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# GENERALIZATION
# OF THERMAL RANDOM PHASE APPROXIMATION

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Обобщение теплового приближения случайной фазы

В рамках формализма термополевой динамики сформулирована новая версия теплового приближения случайной фазы (ПСФ) для системы конечного числа фермионов. В сравнении со стандартным тепловым ПСФ учтены следующие эффекты: наличие в тепловом вакууме нагретой системы некоторого количества тепловых квазичастиц; взаимное влияние коллективных и хартри-фоковских переменных и изменение из-за этого взаимодействия тепловых чисел заполнения; некоторые двухчастичные корреляции в уравнениях движения. В качестве частных случаев обобщенное ТПСФ включает ранее рассмотренные тепловое перенормированное ПСФ и тепловое самосогласованное ПСФ.

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Generalization of Thermal Random Phase Approximation

A general and self-consistent version of a thermal random phase approximation is developed using the formalism of thermo field dynamics. The following effects are taken into account as compared with a standard TRPA: the non-vanishing number of thermal quasiparticles in a thermal vacuum state; the coupling of collective and HF variables and its influence on thermal occupation numbers; some two-particle correlations in equations of motion omitted in TRPA. The generalized TRPA includes, as particular cases, the thermal renormalized RPA and the thermal self-consistent RPA.

# 1. Introduction

In the present paper we continue (see [1-4]) to develop approximate methods of description of a collective motion in heated finite Fermi - systems in the framework of thermo field dynamics.

A standard technique of treating quantum many-body systems at finite temperature $T$ is the temperature-dependent Green function method (see, e.g. [5]). But other approaches exist as well. We apply here the so-called thermo field dynamics (TFD) [6]. In contrast with the temperature-dependent Green function method TFD is a real-time formalism, that is temperature $T$ and time $t$ are independent variables within TFD. The TFD formalism is quite convenient for our purposes since temperature effects arise explicitly as $T$-dependent vertices, thus providing a good starting point for various approximations.

The main idea behind TFD is a construction of a field theory in which the grand canonical statistical average of a quantity $A$ is given by some sort of expectation value rather than the trace operation

$$\ll A \gg = \frac{1}{Tr(\exp(-H/T))} Tr\left[A \exp(-H/T)\right] = \langle 0(T)|A|0(T)\rangle .$$

TFD gives rigorous prescriptions how to construct a representation in which the "vacuum" expectation value coincides with the statistical average. This aim is achieved by a formal doubling of the Hilbert space of a system. One introduces a fictitious system which is of exactly the same structure as the physical one under consideration. The whole Hilbert space of a hot system is spanned by the direct product of the eigenstates of the Hamiltonian $H|n\rangle = E_n|n\rangle$ and those of the "tilde" Hamiltonian having the same eigenvalues $\widetilde{H}|\tilde{n}\rangle = E_n|\tilde{n}\rangle$. With the doubling of the Hilbert space one can write the expression for a "vacuum" state $|\Psi_0(T)\rangle$ that is called "the thermal vacuum state"

$$|\Psi_0(T)\rangle = \frac{1}{\sqrt{Tr(\exp(-H/T))}} \sum_n \exp\left(-\frac{E_n}{2T}\right)|n\rangle \otimes |\tilde{n}\rangle . \tag{1.1}$$

The vectors $|n\rangle$ and $|\tilde{n}\rangle$ appear as a pair, and the function of $|\tilde{n}\rangle$ is merely to pick up the diagonal element of $A$. A tilde conjugate operator $\widetilde{A}$ acting in the tilde space is associated with any operator $A$ acting in the ordinary space in accordance with special rules [6]. The time - translation operator in the hot system appears to be a thermal Hamiltonian $\mathcal{H}$ defined by $\mathcal{H} = H - \widetilde{H}$. Properties of the system excitations are obtained by the diagonalization of $\mathcal{H}$. The thermal vacuum is an eigenstate of $\mathcal{H}$ with a zero eigenvalue. Thus, the dynamical development of the system is carried by the thermal Hamiltonian while the thermal behaviour is controlled by the thermal vacuum.

The TFD approach provides transparent interpretation of a collective motion in a hot Fermi - system. The thermal vacuum can be regarded as a temperature - dependent

wave function of a compound nuclear state. Collective excitation is produced by applying the corresponding collective operator (phonon operator) to this thermal vacuum state. Properties of the collective excitation are dependent on the vacuum correlations.

