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ON A NUMERICAL METHOD FOR SOLVING THE FADDEEV EQUATION WITHOUT DEFORMATION OF CONTOUR

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The Faddeev equations /1/ describing a system of three spinless particles are reduced to a one-dimensional integral equation if the angular variables are separated and the two-particle interaction is taken in separable form. This equation can be written as follows

$$F(x) = F_0(x) + \int_0^\infty dx' K(x, x', E_3) F(x'), \qquad (1)$$

(E_3 - total energy in cms).

The difficulty in solving this integral equation numerically at positive energies E_3 consists in the fact that the kernel of the equation contains a pole (if the interaction is strong enough) and two logarithmic singularities on the path of integration. Mainly two methods are in use to solve the singular integral equation numerically. The first one due to Hetherington and Schick /2/ is based on the deformation of the contour of integration. Although this method has been used successfully in a number of calculations (e.g., /3/), it has the disadvantage that it requires the kernel $K(x, x', E_3)$ and the function $F(x')$ to be extended to complex values of x'.

Sohre and Ziegelmann /4/ proposed another method which avoids the contour deformation. This is achieved by factoring out the singularities of the kernel in the form (with the E_3-dependence omitted in the notations)

$$\int_0^\infty dx' K(x, x') F(x') = \sum_i \int_0^\infty dx' S_i(x' - q_i(x)) f_i(x, x'), \qquad (2)$$

where the functions $S_i(x' - q_i(x))$ describe the singularities and the functions $f_i(x, x')$, the smooth part of the kernel. Now an arbitrary set of mesh points is choosen.

Between the mesh points the functions $f_i(x, x')$ are represented by interpolation polynomials. Then the remaining integrals over the singularities can be taken analytically. The method /4/ has been applied, for example, in paper /5/.

The pole singularities are handled usually by the so-called subtraction method in the form

$$\int dx' K(x,x') F(x') = \int dx' \left[K(x,x') F(x') - \frac{R_p(x) F(x_p)}{(x' - z_p)} \right] + R_p(x) F(x_p) \int \frac{dx'}{(x' - z_p)}; \quad z_p = x_p + i0, \tag{3}$$

Here $R_p(x)$ is the residue of the kernel at $x' = x_p$. The integrand in square brackets is now a smooth function of x' in the region of the pole. The integral can be simply calculated by numerical integration. The second integral on the right-hand side can be taken analytically. The quantity x_p is chosen to be among the mesh points.

As an alternative to the methods mentioned we propose to subtract the logarithmic singularities of the kernel in the same way as the pole singularity is subtracted. Then we have the following splitting of the kernel

$$\int dx' K(x,x') F(x') = \int dx' \left[K(x,x') F(x') - \frac{R_p(x) F(x_p)}{(x' - z_p)} - R_1(x) F(x_1) \ln|x' - x_1| - (2) \right] + R_p(x) F(x_p) \int \frac{dx'}{(x' - z_p)} + R_1(x) F(x_1) \int dx' \ln|x' - x_1| + (2). \tag{4}$$

Here the abbreviation (2) stands for the second logarithmic singularity. The position of the first logarithmic singularity is denoted by $x_1(x)$. The quantity $R_1(x)$ is

the logarithmic residue at this point. As is known, the decisive difference between the pole and the logarithmic singularities in the considered integral equation is the fact that the positions of the logarithmic singularities depend on x (moving singularities) whereas the pole is independent of this variable. Therefore, as is mentioned in /4/, it would be very difficult to construct an integration mesh with $x_1(x_i)$ and $x_2(x_i)$ among the mesh points for any arbitrary mesh point x_i. In practice, however, there is no necessity for x_1, x_2 to belong to the mesh point set since the quantities $F(x_1)$ and $F(x_2)$ in eq. (4) can be easily calculated from the values of F in the neighbouring mesh points by simple interpolation. If the function F to be calculated has a sufficiently smooth behaviour (and only in this case the numerical solution of eq. (1) is meaningful), the interpolation can be done with sufficient accuracy. Thus, in practice there is no essential difference in handling moving and not moving singularities in numerical calculations. But we have to mention another point. After subtraction of the pole term for $x' \approx x_p$ the kernel behaves like $K(.,x') = a_0 + a_1(x' - x_p) + \ldots$. For the logarithmic singularities we get $K(.,x') = b_0(x' - x_1) \ln|x' - x_1| + b_1(x' - x_1)^2 \ln|x' - x_1| + \ldots$. This means that after subtraction the pole singularity is completely extracted, whereas for the logarithmic singularities there remains an irregular behaviour of the difference kernel at $x' = x_1$ (the derivatives turn to infinity at this point). This irregularity leads to increased error of the numerical integration in the region of the logarithmic singularities. However, numerical tests have shown that in most practical applications the accuracy would be within acceptable limits even if Gaussian integration is used (which is not very suited in this case). Of course one can obtain any required accuracy if a suitable integration method is chosen with the step in the numerical integration taken small enough. Here again the values of F between the mesh points are calculated by interpolation from the mesh point values. Moreover the behaviour of the difference kernel at $x' = x_1$ can be improved by

additionally subtracting in eq. (4) for each logarithmic singularity a term of the form
$(R'_L(x)F(x_L) + R_L(x)F'(x_L))(x' - x_L)\ln|x' - x_L|$
with $L = 1,2$. Here $R'_L(x)$ can easily be calculated if the explicit form of the kernel is known. The function $F'(x_L)$ is the first derivative of F at $x'=x$. The derivative can be calculated without difficulty from the interpolation polynomial for $F(x_L)$.

The proposed method has been applied to calculate the eigenvalues of the Faddeev-kernel for separable interaction /6/. The results coincide with those obtained using the described method of Sohre and Ziegelmann. Although it is difficult to define a unique criterion for comparison of the different methods mentioned it seems to us that the proposed method can be used effectively for three-body calculations. Especially if the required accuracy is not very high, the method works very fast.

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