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INQUIRY INTO ANTINEUTRINO ANGULAR  
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ON POLARIZED NEUTRON  $\beta$ -DECAY

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Исследование углового распределения антинейтрино в экспериментах по  $\beta$ -распаду поляризованного нейтрона

Поскольку  $\gamma$ -излучение неизбежно сопровождает  $\beta$ -распад и вследствие этого конечное состояние в  $\beta$ -распаде нейтрона включает фотон наряду с протоном, электроном и антинейтрино, т.е. не три, а четыре частицы, кинематика антинейтрино не может быть восстановлена однозначно, когда даны лишь импульсы электрона и протона, а импульс  $\gamma$ -излучения неизвестен. Соответственно, из экспериментов по распаду поляризованного нейтрона, где наблюдаются лишь импульсные распределения электронов и протонов, а  $\gamma$ -излучение не регистрируется, фактор асимметрии  $B$  углового распределения антинейтрино не может быть получен строго, но значение величины  $B$  следует оценивать лишь в среднем, вводя в рассмотрение среднее, наиболее вероятное значение  $\langle B \rangle$ , и дисперсию  $\Delta B$ . Неизбежные при этом неопределенности в получении  $B$  составляют несколько процентов, что существенно для современных экспериментов, имеющих целью получить значение  $B$  с очень высокой точностью  $\sim (0,1 - 1) \%$ . При учете электромагнитных взаимодействий измерение импульсного распределения электронов и протонов оказывается полезным также для проверки с высокой точностью значения  $g_A$ , полученного ранее из углового распределения электронов.

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Inquiry into Antineutrino Angular Distribution in the Experiments on Polarized Neutron  $\beta$ -Decay

So far as  $\gamma$ -radiation unescapably accompanies  $\beta$ -decay and thereby the final state of neutron  $\beta$ -decay involves photon beside proton, electron and antineutrino, i.e., not three but four particles, the antineutrino kinematics cannot be reconstructed unambiguously when the proton and electron momenta are given only, the  $\gamma$ -radiation momenta being unknown. Consequently, in the experiments on the polarized neutron  $\beta$ -decay where the electron and proton momentum distributions are observed only, without registering the  $\gamma$ -radiation, the asymmetry factor  $B$  of the antineutrino angular distribution cannot be acquired rigorously, but the  $B$  value is to be estimated only on the average by drawing into consideration the expectation (mean) value  $\langle B \rangle$  and the dispersion  $\Delta B$ . The correspondent unavoidable ambiguities in  $B$  attainment amount to several percent which is significant for the nowadays experimental attempts to obtain the  $B$  value with the very high precision  $\sim (0.1 - 1) \%$ . With allowance for the electromagnetic interaction, experimental measurements of the electron and proton momentum distributions is seen also to be instructive to verify with high accuracy the  $g_A$  value obtained previously from the electron angular distribution.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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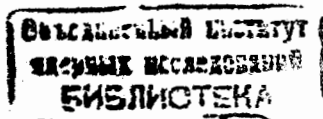
## 1. Introduction.

Recently, there has been a great deal of interest in high-precise acquiring the neutron  $\beta$ -decay characteristics, first, the lifetime  $\tau$  [1], and, with neutron being polarized, the asymmetry factors of electron,  $A$ , [2] and antineutrino,  $B$ , [3] angular distributions in respect to neutron polarization vector  $\xi$ . The quantity  $A$  is obtained merely from the experimentally observed electron angular distribution, whereas to gain the quantity  $B$  is far more subtle thing as there is no means to measure straightforward the antineutrino angular distribution. The germ of the idea of the experiment [3] to acquire  $B$  value ran back to a long-ago. It was as far back as in the 'sixties when the method [4] was launched to disentangle the quantity  $B$  by processing the experimental data on the proton and electron angular distributions, the unique reconstruction of the antineutrino kinematics was presumed, the electron and proton momenta given. However, this assertion had been strictly true only if there would have been no  $\gamma$ -radiation which is well-known to accompany inescapably (see, for instance [5-8])  $\beta$ -decay. Certainly, the necessity of the allowance for this  $\gamma$ -radiation is only a matter of accuracy required in obtaining the  $B$  value in experimental data processing. Yet, in so far as the knowledge of the  $\beta$ -decay characteristics with the precision about  $\sim (0.1-1)\%$  goes [1-3], there sees no reason to take for granted the  $\gamma$ -radiation being negligible in acquiring the  $B$  value in the experiments like those set out in Ref. [3].

In the work presented, our modest purpose is just to visualize the effect of electromagnetic interactions on the electron, proton and antineutrino distributions studied in [3] and, subsequently, to ascertain the accuracy attainable in describing the antineutrino angular distribution [3]. To warrant the necessity of the high-precise, unambiguous determination, both experimental and theoretical, of the  $\beta$ -decay characteristics, especially  $B$ , we shall abstract now some features of the  $\beta$ -decay treatment, the electromagnetic interactions being properly accounted for.

## 2. The polarized neutron $\beta$ -decay characteristics with proper allowance for electromagnetic interactions.

At present, the stringent attainment of semileptonic decay characteristics is well understood to be of fundamental value for the general elementary particle theory which



imposes several strict constraints on the quantities involved in the semiweak interactions. Certainly, the validity of these constrained relations must be thoroughly examined with the accuracy sufficient to judge with full confidence up to what extent, with what precision the underlying principles of the up-to-date elementary particle theory hold true. For that matter, we have at our disposal, in actual fact, no option but to confront the high-precise experimental data with the results of the consistent theoretical calculations, the semiweak interactions being described by means of the tenable effective lagrangian descending from the general field theory.