There are two well-known approximations to describe excitations in hot finite Fermi - systems: the thermal Hartree - Fock approximation (THFA) and the thermal random phase approximation (TRPA). The latter was applied to study a giant dipole resonance in hot nuclei (see the review paper [7] and references therein). Also some approximations going beyond TRPA were considered but merely a coupling of thermal particle-hole or TRPA phonon excitations with more complex ones was studied [4, 8].

At the same time, while studying low-energy nuclear excitations quite elaborate modifications of the standard RPA were suggested [9-11]. And only a few of these new variants of RPA were applied to the $T \neq 0$ case [1-3]. It seems interesting to formulate a generalization of the thermal RPA like it was done for RPA in "cold" many-body systems. Here, we present such a generalization. In our studies we follow general schemes and ideas of refs. [2, 10].

## 2. Mean field approximation in TFD

We start a discussion as in [2] describing a thermal mean field approximation (or a thermal Hartree - Fock approximation). Let us consider a system of N fermions with a two-body interaction. The Hamiltonian is

$$H = \sum_{12} t_{12} a_1^+ a_2 + \frac{1}{4} \sum_{1234} V_{1234} a_1^+ a_2^+ a_4 a_3 , \qquad (2.1)$$

where $a^+$ and $a$ are fermion creation and annhilation operators. The one-body part of the Hamiltonian $t_{12} = T_{12} - \lambda \delta_{12}$ contains a kinetic energy matrix $T_{12}$ and a chemical potential $\lambda$.

To describe properties of the system when it is heated up to finite temperature $T$ within the framework of the thermo field dynamics, one should build the thermal Hamiltonian of the system $\mathcal{H} = H - \widetilde{H}$ and then find the corresponding thermal vacuum state $|\Psi_0(T)\rangle$. The thermal vacuum state should be an eigenstate of $\mathcal{H}$ with a zero eigenvalue. Obviously, one cannot resolve the problem in general. A series of necessary approximations begins with a thermal mean field approximation.

Usually, under statistical consideration of a highly excited (heated) finite Fermi system the unified statistically averaged single-particle mean field is used for all energy states (instead of finding the best single-particle approximation for every state in a partition function). The single-particle Hamiltonian $H_0$ corresponding to this mean potential can be written as

$$H_0 = U_0 + \sum_1 E_1 \alpha_1^+ \alpha_1 , \qquad (2.2)$$

where $U_0$ is a constant, $E_1$ are single-particle (or single-quasiparticle) energies. According to Bogoliubov's variational theorem [12], a model thermodynamic potential defined by

$$\Omega_{\mathrm{mod}}(H) = \Omega_0 + \ll H - H_0 \gg_0 , \qquad (2.3)$$

is the upper limit of the "true" thermodynamic potential $\Omega(H)$ for any Hamiltonian (2.1), i.e. $\Omega_{\mathrm{mod}}(H) \geq \Omega(H)$. $\Omega_0$ and $S_0$ are the thermodynamic potential and the entropy of the system of non-interacting quasiparticles described with the Hamiltonian $H_0$,

$$\Omega_0 = \ll H_0 \gg_0 - T S_0 ,$$

$$\ll H_0 \gg_0 = \frac{Tr[H_0 \exp(-H_0/T)]}{Tr[\exp(-H_0/T)]} .$$

The density matrix of the system with the Hamiltonian $H_0$ (2.2) is

$$n_1 = \ll \alpha_1^+ \alpha_1 \gg_0 ,$$

and the following expression for the entropy $S_0$ is valid:

$$S_0 = - \sum_1 [n_1 \ln(n_1) + (1 - n_1) \ln(1 - n_1)] . \qquad (2.4)$$

The model thermodynamic potential $\Omega_{\mathrm{mod}}$ can be constructed with any single-particle Hamiltonian $H_0$. But certainly to find a Hamiltonian $H_0$ that makes the difference $\Omega_{\mathrm{mod}}(H) - \Omega(H)$ minimal is of special interest. Such a Hamiltonian $H_0$ is called the self-consistent Hartree - Fock Hamiltonian. To find a Hartree - Fock basis, one has to make a unitary transformation $D$ ( $DD^+ = 1$) of the Hamiltonian (2.1) from initial "bare" fermions $a_1^+, a_1$ to new Hartree - Fock (HF) quasiparticles $\alpha_1^+, \alpha_1$

$$a_1^+ = \sum_2 D_{21}^* \alpha_2^+ , \qquad a_1 = \sum_2 D_{21} \alpha_2 . \qquad (2.5)$$

