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О последовательном учете релятивистской кинематики в задаче пион-дейтронного рассеяния

На основе трехчастичный квазипотенциальных уравнений изучается роль релятивистской кинематики в задаче пион-дейтронного рассеяния. Показано, что вклад от последовательного учета релятивистской кинематики того же порядка, что и другие обычно учитываемые поправки.

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The Consecutive Taking into Account of Relativistic Kinematics in the Pion-Deuteron Scattering

The consecutive taking into account of relativistic kinematics in the pion-deuteron scattering length is studied on the basis of three-body quasipotential equations. It is shown, that the correction due to the consecutive taking into account of relativistic kinematics is of the same order as the other corrections usually considered.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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The pion-deuteron scattering at low and medium energy ($\lesssim$300 MeV) is the simplest case of pion-nucleus scattering which gives some possibility to solve several general problems arising in the study of pion-nucleous interaction. For example: the convergence of multiple scattering series; the problem of taking into account the excitation of target nucleus in the intermediate states and connected with this the validity of the fixed scatterer or coherent approximations;the problem of the correct taking into account of the relativistic kinematics and other problems. All these are at present intensively discussed in the literature[1-6].

One of the ways for investigating these problems is the application of the three-body quasipotential equations to the pion-deuteron scattering. These equations guarantee two- and three-body unitarity. One version of quasipotential three-body equations[7] was suggested to use, which seems to be the closest to the non-relativistic approach.

These equations are written in three-body center mass frame (c.m.f.) and involve two-body collision matrix depended not only on the energy of the pair but on its total momentum as well. For reduction of this matrix to the collision matrix in the two-body c.m.f. the Lorentz-transformation for the relative momentum of pair ($j\kappa$) in the three-body c.m.f. $\vec{p}_{j\kappa}$ the relative momentum in the two-body c.m.f. $\vec{q}_{j\kappa}$ is taken defined by [3,7]

$$\vec{q}_{jk} = \vec{p}_j - \vec{\mathcal{P}}_i \varphi_{jk}(\vec{p}_j \cdot \vec{\mathcal{P}}_i) = \vec{p}_{jk} - \vec{\mathcal{P}}_i \tilde{\varphi}_{jk}(\vec{p}_j \vec{\mathcal{P}}_i), \qquad (1)$$

$$\vec{\mathcal{P}}_i = \vec{p}_j + \vec{p}_k = \vec{q}_i ,$$

where $\vec{p}_i = -\vec{q}_i$, $\vec{p}_j$ and $\vec{p}_k$ are the momenta of the particles in three-body c.m.f. and φ_{jk} $\tilde{\varphi}_{jk}$ are known functions. The corresponding transformation for two-body collision matrix has the form [3]

$$\tilde{T}_i(\vec{p}_{jk}{}' \vec{p}_{jk'} \cdot \mathcal{P}_{oi} \vec{\mathcal{P}}_i) =$$

$$a_i(q'_{jk} q'_i \mathcal{P}_o) \sqrt{\omega_j(\vec{q}\,'_{jk})\, \omega_k(\vec{q}\,'_{jk})}\, T_i(\vec{q}\,'_{jk} \vec{q}_{jk'} \cdot \mathcal{P}_i^2) \sqrt{\omega_j(\vec{q}_{jk})\omega_k(\vec{q}_{jk})}\, a_i(q_{jk} q_i \mathcal{P}_o), \qquad (2)$$

$$\mathcal{P}_i^2 = (\mathcal{P}_o - \omega_i(\vec{q}_i))^2 - \vec{q}_i^{\,2}$$

where a_i is the known function which is equal to one if collision matrix is taken on-shell $(\mathcal{P}_o = \omega_i(\vec{q}_i) + \omega_j(\vec{p}_j) + \omega_k(\vec{p}_k))$ or one of the particles of the pair is considered to be nonrelativistic. The two-body collision matrix $T_i(\vec{q}\,'_{jk} \vec{q}_{jk'}, \mathcal{P}_i^2)$ satisfies the Lipman-Schwinger equation with relativistic kinematics. It is worthwhile to note that propagators in the two- and three-body equations we used to have linear energy-dependence and therefore the so-called "clustering" property of three-body equations is not lost. Note also, that the relation of fully-off-shell two-body $\tilde{T}_i$-matrix with arbitrary total momentum $\vec{\mathcal{P}}_i$ to another one - T_i , which has a zero total momentum, is much simpler, than the relation obtained in the reference [9] for the same object.