For the purpose of our work, the relevant effective lagrangian to describe  $\beta$ -decay of baryons with accounting for electromagnetic interactions is known to be set out in the form (see, for instance [9–11]):

$$L_{int} = L_{BfBiw} + L_{e\gamma} + L_{B\gamma}, \quad (1)$$

where

$$L_{BfBiw}(x) = \frac{G_{if}}{\sqrt{2}} (\bar{\psi}_e(x) \gamma^\alpha (1 + \gamma^5) \psi_\nu(x)) \times \bar{\Psi}_{Bf}(x) [\gamma_\alpha g_V^B(q) + g_{WM}^B \sigma_{\alpha\nu} q_\nu + \gamma^5 (\gamma_\alpha g_A^B(q) + g_{IF}^B q_\alpha)] \Psi_{Bi}(x) \quad (2)$$

renders the  $(V - A)$  baryon-lepton weak interaction,  $q$  being the four-momentum transfer in  $\beta$ -decaying, whereas the expression

$$L_{e\gamma}(x) = -e \bar{\psi}_e(x) \gamma^\mu \psi_e(x) \cdot A_\mu(x) \quad (3)$$

stands for electromagnetic field interaction with leptons and  $L_{B\gamma}$ , likewise, with baryons. In (2), (3), the notations are alike ones in Ref. [9], the index  $B$  specifying here the various kinds of baryons and the system of units  $\hbar = c = 1$  being adapted;  $\Psi_{Bi}(x)$ ,  $\Psi_{Bf}(x)$  render baryon fields in the initial and final states, and  $\psi_e, \psi_\nu, A_\mu$  stand for electron (positron), (anti)neutrino, and electromagnetic fields, respectively.

As to the  $g_V^B(0)$  value, we adapt, alike in Ref. [10],  $g_V^p(0) = 1$  for neutron decay (and  $g_V^{\bar{p}}(0) = 0$  for strangeness-conserving decay  $\Sigma^\pm \rightarrow \Lambda^0 + e^\pm + \nu(\bar{\nu}) + \gamma$ ). Then, for the various semiweak decays associated to the certain  $i \rightarrow f$  quarks transitions:  $u \rightarrow d$ ,  $s \rightarrow d$ ,  $b \rightarrow d$ , the amplitudes  $G_{if}$  in Eq. (2) are known [9–11] to be represented as

$$G_{if} = G_F \cdot |V_{if}|. \quad (4)$$

Here,  $G_F = 1.16639(2) \cdot 10^{-5} \cdot GeV^{-2}$  is fixed by muon lifetime [12], whereas the Cabibbo-Kobayashi-Maskawa,  $CKM$ , [13] quark-mixing matrix elements  $V_{if}$  satisfy the unitarity identity

$$|V_{ud}|^2 + |V_{sd}|^2 + |V_{bd}|^2 = 1 \quad (5)$$

which must hold true exactly within the framework of the standard model [10, 11]. Thus, any deviation from Eq. (5), if emerged from experiment, small as it might be, would be of fundamental significance, manifesting some puzzle in the underlying principles of the up-to-date theory of elementary particles. The pivotal, overwhelming contribution to the left side in Eq. (5) is just due to the strangeness-conserving,  $u \rightarrow d$ , transitions;  $|V_{ud}| \approx 0.9744 \pm 0.001$  as asserted, for instance, in Refs. [1, 12]. Thus, properly speaking, the accuracy attained in fixing the  $|V_{ud}|$  value warrants the precision the Eq. (5) is valid with. The thoroughly treatment of neutron  $\beta$ -decay is believed to provide the precise, trustworthy cognizance about the  $CKM$ -matrix element  $|V_{ud}|$  as well as of other quantities incorporated in the semiweak interaction (2); hereafter, we treat the neutron  $\beta$ -decay solely.

Thus, given the high-accuracy,  $\sim 0.1\%$ , experimental data on the neutron lifetime [1] and on the energy and angular final-state particles distributions [2, 3], the point is upon consistent calculating these characteristics accordingly (1)–(3) to confront the experimental results with theoretical ones and thereby to acquire the plausible values of  $|V_{ud}|$  and of the various amplitudes  $g(q)$  in (2). With baryon mass let tend to infinity,  $M \rightarrow \infty$ , the quantities  $g_V(0), g_A(0)$  are known to provide the bulk  $\beta$ -decay probability, whereas the allowance for all other amplitudes  $g$ , for  $M$  finiteness,  $M \neq \infty$ , for the  $q$ -dependence of  $g_V(q), g_A(q)$  and for the electromagnetic interactions (3) would give rise to the small-scale corrections to that. Certainly, what determines the significance of accounting for the different corrections is just the precision the studied physical quantity to be obtained with. In this work, alike in the previous one [8], we focus solely on the effect of electromagnetic interactions on the angular distributions of the particles, especially antineutrino, in the final state of the polarized neutron decay, all other corrections being put aside, important though they might be in their own right. Let us recall that the corrections due to the finiteness of baryon mass,  $M \neq \infty$ , was thoroughly investigated in Ref. [14].

