{165}Ho the position of the isoscalar resonance and its width consistent with experimental data are obtained.

5. Now we turn to the model independent energy weighted sum rule (EWSR). For $\lambda = 2$ we write it as:

$$\sum_i \omega_i B(E\lambda;\omega_i) = 240 \frac{Z}{A^{2/3}} B(E2)_{s.p.} MeV,$$

where $B(E2)_{s.p.} = 3 e^2 A^{4/3}\, 10^{-53}\, cm^4$. (7)

Table 1 represents the model EWSR for each nucleus and the ratio of the model to the model independent values of EWSR. It is seen from the table that this ratio varies from 0.88 to 0.99 when the effective charges are zero. This gives evidence that in our calculations almost all the required part of the configurational space has been taken into account. Table 2 represents the quantities for each value of K . It is seen from the table that the contribution of all K is considerable and they should be taken into account simultaneously.

Table 1 gives the values of the model EWSR in the regions of the isoscalar and isovector GQR and the relative contribution in percent of the model EWSR. Table 3 represents the values of EWSR for ^{160}Dy and ^{232}Th in different energy intervals. The isoscalar and isovector resonances exhaust (18-22)% and (27-40)% of the model EWSR, respectively. A large number of phonons takes part in the formation of the isoscalar and isovector resonances. So, in the region of the isoscalar resonance there are about 500 one-phonon states, and in the region of the isovector resonance - about 1000. The giant isoscalar and isovector quadrupole resonances manifest themselves rather clearly, since in the energy interval of 8 MeV between them only (8-10)% of the EWSR is exhausted.

Experimental detection of GQR especially of its isovector part, encounters pronounced difficulties. The contribution of GQR to the total cross section of photoabsorption is approximately 12 mb·MeV for the isoscalar resonance and about 100 mb·MeV for the isovector resonance.

6. In conclusion it should be noted that our study of giant multipole resonaces is performed within the framework of the semimicroscopic theory pretending to the unified description of few-

quasiparticle components of the wave functions at low, intermediate and high excitation energies. In ref.[17] the possibilities of the unified description from the low-lying states up to the neutron resonances and further up to the giant multipole resonances are demonstrated. For the low-lying states the description of each individual level is obtained. For the states of intermediate and high excitation energies, the few-quasiparticle components are represented as the corresponding wave functions.

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