For the angular momentum reduction of three-body quasipotential equations we need the scalar product of two basic state vectors $|\vec{p}_{jk} \vec{q}_i\rangle_i$ and $|\vec{p}_{ki} \cdot \vec{q}_j\rangle_j$ expressed via the new variables

$${}_i\langle \vec{p}\,'_{jk} \vec{q}\,'_i | \vec{p}_{ki} \vec{q}_j \rangle_j = F_{ij}(\vec{q}\,'_{jk} \vec{q}\,'_i ; \vec{q}_{ki} \vec{q}_j), \qquad (3)$$

where F_{ij} is a known function [3]. Note, that in spite of that the function F_{ij} is a complicated one, it does not lead to any additional principal computing difficulties in solving three-body equations. By making use of the separable model for interactions, the system of one-dimensional integral equations was obtained [3] in the representation of total angular momentum and the isospin of the pion-deuteron system taking into account the identity of nucleons. As a first step of using these equations we have calculated the pion-deuteron scattering length. The pion-deuteron scattering length was calculated by several authors before summing nonrelativistic graphs or solving Faddeev equations. The review of the results of these calculations is given in the reference [9], where the correction from relativistic kinematics is considered too, however, it is done for pion only in the propagators. Our main aim is to investigate the role of the relativistic kinematics in the pion-deuteron scattering length, when it is taken into account not only in the propagators for pions but in the relativistic transformation between three- and two-body c.m.f. as well. That is, when the relativistic kinematics is considered consequently.

In the table some results of our calculations are given. The results in the case A and B correspond to the pion-nucleon potential in the paper [6] describing scattering phase shifts up to 300 MeV, when nucleon is considered to be nonrelativistic one.

As to nucleon-nucleon interaction, we have used unitary pole approximation (UPA) to Bryan-Scott or Reid potentials. In the case A the relativistic kinematics for pion was taken into account not only in the propagators but in the transformation between three- and two-body c.m.f. as well; in the case B only in the propagators. From the comparison of the results obtained in the cases A and B we can see that the consecutive taking into account of the relativistic kinematics for pion perceptibly changes the leading terms in the multiple scattering series for the pion-deuteron scattering length and that corrections which come from the consecutive taking into account of the relativistic kinematics are of the same order as corrections due to the $^3D_1$ - wave nucleon-nucleon interaction, the P_{33}- wave pion-nucleon interaction correction to and from P_{11}, P_{31} and P_{13} wave interactions are negligible and the other corrections are considered as usual /9/.

The value of the calculated pion-deuteron scattering length $a_{\pi d} = -0.0260 fm$ differs very much from the experimental value $Re\, a_{\pi d} = -(0.074^{+0.031}_{-0.024})\, fm$ /10/. This difference is mainly connected with the values of pion-nucleon scattering lengths given in the table and corresponds to the pion-nucleon potential used in our calculations. As was mentioned above, this potential is obtained by fitting to experimental pion-nucleon scattering data up to 300 MeV. In this respect it seems to be interesting the result we have got from calculation of single scattering term by making use of the solution of the inverse problem for pion-nucleon scattering given in the reference /10/. The result is shown in the table (case C). Note, that here the relativistic kinematics is taken into account for both particles: pion and nucleon. From the comparison of results in case C with the results in cases A

and B we can conclude that taking into account of the high energy part of pion-nucleon interaction leads to the considerable increasing of the value of pion-deuteron scattering lengths.

The consecutive taking into account of relativistic kinematics is expected to be very important for pion-deuteron scattering in the (3.3) resonance region especially when the solution of inverse problem for pion-nucleon-collision matrix is used. This problem is now under study.

Table

$a\ (fm)$		$a_{\pi d}^{(S1)}$	$a_{\pi d}^{(S2)}$	$a_{\pi d}^{(S)}$	$a_{\pi d}^{(P1)}$	$a_{\pi d}^{P}$
A	$^3S_1+^3D_1$	-0.0204	-0.0096	-0.0280		
	$^3S_1$	-0.0181	-0.0087	-0.0261	+0.0007	0.0021
B	$^3S_1+^3D_1$	-0.0174	-0.0095	-0.0252		
	$^3S_1$	-0.0161	-0.0078	-0.0240	-0.0010	0.0008
C	$^3S_1$	-0.0360	$a_{\pi N}(m_\pi^{-1})$	$a_1 = 0.171$	$a_3 = -0.091$	$a_{33} = 0.210$

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