With the electromagnetic interactions properly accounted for [8], the probability of

the polarized neutron  $\beta$ -decay with the electron energy-momentum  $(\varepsilon, \mathbf{p})$  and antineutrino emitted in the direction  $\mathbf{n}_\nu$ , accompanied by the  $\gamma$ -radiation of the energy  $\omega \equiv k$  less than some given value  $k_m$ , ( $k \leq k_m \leq \Delta - \varepsilon$ ), proves to be set out generally in the form [8]

$$dW(\varepsilon, \mathbf{p}, \mathbf{n}_\nu, k_m, \boldsymbol{\xi}) = dW \frac{dn_\nu}{4\pi} e^{\beta(\varepsilon, k_m)} \times \left\{ W_0(\varepsilon, \mathbf{p}, k_m, g_V, g_A) + (\mathbf{v}\boldsymbol{\xi}) \cdot W_{\nu\xi}(\varepsilon, \mathbf{p}, k_m, g_V, g_A) + (\mathbf{n}_\nu \boldsymbol{\xi}) \cdot W_{\nu\xi}(\varepsilon, \mathbf{p}, k_m, g_V, g_A) + (\mathbf{n}_\nu \mathbf{v}) \cdot W_{\nu\nu}(\varepsilon, \mathbf{p}, k_m, g_V, g_A) \right\}. \quad (6)$$

Here

$$B = \frac{2\alpha}{\pi} \mathcal{L} \ln\left(\frac{2k_m}{m}\right), \quad \mathcal{L} = \frac{1}{v} \ln\left(\frac{p+\varepsilon}{m}\right) - 1, \\ W_a = w_a^0 [1 + \tilde{C}_a(\varepsilon, \mathbf{p}, k_m)] + C_a(g_V, g_A, \varepsilon, \mathbf{p}), \quad a \equiv 0, \quad v\xi, \nu\xi, \nu\nu, \\ w_a^0 = g_V^2 + 3g_A^2, \quad w_{\nu\xi}^0 = 2g_A(g_V - g_A), \quad w_{\nu\xi}^0 = 2g_A(g_V + g_A), \quad w_{\nu\nu}^0 = g_V^2 - g_A^2 \quad (7) \\ dw = \frac{G_{if}^2}{2\pi^3} \varepsilon p \omega_{\nu 0}^2 d\varepsilon \frac{dn_e}{4\pi}, \quad \omega_{\nu 0} = \Delta - \varepsilon, \quad \Delta = M_n - M_p, \quad n_e = \frac{p}{p}, \quad v = \frac{p}{\varepsilon}, \quad \mathbf{n}_\nu = \frac{p_\nu}{\omega_{\nu 0}}.$$

The exponent  $e^{\beta(\varepsilon, k_m)}$  emerging in (6) governs the true infrared behaviour of the decay probability:  $dW(\varepsilon, \mathbf{p}, \mathbf{n}_\nu, k_m, \boldsymbol{\xi}) \rightarrow 0$ , when the boundary  $\gamma$ -radiation energy  $k_m \rightarrow 0$ , which means, in accordance to the general theory [9, 15], that there is no  $\beta$ -decay without the infrared(soft)  $\gamma$ -radiation. For the purposes of our present work, it is not necessary to set out the explicit expressions of the functions  $\tilde{C}, C$ , multiples of fine-structure constant  $\alpha$ , which are calculated accordingly [8].

On substituting the maximum  $k_m$  value,  $k_m = \Delta - \varepsilon$ , Eq. (6) gives the correspondent decay probability accounting for all the possible  $\gamma$ -radiation at some given  $\varepsilon$ . Obviously, the quantities

$$A = \frac{W_{\nu\xi}(\varepsilon, \mathbf{p}, k_m, g_V, g_A)}{W_0(\varepsilon, \mathbf{p}, k_m, g_V, g_A)}, \quad B = \frac{W_{\nu\xi}(\varepsilon, \mathbf{p}, k_m, g_V, g_A)}{W_0(\varepsilon, \mathbf{p}, k_m, g_V, g_A)}, \quad a = \frac{W_{\nu\nu}(\varepsilon, \mathbf{p}, k_m, g_V, g_A)}{W_0(\varepsilon, \mathbf{p}, k_m, g_V, g_A)} \quad (8)$$

stand for the asymmetry of the electron and antineutrino angular distributions and for the electron-antineutrino angular correlation, respectively. It is to point out that when proton mass was not let tend to infinity (i. e.  $M \neq \infty$ ), the effects entailed thereby: the proton recoil, the  $q$ -dependence of  $g_V, g_A$ , and the contributions of the terms with  $g_{WM}, g_{IP}$ , would not spoil the general forms (6), (8), the correspondent corrections to the quantities (8) evaluated in Ref. [14]. Surely, all the aforesaid corrections having been abandoned,

the quantities (8) take the familiar well-known uncorrected form

$$A_0 = \frac{2g_A(g_V - g_A)}{g_V^2 + 3g_A^2}, \quad B_0 = \frac{2g_A(g_V + g_A)}{g_V^2 + 3g_A^2}, \quad a_0 = \frac{g_V^2 - g_A^2}{g_V^2 + 3g_A^2}, \quad (9)$$

and Eq. (6) itself reduces to

$$dW(\varepsilon, \mathbf{p}, \mathbf{n}_\nu, \boldsymbol{\xi}) = dW \frac{dn_\nu}{4\pi} (g_V^2 + 3g_A^2) \left\{ 1 + (\mathbf{v}\boldsymbol{\xi})A_0 + B_0(\mathbf{n}_\nu \boldsymbol{\xi}) + a_0(\mathbf{n}_\nu \mathbf{v}) \right\}. \quad (10)$$

Upon integrating Eq. (6) over  $dp dn_\nu$ , the total neutron  $\beta$ -decay probability,  $W = 1/\tau$ , is obtained. Subsequently, with equating this calculated lifetime  $\tau$  value and one measured in experiment [1], the first relation emerges to determine the quantities incorporated in (2), in particular  $|V_{ud}|$ . The experimental  $A$  value [2] is obtained in studying the electron momentum distribution which corresponds with Eqs. (6), (10) having been integrated over the antineutrino emission direction  $dn_\nu$ , the terms with the coefficients  $B, a$  in Eqs. (6), (10) disappeared thereby. The relation to acquire  $g_A$  value follows then from equating the calculated and experimentally observed  $A$  values (8) [2].