Unitarity of the transformation provides a conservation of the commutation rules. Since $\Omega_{mod}(H)$ is an upper limit for $\Omega(H)$ equations for the coefficients $D$ can be derived from the requirement that $\Omega_{mod}(H)$ is minimal under the constraint $DD^+ = 1$.

$$\frac{\delta}{\delta D_{12}^*} \left( \Omega_{\mathrm{mod}}(H) - \sum_2 \xi_2 \sum_3 D_{23}^* D_{23} \right) = \frac{\delta}{\delta D_{12}^*} \left( \ll H \gg_0 - \sum_2 \xi_2 \sum_3 D_{23}^* D_{23} \right) = 0$$
$$(2.6)$$

$$\frac{\delta}{\delta D_{12}} \left( \Omega_{\mathrm{mod}}(H) - \sum_2 \xi_2 \sum_3 D_{23}^* D_{23} \right) = \frac{\delta}{\delta D_{12}} \left( \ll H \gg_0 - \sum_2 \xi_2 \sum_3 D_{23}^* D_{23} \right) = 0 ,$$

which should be completed by the number conserving condition

$$\sum_1 \ll a_1^+ a_1 \gg_0 = N .$$

Moreover, $\Omega_{\mathrm{mod}}(H)$ is supposed to be stable under variations of a quasiparticle number density $n_1$. From the requirement

$$\frac{\delta \Omega_{\mathrm{mod}}(H)}{\delta n_1} = 0 , \qquad (2.7)$$

using the expression for the entropy (2.4), and under the assumption that the thermal distribution of non-interacting HF quasiparticles has the Fermi - Dirac shape

$$n_1 = \left[1 + \exp\left(\frac{E_1}{T}\right)\right]^{-1} , \qquad (2.8)$$

the following equation for HF quasiparticle energies $E_1$ can be evaluated

$$E_1 = \frac{\delta \ll H \gg_0}{\delta n_1} . \qquad (2.9)$$

Let us now formulate the thermal HF approximation within TFD. Quantities $\ll H \gg_0$, $\ll \alpha_1^+ \alpha_1 \gg_0$ or $\ll a_1^+ a_1 \gg_0$ are evaluated within TFD as expectation values with respect to a thermal vacuum state $|0(T)\rangle$ of the thermal Hamiltonian $\mathcal{H}_0 = H_0 - \tilde{H}_0$. The equations for a HF basis can be derived in the following way [2]. First, one should rewrite the Hamiltonian (2.1) in the normal ordered form with respect to a thermal ground state. The result is

$$H = U_0 + H_1 + H_2$$

$$U_0 = \sum_{12} t_{12} \rho_{21} + \frac{1}{2} \sum_{1234} V_{1234} \rho_{31} \rho_{42} ;$$

$$H_1 = : \sum_{12} t_{12} a_1^+ a_2 + \sum_{1234} V_{1234} \rho_{42} a_1^+ a_3 : , \qquad (2.10)$$

$$H_2 = : \frac{1}{4} \sum_{1234} V_{1234} a_1^+ a_2^+ a_4 a_3 : ,$$

where $\rho_{ij}$ – is a one-body density matrix

$$\rho_{ij} = \langle 0(T)|a_j^+ a_i|0(T)\rangle .$$

Then one needs to perform the unitary transformation (2.5) and to find $D$ from the requirement that the item $H_1$ (2.10) has to be diagonal in terms of the HF quasiparticles. The equations for $D$ take the form (2.6) and

$$U_0 + H_1 = U_0 + \sum_1 E_1 \alpha_1^+ \alpha_1 = H_0 \equiv H_{HF} .$$

Number conserving condition appears to be

$$\sum_1 \rho_{11} = N .$$

It is quite easy to find the wave function $|0(T)\rangle$ of a thermal HF Hamiltonian. For example, the expression for $|0(T)\rangle$ can be obtained from (1.1) if one takes as $|n\rangle$ and $|\tilde{n}\rangle$ single-quasiparticle and tilde single-quasiparticle states with the energies $E_1$. But more fruitful is to use the following fact. In accordance with the definition,

$$\mathcal{H}_{HF}|0(T)\rangle = \left(H_{HF} - \tilde{H}_{HF}\right)|0(T)\rangle = \sum_1 E_1 \left(a_1^+ a_1 - \tilde{a}_1^+ \tilde{a}_1\right) \cdot |0(T)\rangle = 0 .$$

