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G.N.Afanasiev, M.B.Dobromyslov, Kim Eung Poong,
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ON THE DEFINITION OF THE REACTION AND COMPLETE FUSION CHANNELS IN HEAVY ION REACTIONS

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It is quite important to have a clear answer to the following question: what is understood under the process of fusion of heavy ions. Theoretical considerations concerning this subject may be conventionally divided into three different categories. First of them includes theoretical efforts which make use, in explicit or implicit form of the concepts of the optical model. As an example we mention here the papers by T.D.Thomas [1] and V.Babikov [2]. The main point of these papers is as follows. Beyond some distance identified with the nucleus radius R one assumes the existence of the pure Coulomb interaction. For $r \leq R$ there is the complete absorption, i.e., the solution of this area is the convergent wave. The sewing at $r = R$ of the Coulomb function with the solutions inside the well determines in the unique way the S-matrix. The calculated cross section was identified in [1,2] with that of fusion. Later it was established that this cross section is that of reaction rather than of fusion. We mention also paper [3] where the process of fusion is identified with the formation of the quasi-stationary states. The second class incorporates the papers which having no sufficiently good quantum-mechanical grounds are successful prescriptions widely used in fitting the experimental data. Typical are papers[4,6]. In ref.[4] the Brueckner method was used to calculate the real part of the ion-ion potential in sudden approximation. The fusion cross section was defined as a sum of partial penetrabilities (assumed to equal unity) up to some value of ℓ_c.This one in turn was defined as the maximal angular momentum at which the turning point still lies to the left from the critical distance r_c. It has been found empirically that r_c does not depend on the energy of incident ions but depends only on the atomic weights of ions: $r_c = r_0(A_1^{1/3} + A_2^{1/3})$, $r_0 = 1$ fm. In[5] for the one-dimensional potential in the form of the inverted parabola an analytic expression has been obtained for partial penetrabilities $T_\ell(E)$. These were used in paper[6] to calculate penetrabilities through the potential barrier formed by the Coulomb and the Saxon-Woods-type potentials. The oscillator constant was found from the condition of equality of the radii of curvature near the top of the barrier of the real and approximating potentials. The model used in[6] seems to be unsufficiently grounded for the following reason. The potential used there is real. For an arbitrary real potential the scattering matrix is unitary and, consequently, the cross sections of reaction and fusion are identically equal to zero. A nonzeroth result was obtained in[6] due to the use of the one-dimensional potential barrier. We have no objection

against the replacement of the real potential near the top of the barrier by the inverted parabola but only with taking into account the 3-dimensional aspect of the treated problem. In the three-dimensional case for the real potential the wave which penetrates the barrier after reflections and refractions on inhomogeneities of the potential and after the subsequent penetration of the barrier interferes with the reflected wave completing its amplitude in magnitude up to unity. In the one-dimensional case the wave which penetrates the barrier cannot return to the initial point (if there is no reflecting wall). However, the formulas for the cross sections of reaction and fusion derived in[6] may be regarded as comfortable parametrizations.

It is important to have the physical justification of these formulas in the three-dimensional case. More exactly, the problem may be stated as follows: what conditions should be imposed on the potential (i.e., to introduce the complete or partial absorption, imaginary part, etc.) in order to make the formulas of ref.[6] to be valid also in the three-dimensional case.

The third category is formed by the papers which use the concepts typical only for reactions with heavy ions: friction, energy dissipation, etc. Here we go beyond the usual quantum-mechanical phenomena and enter a completely new region. Without rejecting the value of such an approach we prefer here to understand and justify the results of papers[4,6] in the framework of the usual quantum-mechanical concepts. The following is based on the results of refs. /7,8/.

Since the potential we use further contains the imaginary part in addition to the real one, we discuss now the influence of the value of the imaginary part of the potential on the reaction cross sections. Intuitively, one might expect the growth of the cross section with increasing W_0 due to the rise of absorption. However, calculations show (fig. 1) that the reaction cross section (equal to zero at $W_0 = 0$) increases with increasing W_0 in certain interval and then decreases. As is shown in detail in ref. /7/ , the mathematical reason for such dependence is due to the unitarization of the scattering matrix both for $W_0 \to 0$ and for $W_0 \to \infty$. The physical reason is as follows. At small W_0 the incident wave passes through the area of action of the imaginary potential without substantial attenuation, and the sum of coefficients of reflection and penetration only slightly differs from unity. At large values of W_0 the unitary is restored because the large value of the complex potential barrier prevents penetration into the interior region, i.e., there is the practically complete reflection.