How precise are the measurements of the  $\tau, A$  values [1, 2] and their theoretical evaluations, even with all the aforesaid corrections included, it is, nevertheless, extremely desirable to draw into consideration beside  $\tau$  and  $A$  another  $\beta$ -decay characteristics, in particular the antineutrino angular distribution in respect to the neutron polarization vector  $\boldsymbol{\xi}$  and the angular correlation between antineutrino and electron emission directions which are figured by the coefficients  $B, a$  prefixed to  $(\mathbf{n}_\nu \boldsymbol{\xi}) = \cos\Theta_{\nu\xi}$  and to  $(\mathbf{n}_\nu \mathbf{v}) = v\cos\Theta_{\nu\nu}$  in (6), (10), respectively. Such evolving is expedient both to check ones more the precision the quantities  $G_{if}, g_A, g_{IP}, g_{SM}$  in (2) are determined with (see [16]), and to ascertain how stringent is the general form (2) itself. As has been asserted in Ref. [17], pursuing the ideas argued in [18], the  $B$  magnitude, if known with the accuracy  $\sim 0.1\%$ , is pertinent as the experimental input to test the possibility that weak interaction may be left-right symmetric at the lagrangian level, parity violation arising exclusively from spontaneous breakdown of this symmetry. According to the assertion of Refs. [17, 18], if  $B$  value was fixed with such a high precision, the value of an admixture to the lagrangian (2) could be revealed, having the same transformation properties as (2) has, but differing from (2) via replacing  $\gamma^5 \rightarrow -\gamma^5$ . Thereby, the point is to check the feasible right-handed currents contribution to the effective semiweak interaction and to estimate the restriction on the magnitude of the mass of the right gauge boson associated with those [19]. Certainly, all

this conception does make sense only when the  $B$  value is known from experiment with the accuracy  $\sim 0.1\%$ .

Evidently, to ascertain  $B$  value in (6), (10) one has to attain the antineutrino angular distribution which corresponds with Eq. (6) having been integrated over electron momentum  $dp$ , the terms with the coefficients  $A, a$  in (6) disappeared thereby. Yet, to observe the antineutrino angular distribution in experiment immediately is quite impossible because there is no means to register antineutrino itself, alas.

The very ingenious fair way to sidestep this obstacle seemed to have been managed all the time ago in Ref. [4].

### 3. The electron and proton momentum distributions and the angular distribution of antineutrino.

It is just the Eqs. (6) - (10) that usually haunts us every time when we conceive  $\beta$ -decay process, but they themselves turn out to be not applicable in acquiring the quantity  $B$  from experiment. Describing the electron and antineutrino distributions in the final state in polarized neutron  $\beta$ -decay, these Eqs. (6) - (10) imply, obviously, the integration of the general decay probability has been carried out over outgoing proton momentum as well as over momentum of  $\gamma$ -radiation. This treatment would correspond to the experiment where the outgoing proton as well as the  $\gamma$ -radiation were not registered at all, that is we would be dealing with the decay probability involving proton and  $\gamma$ -radiation of all the momenta allowed at given  $dpdn_\nu$ . In the experiment to acquire  $A$  value [2], the electron momentum distribution is observed solely without registering antineutrino, that is including all the allowed antineutrino momenta. This observed distribution is to be described by Eq. (6) upon integrating over  $dn_\nu$  which would contain then only the term  $\propto (\nu\xi)A$ . To attain the quantity  $B$  prefixed to  $(n_\nu\xi)$  in (6), (10), the experiment should have been arranged to measure, the other way round, just the antineutrino angular distribution, regardless of the momenta of proton, electron, and  $\gamma$ -radiation, the consequent integration of the Eq. (6) over  $dp$  to be carried out, the terms with  $A$  and  $a$  disappeared thereby, and we left, respectively, with the term  $\propto (n_\nu\xi)B$  only. But this advisable experiment is well known to be unfeasible, so far as antineutrino can't be registered, hereupon the Eqs. (6), (10), immediately as they stand, are thought to be fruitless to obtain the quantity  $B$  from experiment.

To have got at our disposal the antineutrino angular distribution without the antineutrino itself registration we are in need of the momentum distributions of electrons, protons, and  $\gamma$ -radiation, the initial neutron being presumed to be at rest. Whereas the electron momentum can be determined in up-to-date experiment with high enough accuracy [2], the unsophisticated way to measure the desirable momenta of the outgoing proton and  $\gamma$ -radiation is inferred to be rather as good as impossible for now. So, it might seem, we have deadlocked trying to acquire  $B$  value from experiment. Nevertheless, mazy as came out this problem, the research [4] blazed the trail to resolve it. In these investigations and subsequently in [3], the method had been asserted and the respective experiment profoundly elaborated to reconstruct kinematics of antineutrino and to acquire, consequently, the quantity  $B$  which resides in Eqs. (6), (10) from measurements of the electron momentum distribution and the distribution of the values  $P_x$  of the proton momentum projection on the  $x$  axis, the initial neutron being polarized along or opposite the direction of this  $x$  axis. With the real experiment thoroughly expounded in Ref. [3], we only recall here that in its ideal scheme, just sufficient for our purposes, the registered electron momentum  $\mathbf{p}$  is directed strictly along the  $x$  axis, the rested neutron polarization vector  $\xi$  is also directed exactly along or opposite the  $x$  axis direction, and the proton momentum projection on the  $x$  axis,  $P_x$ , is registered in coincidence with the electron momentum  $\mathbf{p}$ , the perpendicular to  $x$  components of the proton momentum  $\mathbf{P}$  not observed at all, as well as all the  $\gamma$ -radiation. The kinematics of the particles participating in the process described is set out in the Fig. 1.