The solution of the above equation is the vacuum for thermal quasiparticle operators $\beta, \tilde{\beta}$ [6] that are defined by

$$\begin{aligned} \beta_1 &= x_1 \alpha_1 - y_1 \tilde{\alpha}_1^+ \\ \tilde{\beta}_1 &= x_1 \tilde{\alpha}_1 + y_1 \alpha_1^+ , \end{aligned} \qquad (2.11)$$

with the coefficients $x_1 = \sqrt{1 - n_1}$ , $y_1 = \sqrt{n_1}$ ($n_1$ is defined by (2.8). A unitary transformation $\{x, y\}$ is called the thermal Bogoliubov transformation.

Useful to note that the value $\Omega_{\mathrm{mod}}(H)$ (2.3) is minimal for the thermal HF vacuum state $|0(T)\rangle$. This is seen from the following consideration. Since $\beta|0(T)\rangle = \tilde{\beta}|0(T)\rangle = 0$, the relations are valid

$$\langle 0(T)|H|0(T)\rangle = U_0 , \qquad n_k = \langle 0(T)|\alpha_k^+ \alpha_k|0(T)\rangle = y_k^2 .$$

And the one-body density matrix $\rho_{12}$ takes the form

$$\rho_{12} = \langle 0(T)|a_2^+ a_1|0(T)\rangle = \sum_k D_{2k}^* D_{1k} n_k = \sum_k D_{2k}^* D_{1k} y_k^2 . \qquad (2.12)$$

The entropy is

$$S_0 = -\sum_k \left[y_k^2 \ln y_k^2 + (1 - y_k^2) \ln(1 - y_k^2)\right] .$$

Then from the condition (2.7) one can see that

$$y_k^2 = [1 + \exp(E_k/T)]^{-1} ,$$

where $E_k = \delta U_0/\delta y_k^2$. It means that the values of $x$ and $y$, satisfying the relation $\beta|0(T)\rangle = \tilde{\beta}|0(T)\rangle = 0$, also produce the minimum of $\Omega_{\mathrm{mod}}(H)$. From a requirement of a minimum of a grand thermodynamic potential one can determine coefficients of the usual and thermal Bogoliubov transformations for a hot system of N nucleons with the BCS pairing [13].

# 3. A generalization of TRPA

Within THFA a residual interaction between thermal quasiparticles is omitted. But evidently this interaction exists and provides some types of correlations. As a result, a structure of the thermal HF vacuum state $|0(T)\rangle$ may be quite far from the "true" one. Here, we try to take into account the residual interaction.

At the first step, we express the initial thermal Hamiltonian $\mathcal{H} = H - \widetilde{H}$ ( $H$ is defined in (2.1)), in terms of thermal quasiparticles. We only suppose that the transformations $D$ and $\{x, y\}$ are unitary.

The Hamiltonian $\mathcal{H}$ can be expressed as a sum of items $\mathcal{H}_{mn}$. Each of them contains $m$ creation operators and $n$ annihilation operators of thermal quasiparticles.

$$\mathcal{H} = \mathcal{H}_{11} + \mathcal{H}_{22} + \mathcal{H}_{20} + \mathcal{H}_{02} + \mathcal{H}_{40} + \mathcal{H}_{04} + \mathcal{H}_{31} + \mathcal{H}_{13} \qquad (3.1)$$

We define the bifermionic operators as follows

$$A_{12}^+ = \widetilde{A}_{21}^+ = \beta_1^+ \widetilde{\beta}_2^+, \quad A_{12} = \widetilde{A}_{21} = \widetilde{\beta}_2 \beta_1, \quad B_{12} = \beta_1^+ \beta_2, \quad \widetilde{B}_{12} = \widetilde{\beta}_1^+ \widetilde{\beta}_2.$$