The model discussed below is a modification of the Thomas model /1/ (fig. 2). The real part of the potential is taken in the form

$$V = \begin{cases} \dfrac{Z_1 Z_2 e^2}{r} & r > R \\ -V_0 & r < R \end{cases}$$

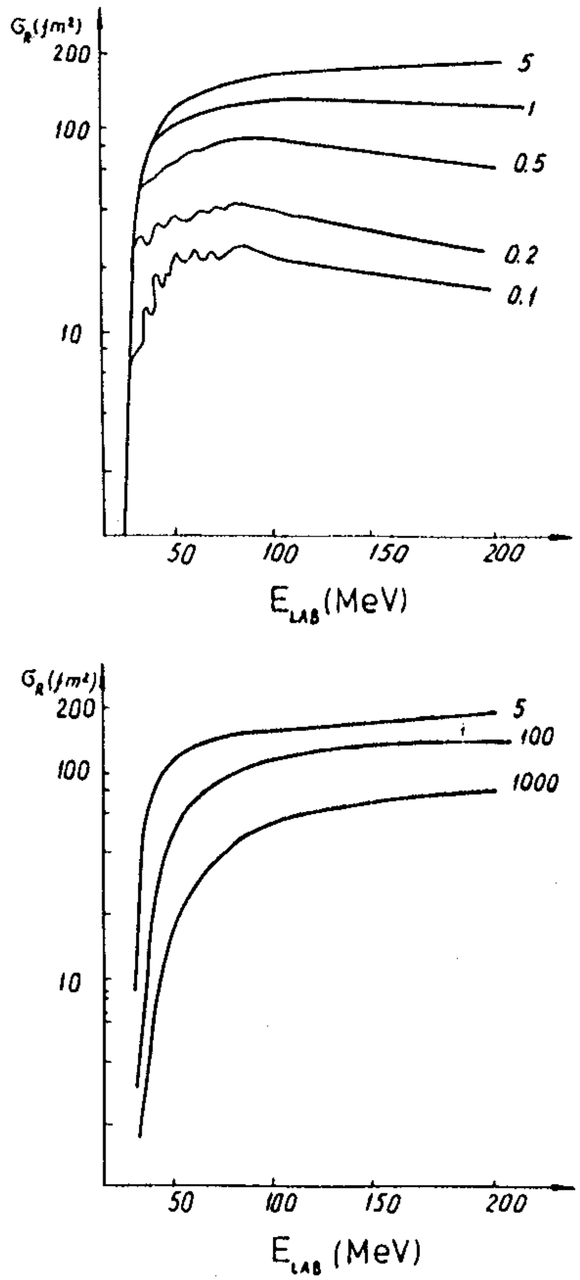

Fig. 1. Cross section of the reaction plotted as a function of energy. The numbers at the right denote the value of imaginary potential.

In the range $a < r < R$ the potential contains also the imaginary part $-V_0 - iW_0$.

Then it is assumed that for $r \le a$ there is the complete fusion. We define the fusion as the existence, in the region $r < a$, only of the convergent spherical wave:

$$\Psi_{in}(r) = A_\ell \, h_\ell^{(2)} (k_0 r),$$

where

$$k_0 = \sqrt{\frac{2\mu(E+V_0)}{\hbar^2}}.$$

However, as in the Thomas model, in this case the behaviour of the wave function for $r<a$ is not important, only the boundary condition at $r=a$ is essential. It is clear that the quantity $C_\ell = |A_\ell|^2 \sqrt{\frac{E+V_0}{E}}$ defines what portion of the initial flux (equal to unity) reaches the region $r=a$, i.e., in our terminology, the region of fusion. Then it is natural to define the cross section of fusion in the following way:

$$\sigma_F = \pi \lambda^2 \cdot \Sigma (2\ell+1) C_\ell .$$

Now, let us analyse some physical consequences of the given definition of the fusion cross section. Note first, that the cross section of reaction always exceeds that of fusion. This is due to the processes in the surface layer (the reaction of stripping, pickup, etc.). Exactly, the imaginary part of the potential being concentrated in the surface region is responsible for these processes. It is clear that at $W_0 = 0$ for the physical reason we should have $\sigma_R = \sigma_F$, that is confirmed by direct calculation. The difference between σ_R and σ_F should diminish with decreasing imaginary potential, with decreasing thickness of the surface layer (where W_0

is different from zero), with increasing energy of the incident ion and increasing depth of the real potential V_0. The latter is due to the fact that for large values of E (or V_0) one can make the expansion in parameter $\frac{W_0}{E+V_0}$. In this case σ_R and σ_F differ by the same order of magnitude. At very large value of W_0 the scattering matrix again approaches the unitary limit. In this case the greater part of the incident wave is reflected. A small portion of it, which reaches the fusion region, will be significantly attenuated due to the large value of the W_0. In this case both the reaction and the fusion cross sections are tending to zero, the latter much faster. For $E \to \infty$ both the cross sections tend to the same limit equal to πa^2. We mention also that at $R=a$ or $W_0=0$ the model under consideration does not differ from the Thomas model.

Let us illustrate our consideration by the reaction $^{16}O + {}^{40}Ca$. The parameters R and a are taken to be equal to the Thomas and Lefort's radii, respectively:

$$R = r_0 (A_1^{1/3} + A_2^{1/3}),$$

$$a = r_f (A_1^{1/3} + A_2^{1/3}),$$

$$r_0 = 1.5 \text{ fm}, \quad r_f = 1 \text{ fm}.$$

The real part of the potential is the same as in the Thomas paper /1/, i.e., $V_0 = 2.063$ MeV. Fig. 3 shows the cross sections of the reaction and fusion as

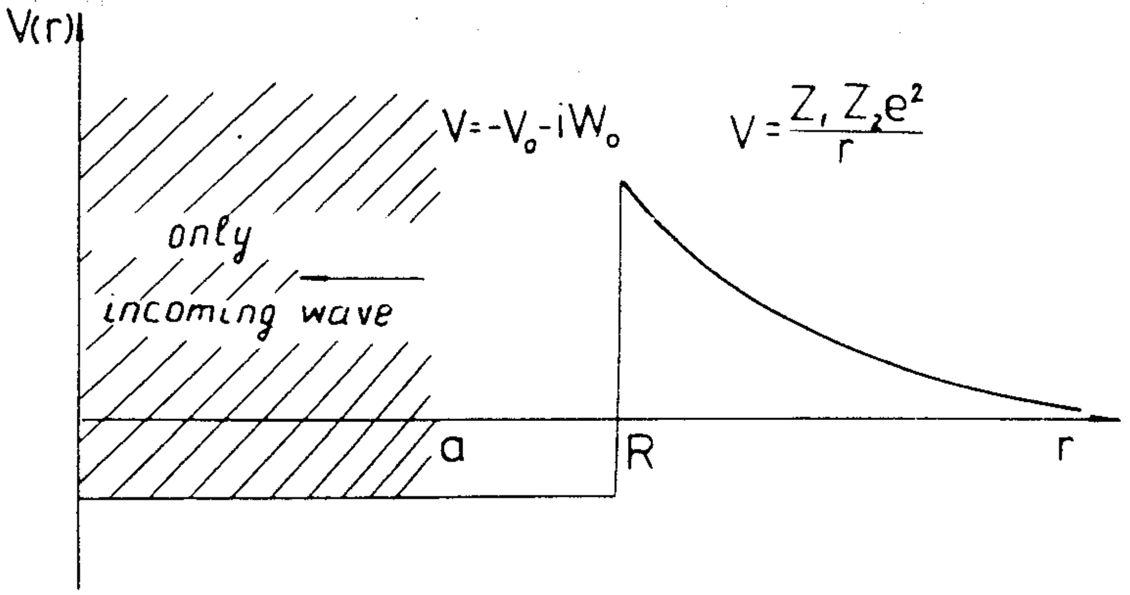

Fig. 2. Schematic representation of the present model.