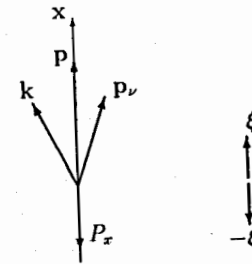


FIG.1

Fixed the electron energy  $\varepsilon$ , the  $P_x$  value varies within the limits

$$|p| - (\Delta - \varepsilon) \leq P_x \leq |p| + (\Delta - \varepsilon). \quad (11)$$

For the sake of visualization, we find convenient, alike done in [3], the quantity  $P_x$  in (11) and hereafter standing for the  $x$ -component of the proton momentum with opposite sign, that is, the value  $P_x > 0$  is figured in the Fig. 1. With leaving aside, for a moment, the  $\gamma$ -radiation and neglecting the proton kinetic energy on account of the very large proton mass, the antineutrino energy  $\omega_{\nu 0}$  and the cosine of the angle between the  $x$  axis and the direction of antineutrino emission are evidently

$$\omega_{\nu 0} = \Delta - \varepsilon, \quad y_0 \equiv \cos \Theta_{\nu x} = \frac{P_x - |\mathbf{p}|}{\omega_{\nu 0}}. \quad (12)$$

In the real experiment in Ref. [3], the measurements consisted in counting up the events with the given values  $\mathbf{p}$ ,  $P_x$  and the neutron polarization vector  $\boldsymbol{\xi}$  directed along or opposite the  $x$  axis direction. What is measured in this experiment is the probability of the polarized neutron  $\beta$ -decay with the electron momentum  $\mathbf{p}$  and the value of the proton momentum projection on the  $x$  axis,  $P_x$ , regardless of the antineutrino and  $\gamma$ -radiation momenta and of the proton momentum components perpendicular to the  $x$  axis. Thus, we arrive at the electron momentum  $\mathbf{p}$  distribution simultaneous with the distribution of the values  $P_x$ ,

$$dW_{exp}^z(P_x, \mathbf{p}) = w_{exp}^z(P_x, \mathbf{p}) \cdot d\mathbf{p} dP_x, \quad (13)$$

the contributions of the antineutrino,  $\gamma$ -radiation, and proton with all the allowed momenta incorporated therein. In (13) and hereafter, the value  $z = +$  stands for the neutron polarization along the  $x$  axis, whereas  $z = -$  for the opposite one.

The general expression to describe the polarized neutron  $\beta$ -decay probability with the electron momentum  $\mathbf{p}$  and the  $x$ -component of proton momentum  $P_x$ , accompanied by the  $\gamma$ -radiation of all the energies  $\omega = k = |\mathbf{k}|$  less than some given value  $k_m$ ,  $k \leq k_m \leq \Delta - \varepsilon$ , is deduced from the lagrangian (1)-(3) in much the same way as Eq. (6) was obtained in Ref. [8],

$$dW^z(P_x, \mathbf{p}, k_m) = dP_x \frac{dw}{2\omega_{\nu 0}} w^z(P_x, \mathbf{p}, k_m),$$

$$w^z(P_x, \mathbf{p}, k_m) = e^{\beta(\varepsilon, k_m)} \cdot \left\{ w_0^0 [1 + \tilde{C}_0(\mathbf{p}, k_m)] + C_0(P_x, \mathbf{p}, g_V, g_A) + \right.$$

$$\left. (\mathbf{v}\boldsymbol{\xi}) \{ w_{v\xi}^0 [1 + \tilde{C}_{v\xi}(\mathbf{p}, k_m)] + C_{v\xi}(P_x, \mathbf{p}, g_V, g_A) \} + \right.$$

$$\left. + w_{\xi\nu}^0 z y_0 [1 + \tilde{C}_{\xi\nu}(P_x, \mathbf{p}, k_m)] + v w_{\nu\nu}^0 z y_0 [1 + \tilde{C}_{\nu\nu}(P_x, \mathbf{p}, k_m)] + z C(P_x, \mathbf{p}, g_V, g_A) \right\}. \quad (14)$$

For our purposes in this work, there is no need to pull out the explicit expressions of the functions  $\tilde{C}, C$ , multiples of fine-structure constant  $\alpha$ . At  $k_m = \Delta - \varepsilon$ , Eq. (14)

renders the aforecited experimental distribution (13). Instead of Eq. (13) immediately, the treatment of the quantity

$$X = \frac{w_{exp}^+(P_x, \mathbf{p}) - w_{exp}^-(P_x, \mathbf{p})}{w_{exp}^+(P_x, \mathbf{p}) + w_{exp}^-(P_x, \mathbf{p})} \quad (15)$$

is known to be convenient in experimental data processing [2, 3]. With Eq. (14) used, the quantity (15) is presented as:

$$X = \frac{v[w_{v\xi}^0(1 + \tilde{C}_{v\xi}) + C_{v\xi}] + w_{\xi\nu}^0 y_0(1 + \tilde{C}_{\xi\nu}) + v w_{\nu\nu}^0 y_0(1 + \tilde{C}_{\nu\nu}) + C}{w_0^0[1 + \tilde{C}_0] + C_0}. \quad (16)$$

Thus, confronting (14) with the correspondent experimental data (13), (16) [3], we arrive at the equation, additional to ones set out in [1], [2], to specify the quantities  $G_{if}, g_A, \dots$  which reside in (6), (14). So, the  $g_A$  value obtained from experiments [2] might be checked once again [16].