Their exact commutation relations are

$$\begin{aligned}
\left[A_{12}, A_{34}^+\right] &= \delta_{13}\delta_{24} - \delta_{13}\widetilde{B}_{42} - \delta_{24}B_{31}, \\
\left[B_{12}, B_{34}\right] &= \delta_{23}B_{12} - \delta_{14}B_{32}, \\
\left[A_{12}, B_{34}\right] &= \delta_{13}A_{42}, \\
\left[A_{12}, \widetilde{B}_{34}\right] &= \delta_{23}A_{14}, \\
\left[A_{12}^+, B_{34}\right] &= -\delta_{14}A_{32}^+, \\
\left[A_{12}^+, \widetilde{B}_{34}\right] &= -\delta_{24}A_{13}^+, \\
\left[B_{12}, \widetilde{B}_{34}\right] &= 0.
\end{aligned} \qquad (3.2)$$

In terms of the bifermionic operators the items $\mathcal{H}_{mn}$ are

$$\begin{aligned}
\mathcal{H}_{11} &= \sum_{12}\left(P_{12} + \sum_3 y_3^2 U_{1323}\right)(x_1 x_2 + y_1 y_2)\left(B_{12} - \widetilde{B}_{12}\right) \\
\mathcal{H}_{20} &= \sum_{12}\left(P_{12} + \sum_3 y_3^2 U_{1323}\right)(x_1 y_2 - x_2 y_1) A_{12}^+ \\
\mathcal{H}_{22} &= \sum_{1234} U_{1234}\left[(x_1 x_4 y_2 y_3 - x_2 x_3 y_1 y_4) A_{13}^+ A_{42}\right. \\
&\quad\left. + \frac{1}{4}(x_1 x_2 x_3 x_4 - y_1 y_2 y_3 y_4)\left(: B_{13}B_{24} - \widetilde{B}_{13}\widetilde{B}_{24}:\right)\right] \\
\mathcal{H}_{40} &= \sum_{1234} U_{1234}(x_1 x_2 y_3 y_4 - x_3 x_4 y_1 y_2) A_{13}^+ A_{24}^+ \\
\mathcal{H}_{31} &= \frac{1}{2}\sum_{1234} U_{1234}(x_1 x_2 x_3 y_4 + y_1 y_2 y_3 x_4)\left(A_{24}^+ B_{12} - A_{42}^+\widetilde{B}_{12}\right)
\end{aligned} \qquad (3.3)$$

$$\mathcal{H}_{20} = \mathcal{H}_{02}^+, \quad \mathcal{H}_{40} = \mathcal{H}_{04}^+, \quad \mathcal{H}_{13} = \mathcal{H}_{31}^+.$$

The coefficients $P_{12}$ and $U_{1234}$ in (3.3) are defined by

$$\begin{aligned}
P_{12} &= \sum_{34} t_{34} D_{13}^* D_{24}, \\
U_{1234} &= \sum_{5678} V_{5678} D_{51}^* D_{62}^* D_{73} D_{84}.
\end{aligned}$$

To find an excitation spectrum of a hot system, one should diagonalize the Hamiltonian (3.1). We use the equation of motion method [14] and define a ground and excited states of the system by the relations:

$$|\Psi_\nu(T)\rangle = Q_\nu^+|\Psi_0(T)\rangle, \qquad Q_\nu|\Psi_0(T)\rangle = 0. \qquad (3.4)$$

A composition of an operator $Q_\nu^+$ in terms of thermal quasiparticle operators is given below. The Raleigh-Ritz variational principle for $\mathcal{H}$ has the form

$$\langle\Psi_0(T)|\left[\delta Q_\nu, [\mathcal{H}, Q_\nu^+]\right]|\Psi_0(T)\rangle = \omega_\nu\langle\Psi_0(T)|\left[\delta Q_\nu, Q_\nu^+\right]|\Psi_0(T)\rangle, \qquad (3.5)$$

$\omega_\nu$ is an energy of an excited state $|\Psi_\nu(T)\rangle$.

Eq. (3.5) is exact if the variation $\delta Q_\nu|\Psi_0(T)\rangle$ exhausts the whole Hilbert space of the system. We restrict ourselves to a trial wave function of the TRPA type

$$Q_\nu^+ = \sum_{12}\psi_{12}^\nu A_{12}^+ - \phi_{12}^\nu A_{12}. \qquad (3.6)$$

But we don't use other TRPA assumptions. For example, the bifermionic operators $A_{12}$ and $A_{34}^+$ are not supposed to be quasibosons etc.