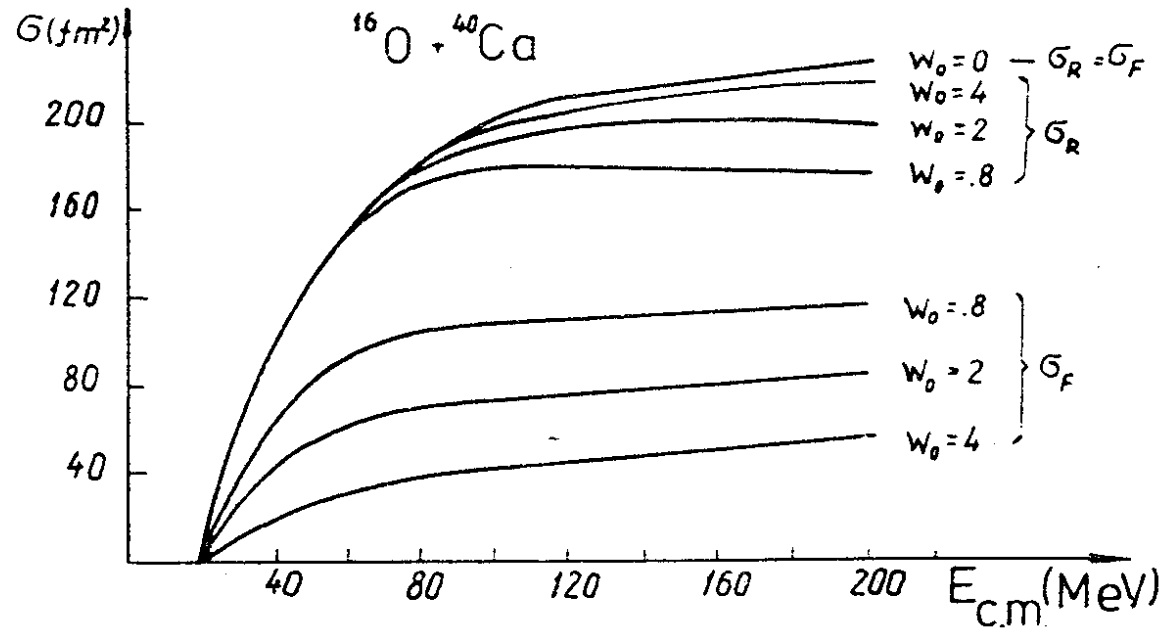

Fig. 3. Reaction and fusion cross sections plotted as a function of energy. The numbers on the right denote the value of the imaginary potential.

functions of the energy of incident ions for different values of the imaginary part of the potential. In this way, earlier qualitative predictions are justified: in the interval of the imaginary potential $0 \leq W \leq 12$ MeV the reaction cross section changes slightly, whereas that of fusion decreases by about two orders. This fact may be used in order to achieve the decrease for the fusion cross section starting from a certain energy. A weak energy dependence of the imaginary part of the potential will do this.

The present treatment being qualitative one, is not free of drawbacks. The most important of them is the use of the optical model. However, this may be justified because of the fact that the conception of the optical model is exploited in most of papers concerning the fusion of nuclei. Further, extremely simplified shape of the potential (square well + Coulomb) may significantly distort the energy dependence of the fusion cross section and shade various subtle effects. The optimal approach, apparently, consists in the following. From experiments on the elastic scattering of heavy ions we extract the optical potential, then distort it introducing the condition of the complete fusion on the Lefort's distance. Since the elastic scattering of heavy ions (and of α-particles) is sensitive only to the potential shape near the surface, one may hope that the distorted optical potential will describe the elastic scattering as good as the initial one. The boundary conditions at the Lefort's radius and sewing of the calculated at computer wave functions with the linear combinations of the Coulomb functions in the area, where the nuclear potential can be neglected, determine uniquely the partial scattering matrix and the flux penetrating into the interior region. This, in turn, defines the reaction and fusion cross sections.

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