However, along this line, our desirable quantity  $B$ , the coefficient prefixed to  $(\mathbf{n}, \boldsymbol{\xi})$  in (6), (10), can't come into picture at all, so far as the integration over all the allowed antineutrino momenta has been carried out in obtaining (14) and, respectively, the observed in the experiment [3] distribution (13) does incorporate the contribution of antineutrino with all the allowed momenta. Thus, we are to pursue another way, properly accounting for the  $\gamma$ -radiation effect in acquiring the quantity  $B$  from the experimental distribution (13) observed in [3].

#### 4. The effect of $\gamma$ -radiation on attainment of the coefficient $B$ through the electron and proton momentum distributions.

Evidently, in the conceivable simplified case, where the relations (12) were valid, the antineutrino kinematics would be uniquely predetermined by the values  $\mathbf{p}, P_x$ , registered in the experiment [3]. Consequently, with  $\gamma$ -radiation left aside and proton mass suggested to be infinite,  $M = \infty$ , the distribution (14) reduces to

$$dW^z(P_x, \mathbf{p}) = dP_x \frac{dw}{2\omega_{\nu 0}} w^z(P_x, \mathbf{p})$$

$$w^z(P_x, \mathbf{p}) = w_0^0 [1 + Azv + B_0 y_0 z + a y_0 v], \quad (17)$$

whereas the antineutrino angular distribution has got the form (10). Thus, in this simplified case, there would be exist the one-to-one correspondence between the distribution



(13), (17) treated in the experiment [3] and the antineutrino angular distribution (10), with the quantity  $(\xi n_\nu)$  in Eqs. (10) having got the value  $zy_0$  in Eq. (17), as the quantity  $dP_x/2\omega_{\nu 0}$  in (17) replacing  $dn_\nu/4\pi$  in (10). Then, on equating (17) to the experimental distribution (13) [3], the equation

$$w_{exp}^z(P_x, \mathbf{p}) = f_0(\omega_{\nu 0}) \cdot (1 + zAv) + f_0(\omega_{\nu 0}) \cdot y_0 \cdot (zB_0 + av) \quad (18)$$

would be inferred, and, consequently, one would arrive at the expression of the coefficient prefixed to  $(\xi n_\nu)$  in (10)

$$B_0 = \frac{1}{zy_0 f_0} [w_{exp}^z - f_0(1 + zAv) - f_0 av y_0], \quad f_0 = \frac{G_{if}^2 \omega_{\nu 0}}{16\pi^4} w_0^0 \quad (19)$$

through the  $w_{exp}^z$  (13), or, more convenient, as was done in [3], via the quantity  $X$  (15)

$$B_0 = [X(1 + av y_0) - Av]/y_0. \quad (20)$$

Of course, we have put aside herein the uncertainties emerging in the real experiment on account of the poor statistics, the geometrical shortcomings of devices, and so on. It does not mean to say all the correspondent corrections must be conceived as being negligible, yet here we are about to scrutinize solely the effect of  $\gamma$ -radiation itself on  $B$  obtaining in experimental data processing [3]. In (17)-(20), we prescribe the subscript 0 to the  $B$  value to stress that it is obtained provided  $\gamma$ -radiation would be turned off, alike in (9), (10). As seen, the quantity  $B_0$  shows up to be expressed via the quantities  $f_0, y_0 f_0, \omega_{\nu 0}$  (12), (19) which would be strictly known in this case for each event with the given  $P_x, \mathbf{p}$  values registered in the experiment [3].

However, so far as the neutron  $\beta$ -decay has long been well-known to be accompanied by  $\gamma$ -radiation, the real experiment [3] deals with the  $\beta$ -decay probability with given  $P_x, \mathbf{p}$  values, involving the  $\gamma$ -radiation of all the allowed momenta  $\mathbf{k}$ . In describing each single event, the expressions for  $y_0, \omega_{\nu 0}$  in (12) will give place to the following ones:

$$\begin{aligned} y_0 \longrightarrow y(\omega) &= \cos\Theta_{\nu x} = \frac{P_x - |\mathbf{p}| - x\omega}{\omega_\nu}, \quad x = \cos\Theta_{\gamma x}, \\ f_0 \longrightarrow f(\omega) &= \frac{G_{if}^2 \omega_\nu}{16\pi^4} w_0^0, \quad \omega_{\nu 0} \longrightarrow \omega_\nu(\omega) = \Delta - \varepsilon - \omega, \end{aligned} \quad (21)$$

where  $\omega = |\mathbf{k}|$  is the  $\gamma$ -radiation energy, and  $\Theta_{\gamma x}$  stands for the angle of the  $\gamma$ -radiation direction relative to the  $x$  axis direction. Thus, to reconstruct uniquely the antineutrino

kinematics we ought to have known the quantities  $\omega, x$  of the  $\gamma$ -radiation, accompanying every one single  $\beta$ -decay event with the given  $\mathbf{p}, P_x$  values which are registered in experiment [3].

Certainly, in the real case of the  $\beta$ -decay accompanied by  $\gamma$ -radiation, our desirable quantity  $B$  can't be expressed accordingly (19), (20) through the quantities  $f_0, f_0 y_0, \omega_{\nu 0}$  which themselves, strictly speaking, do get no longer the rigorous physical sense. For that matter, it is natural to estimate the quantity  $B$  in (6) by drawing into consideration the expectation values  $\langle yf \rangle, \langle f \rangle$  of the quantities (21)  $f(\omega), f(\omega)y(\omega, x)$ , replacing  $f_0, f_0 y_0$  in the state with certain  $P_x, \mathbf{p}, z$  values. These quantities  $\langle f \rangle, \langle fy \rangle$  are to be calculated by averaging  $f(\omega), f(\omega)y(\omega, x)$  over the momentum distribution  $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})$  of the  $\gamma$ -radiation accompanying the decay event with given  $P_x, \mathbf{p}, z$ . Thereby, the expectation value  $\langle B \rangle$ , expressed via  $\langle yf \rangle, \langle f \rangle$ , is to be introduced to evaluate the desideratum quantity  $B$  (6)-(10).