Instead, we assume that the number of thermal quasiparticles in a thermal vacuum state does not vanish and can be approximated by c-number, i.e.

$$\langle\Psi_0(T)|B_{12}|\Psi_0(T)\rangle = \langle\Psi_0(T)|\widetilde{B}_{12}|\Psi_0(T)\rangle = q_1\delta_{12}. \qquad (3.7)$$

Then, for the value $M_{1234} = \langle\Psi_0(T)|\left[A_{12}, A_{34}^+\right]|\Psi_0(T)\rangle$ the following expression is valid:

$$\begin{aligned}
M_{1234} &= \langle\Psi_0(T)|\delta_{13}\delta_{24} - \delta_{13}\widetilde{B}_{42} - \delta_{24}B_{31}|\Psi_0(T)\rangle \\
&= \delta_{13}\delta_{24}(1 - q_1 - q_2) = \delta_{13}\delta_{24}(1 - q_{12}). \qquad (3.8)
\end{aligned}$$

If one takes into account (3.7) and (3.8), then from eq. (3.5) a system of equations for phonon amplitudes $\psi_{12}^\nu, \phi_{12}^\nu$ and energies $\omega_\nu$ can be derived [1]

$$\sum_{34}\psi_{34}^\nu\langle|\left[A_{12}, [\mathcal{H}, A_{34}^+]\right]|\rangle - \sum_{34}\phi_{34}^\nu\langle|\left[A_{12}, [\mathcal{H}, A_{34}]\right]|\rangle = \omega_\nu\sum_{34}\psi_{34}^\nu M_{1234}$$

$$\qquad (3.9)$$

$$\sum_{34}\psi_{34}^\nu\langle|\left[A_{12}^+, [\mathcal{H}, A_{34}^+]\right]|\rangle - \sum_{34}\phi_{34}^\nu\langle|\left[A_{12}^+, [\mathcal{H}, A_{34}]\right]|\rangle = -\omega_\nu\sum_{34}\phi_{34}^\nu M_{1234}.$$

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[1] Hereafter, if there are no special comments, the expectation values are taken over the thermal vacuum state defined in (3.4)

The one-phonon states (3.6) have to be orthonormalized

$$\langle |Q_\nu Q^+_{\nu'}|\rangle = \sum_{12} (1 - q_{12}) \left( \psi^\nu_{12} \psi^{\nu'}_{12} - \phi^\nu_{12} \phi^{\nu'}_{12} \right) = \delta_{\nu\nu'}. \qquad (3.10)$$

In the current approximation only the terms $\mathcal{H}_{11}$, $\mathcal{H}_{22}$, $\mathcal{H}_{04}$ and $\mathcal{H}_{40}$ contribute to expectation values of double commutators in (3.9) because diagonal matrix elements from odd numbers of creation and annihilation quasiparticle operators vanish.

Evaluating the coefficients of the phonon amplitudes $\psi$ and $\phi$ in eqs. (3.9), one needs to know four types of two-body matrix elements: $\langle |A^+_{12} A_{34}|\rangle$, $\langle |A_{12} A_{34}|\rangle$, $\langle |A^+_{12} A^+_{34}|\rangle$ and $\langle |B_{12} B_{34}|\rangle$. Three of them can easily be evaluated using the inverse transformation to (3.6)

$$A^+_{12} = \sum_\nu \sum_{34} M_{3412} \left( \psi^\nu_{12} Q^+_\nu + \phi^\nu_{12} Q_\nu \right) = \sum_\nu (1 - q_{12}) \left( \psi^\nu_{12} Q^+_\nu + \phi^\nu_{12} Q_\nu \right). \qquad (3.11)$$

Taking into account (3.11), one gets

$$\begin{aligned}
\langle |A^+_{12} A_{34}|\rangle &= \sum_\nu (1 - q_{12})(1 - q_{34}) \phi^\nu_{12} \phi^\nu_{34} \\
\langle |A^+_{12} A^+_{34}|\rangle &= \sum_\nu (1 - q_{12})(1 - q_{34}) \phi^\nu_{12} \psi^\nu_{34} \\
\langle |A_{12} A_{34}|\rangle &= \sum_\nu (1 - q_{12})(1 - q_{34}) \psi^\nu_{12} \phi^\nu_{34}.
\end{aligned} \qquad (3.12)$$