Up to now, in the existing experiment [3], only the total number of the  $\beta$ -decay events with the given values of  $\mathbf{p}, P_x$  is registered, accompanied by the  $\gamma$ -radiation including all kinematically admissible  $\omega, x$ , that is the integral

$$w_{exp}^z(\mathbf{p}, P_x) = \int d\mathbf{k} w^z(\mathbf{p}, P_x, \mathbf{k}) \quad (22)$$

over all the permissible  $\mathbf{k}$ . The purpose is to manage and disentangle from the experimentally observed distribution  $w_{exp}^z(\mathbf{p}, P_x) d\mathbf{p} dP_x$  (13) [3] the tenable knowledge about the quantity  $B$  which resides in (6)-(10). Every one single decay event with given  $\mathbf{k}$  value enters into the experimental  $w_{exp}^z(\mathbf{p}, P_x)$  value with its own weight, its own probability  $W_\gamma^z(\mathbf{p}, P_x, \mathbf{k}) d\mathbf{k}$ , that is the probability of the  $\gamma$ -radiation with given momentum  $\mathbf{k}$  accompanying the  $\beta$ -decay with the given  $\mathbf{p}, P_x$  values. Consequently, Eq. (18) gives place to new one, where the experimentally observed quantity  $w_{exp}^z(\mathbf{p}, P_x)$  is equated to the  $\beta$ -decay probability averaged with the weight  $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})$ , namely:

$$\begin{aligned} w_{exp}^z(\mathbf{p}, P_x) &= \frac{\int d\mathbf{k} W_\gamma^z(P_x, \mathbf{p}, \mathbf{k}) f(\omega) [1 + zAv + z \langle B \rangle^z y(\omega, x) + av y(\omega, x)]}{\int d\mathbf{k} W_\gamma^z(\mathbf{p}, P_x, \mathbf{k})} = \\ &= \langle f \rangle^z (1 + zAv) + \langle yf \rangle^z (z \langle B \rangle^z + av), \end{aligned} \quad (23)$$

where the familiar notation of averaging is introduced:

$$\langle F \rangle^z (P_x, \mathbf{p}) = \frac{\int d\mathbf{k} W_\gamma^z(P_x, \mathbf{p}, \mathbf{k}) F(\mathbf{k})}{\int d\mathbf{k} W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})} =$$



$$= \frac{\int_0^{\Delta-\epsilon} d\omega \int_{x_1}^{x_2} dx F(P_x, p, \omega, x) \int_0^{2\pi} d\phi W_\gamma^z(P_x, p, \omega, x, \phi)}{\int_0^{\Delta-\epsilon} d\omega \int_{x_1}^{x_2} dx \int_0^{2\pi} d\phi W_\gamma^z(P_x, p, \omega, x, \phi)} \quad (24)$$

Here, the limits

$$x_1 = 1 - (\Delta + |p| - \epsilon - P_x)/\omega \geq -1,$$

$$x_2 = -1 + (\Delta - |p| - \epsilon + P_x)/\omega \leq 1$$

emerge merely from kinematics of the considered process, the quantities to be average,  $f(\omega), f(\omega)y(\omega, x)$ , being independent on the azimuth  $\phi$  of  $\gamma$ -radiation, see Fig. 1. The quantity  $P_x$  itself varies within the limits (11) at the given electron energy  $\epsilon$ . We shall specify the form of the distribution  $W_\gamma^z(P_x, p, k)$  and discuss concisely its properties a bit later. Once  $W_\gamma^z(P_x, p, k)$  depends on the neutron polarization,  $z = \pm$ , all the average values (24), (23) do depend on  $z$  in turn, being different for the different polarizations, i.e., for  $z = +, z = -$ .

Thus, we have worked out Eq. (23) replacing former Eq. (18). Being prefixed to the quantity  $zy_0f_0$  in Eq. (18), the coefficient  $B_0$  would be just equal, as argued before, to the antineutrino angular distribution coefficient  $B_0$  in Eqs. (6)-(10). Dealing with the real experiment description, the Eq. (23) defines the quantity  $\langle B \rangle^z$  prefixed to the mean, expectation value  $\langle fy \rangle^z$  replacing the quantity  $f_0y_0$  in Eq. (18). In the lack of the immediate one-to-one correspondence between the distribution (23) involving  $\langle B \rangle^z$  and the antineutrino angular distribution (6) involving  $B$ , the quantity  $\langle B \rangle^z$  is seen, nevertheless, to be relevant to estimate, on the average, the  $B$  value in (6)-(10) which is our goal. The expression of  $\langle B \rangle^z$  through  $B_0$  results from Eqs. (17)-(19), (23) straightforward

$$\langle B \rangle^z = z[(1 + zAv)(f_0 - \langle f \rangle^z) + y_0f_0(av + zB_0)] / \langle yf \rangle^z - zav. \quad (25)$$

As seen, two different  $\langle B \rangle^z$  values to describe the antineutrino angular distribution have been obtained for two different polarizations of neutron,  $z = \pm$ . Evidently, Eq. (25) reduces to  $\langle B \rangle^z = B_0$  provided  $\langle f \rangle^z = f_0, \langle fy \rangle^z = f_0y_0$ . As seen, the expectation value  $\langle B \rangle^z$  shows up to be presented as a function of the mean, expectation values of the quantities  $f(\omega), f(\omega)y(\omega, x)$ . Thus, to judge with full confidence about the accuracy, and even validity itself, of the aforesaid  $B$  estimation via  $\langle B \rangle^z$  we are to visualize the distributions of the quantities  $f(\omega), f(\omega)y(\omega, x)$  around their mean, expectation values