Evaluation of the fourth matrix element is more complicated. To this aim, an expansion over the complete phonon basis is used

$$\langle |B_{12} B_{34}|\rangle = \langle |B_{12}|\rangle \langle |B_{34}|\rangle + \sum_{\nu_1 \nu_2} \frac{\langle |B_{12} Q^+_{\nu_1} Q^+_{\nu_2}|\rangle \langle |Q_{\nu_2} Q_{\nu_1} B_{34}|\rangle}{\langle |Q_{\nu_2} Q_{\nu_1} Q^+_{\nu_1} Q^+_{\nu_2}|\rangle} + \sum_{\nu_1 \nu_2 \nu_3 \nu_4} \ldots + \ldots \qquad (3.13)$$

Only the lowest two terms of the expansion (3.13) are taken (see [10])

$$\langle |B_{12} B_{34}|\rangle = \delta_{12} \delta_{34} q_1 q_2 + \\
\sum_{\nu_1 \nu_2} \sum_{ik} \frac{[\psi^{\nu_1}_{2i} \phi^{\nu_2}_{1i} (1 - q_{1i}) + \psi^{\nu_2}_{2i} \phi^{\nu_1}_{1i} (1 - q_{2i})][\psi^{\nu_1}_{3k} \phi^{\nu_2}_{4k} (1 - q_{4k}) + \psi^{\nu_2}_{3k} \phi^{\nu_1}_{4k} (1 - q_{3k})]}{\langle |Q_{\nu_2} Q_{\nu_1} Q^+_{\nu_1} Q^+_{\nu_2}|\rangle}.$$

An expression for a norm of a two-phonon state seems to be quite complicated and we don't show it here. The leading correction to its "quasiboson" value $(1 + \delta_{\nu_1 \nu_2})$ is of an order of $\psi^2 \phi^2$. An equation for $q_1$ is derived by applying the "number operator method". Denote the operator of a number of HF quasiparticles $\widehat{N} = \sum_1 \alpha^+_1 \alpha_1$. Then

$$q_1 = \langle |B_{11}|\rangle = \langle |\beta^+_1 \widetilde{N} \beta_1|\rangle - \langle |\beta^+_1 \widehat{N} \beta_1|\rangle = \sum_2 \langle |A^+_{12} A_{12}|\rangle - \langle |:B_{12} B_{21}:|\rangle. \qquad (3.14)$$

Actually, in contrast with the $T = 0$ case we use the difference $\widetilde{N} - \widehat{N}$ instead of $\widehat{N}$. This helps us to exclude from the expression for $q$ the thermal quasiparticle number fluctuations.

Eq. (3.14) closes the system of equations for $\psi, \phi, \omega$ and $q$. But it is still unclear how to calculate the coefficients of transformations $D$ and $\{x, y\}$ and single-quasiparticle energies $E_i$ which are input values for the equations of motion (3.9). In Sect. 2, the equations for these values were obtained in THFA, i.e. with the thermal vacuum state $|0(T)\rangle$ which is different from $|\Psi_0(T)\rangle$. Certainly, those values of $D, E_1, x$ and $y$ can be used in eqs. (3.9), (3.10), (3.14) as the first iteration although this procedure is not fully consistent. This variant would correspond to a slightly improved thermal renormalized RPA (TRRPA) [1, 3, 15] [2]. More consistent is the so-called thermal self-consistent RPA (TSRPA) which has been developed in [2]. TSRPA takes into account an intimate coupling of collective and HF variables which appears due to non-vanishing values of $q_i$. The reason is a new expression for the one-body density matrix in TSRPA

$$\rho_{12} = \langle 0(T)|a^+_2 a_1|0(T)\rangle = \sum_k D^*_{2k} D_{1k} n_k = \sum_k D^*_{2k} D_{1k} \left[ x^2_k q_k + y^2_k (1 - q_k) \right]. \qquad (3.15)$$

Since $\rho_{12}$ appears in the equations for $D$ (see [2]), it becomes necessary to solve the systems (2.6) and (3.9), (3.10), (3.14) together.

Now we try to formulate a more exact and consistent version of TRPA than TSRPA. First, we suggest using more exact equations of motion and equations for $q$ by inclusion of the terms of an order of $\langle |B_{12} B_{34}|\rangle$. The second improvement seems to be more principal and important. Namely, we suggest using in equations (2.6) and (2.7) a new model thermodynamic potential $\Omega^{ph}_{mod}(H)$ where averaging is performed with respect to the phonon thermal vacuum state (3.4), i.e.

$$\Omega^{ph}_{mod}(H) = \langle \Psi_0(T)|H|\Psi_0(T)\rangle - TS.$$

New equations are valid for the coefficients $D$

$$\frac{\delta}{\delta D^*_{12}} \left( \Omega^{ph}_{mod} - \sum_2 \xi_2 \sum_3 D^*_{23} D_{23} \right) = \frac{\delta}{\delta D^*_{12}} \left( \langle |H|\rangle - \sum_2 \xi_2 \sum_3 D^*_{23} D_{23} \right) = 0$$

$$\frac{\delta}{\delta D_{12}} \left( \Omega^{ph}_{mod} - \sum_2 \xi_2 \sum_3 D^*_{23} D_{23} \right) = \frac{\delta}{\delta D_{12}} \left( \langle |H|\rangle - \sum_2 \xi_2 \sum_3 D^*_{23} D_{23} \right) = 0.$$

$$(3.16)$$

In the above equations the value $\langle |H|\rangle$ has to be calculated with the same exactness as the double commutators in the equations of motion (3.9). Due to this a non-vanishing

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[2] A modest improvement is due to a matrix element $\langle |B_{12} B_{34}|\rangle$ which was not taken into account in refs. [1, 3, 15].

contribution to $\langle|H|\rangle$ will be given not only by the $U_0$ term as in the standard THFA but the terms $\langle|H_{11}|\rangle$ and $(|H_{22}|)$ from (3.3) as well.

In the particle number conservation condition the one-body density matrix has to be as in (3.15).

The coupling of collective and HF variables affects the HF basis and hence affects the thermal occupation numbers of HF quasiparticles (i.e. $x$ and $y$). So to be fully consistent we should include determination of $x$ and $y$ in our general scheme. So we require the stability of $\Omega_{mod}^{ph}(H)$ with respect to small variations of $x$ and $y$

$$\frac{\delta\Omega_{mod}^{ph}}{\delta x_1} = 0, \qquad \frac{\delta\Omega_{mod}^{ph}}{\delta y_1} = 0. \tag{3.17}$$

The expression for the entropy $S$ also has to be changed. The definition (2.4) for the entropy of a system of independent quasiparticles cannot be used any longer. The right expression can be obtained from the general formula

$$S = \int_0^T \frac{1}{T'} \frac{\partial\langle|H|\rangle}{\partial T'} dT'.$$

Using the same expression for $\langle|H|\rangle$ as in $\Omega_{mod}^{ph}(H)$ we get

$$S = -\sum_1 (1 - 2q_1) \left[x_1^2 \ln x_1^2 + y_1^2 \ln y_1^2\right] = 0. \tag{3.18}$$

Unfortunately, eqs. (3.17) cannot help us find a functional dependence of $x$ and $y$ on $T$. It seems natural to make an assumption that $y_i^2$ as a function of $T$ and $E_i$ has the Fermi - Dirac form

$$y_1^2 = \left[1 + \exp\left(\frac{E_1}{T}\right)\right]^{-1}, \quad x_1^2 = \exp\left(\frac{E_1}{T}\right) \times \left[1 + \exp\left(\frac{E_1}{T}\right)\right]^{-1},$$

and eqs. (3.17) are additional constraints for a dependence of HF energies $E_1$ on $\psi, \phi, q$.

So the complete new system of equations includes new equations of THFA (3.16) and (3.17) (together with the expressions for $\langle|H|\rangle$ and $S$ (3.18)), equations of motion (3.9) and equations for the thermal quasiparticle occupation numbers (3.14). It is a very complicated task to solve this system of nonlinear equations for a realistic case. Our preliminary estimations with a solvable two-level Lipkin model [16] demonstrate that the equations of the generalized TRPA are compatible and new corrections are important in the vicinity of a phase transition point (see also [3, 15]).

## 4.   Conclusions

By the use of the TFD formalism and some ideas of ref. [10] we have developed a new, more general and consistent version of the thermal random phase approximation. The following effects are taken into account: the non-vanishing number of thermal quasiparticles in a thermal vacuum state; the coupling of collective and HF variables and its influence on the thermal occupation numbers; some two-particle correlations in equations of motions which were omitted in the previous versions of TRPA. The generalized TRPA includes, as particular cases, the thermal renormalized RPA [1, 3, 15] and the thermal self-consistent RPA [2, 4].

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