$\langle f \rangle, \langle fy \rangle$ , that is, the dispersions of  $f(\omega), f(\omega)y(\omega, x)$  are to be evaluated. For that matter, the mean square deviations of  $f(\omega), f(\omega)y(\omega, x)$  from their expectation values  $\langle f \rangle, \langle fy \rangle$ :

$$\langle (\Delta f)^2 \rangle^z = \langle f^2 \rangle^z - (\langle f \rangle^z)^2, \quad \langle (\Delta(yf))^2 \rangle^z = \langle (yf)^2 \rangle^z - (\langle yf \rangle^z)^2,$$

$$\langle \Delta(f \cdot yf) \rangle^z = \langle f \cdot yf \rangle^z - \langle f \rangle^z \cdot \langle yf \rangle^z, \quad (26)$$

i. e. just their dispersions, must be calculated beside the quantities  $\langle f \rangle, \langle fy \rangle$  themselves. Respectively, the attainable accuracy

$$\Delta B^z \equiv \sqrt{\langle (\Delta B^z)^2 \rangle} = \sqrt{\langle B^2 \rangle^z - (\langle B \rangle^z)^2} \quad (27)$$

of the  $B$  value estimation (25) is expressed in the usual way (see, for instance, [20]) through the quantities (26) and the derivatives

$$\frac{\partial \langle B \rangle^z}{\partial \langle f \rangle^z}, \quad \frac{\partial \langle B \rangle^z}{\partial \langle yf \rangle^z}. \quad (28)$$

Surely, in (26), the mean values  $\langle f^2 \rangle^z, \langle (yf)^2 \rangle^z, \langle f^2y \rangle^z$  are obtained by averaging (24) of the quantities  $f^2(\omega), [f(\omega)y(\omega, x)]^2, f^2(\omega)y(\omega, x)$  with the weight  $W_\gamma^z(P_x, p, k)$ .

Thus, the ambiguities in estimating the genuine quantity  $B$  via the expectation values  $\langle B \rangle^\pm$  stem from the difference between these  $\langle B \rangle^+, \langle B \rangle^-$  quantities themselves and out of the dispersion  $\Delta B^\pm$  emergence.

Handling the expectation value  $\langle B \rangle^z$  to ascertain the genuine quantity  $B$  in (6) is realized to be relevant in the case where the distributions of the values of the quantities  $f(\omega), f(\omega)y(\omega, x)$  show up to be sharp enough, that is, when, at given  $P_x, p$ , the ratios  $\Delta f / \langle f \rangle, \Delta(fy) / \langle fy \rangle$ , and thereby  $\Delta B / \langle B \rangle$  prove to be substantially smaller, rather negligible, as compare to the desirable accuracy of the  $B$  determination [3]. As a matter of course, the magnitude of the ratio  $\Delta B / \langle B \rangle$  sets the bound of the precision to acquire the  $B$  value (6), (8) from experimental data (13) processing [3]. Yet, when, at certain  $P_x, p$ , the distributions of  $f(\omega), f(\omega)y(\omega, x)$  around  $\langle f \rangle, \langle fy \rangle$  turn out to be so smoothed as  $\Delta f / \langle f \rangle \sim 1, \Delta(fy) / \langle fy \rangle \sim 1$ , and, subsequently,  $\Delta B / \langle B \rangle \sim 1$ , there will be no reason, evidently, to estimate the quantity  $B$  (6), (8) via  $\langle B \rangle^z$  at all. In that case, the antineutrino kinematics, antineutrino angular distribution (6)-(10) can't be reconstructed from the experimentally observed [3] distribution (13) even on the average.

Under such circumstances, at such  $P_x, \mathbf{p}$ , there is no sensible way to estimate trustworthy the quantity  $B$  in (6)-(10) utilizing the experimental data [3].

Thus, we are to realize up to what extent, with what accuracy we are in position to reconstruct the antineutrino kinematics, having at our disposal the experimentally observed [3] quantities  $\omega_{exp}^z(\mathbf{p}, P_x)$  (13) at given  $\mathbf{p}, P_x$  values only,  $\gamma$ -radiation being put aside. Consequently, the task we are faced is to evaluate the ambiguities in estimating  $B$  via  $\langle B \rangle^z$  and to visualize the physical sense of the  $\langle B \rangle^\pm$  introduction itself, as a matter of fact. Surely, if the differences  $B^\pm - B_0$  between the calculated values of  $B^\pm$  (25) and  $B_0$  asserted in [3] as well as the  $\Delta B^\pm$  values (27) had been as good as negligible, for any  $\mathbf{p}, P_x$  values, as compared with the proclaimed in [3] uncertainties  $\Delta B \approx 0.4\%$  of the  $B$  experimental measurement, the allowance for  $\gamma$ -radiation would have been superfluous, but it is not a case, in actual fact.

## 5. Evaluating the quantities $\langle B \rangle^z$ , $\Delta B^z$ and discussion of the results.

To the first  $\alpha$ -order, the distribution  $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})$  in (23), (24) is obtained accordingly [8] for polarized neutron  $\beta$ -decay straightforward from (1)-(3) in much the same way, properly speaking, as the probability of  $\gamma$ -radiation  $W_\gamma(\omega)$  with the energy  $\omega$ , regardless of the  $P_x, \mathbf{p}, \mathbf{k}/k$  values, was calculated for unpolarized neutron  $\beta$ -decay, as far back as in [5]. Upon integrating over  $d\phi$ , as it stands in (24), the  $\gamma$ -radiation distribution takes the form

$$\begin{aligned} & \omega^2 d\omega dx dp dP_x \cdot \int_0^{2\pi} d\phi W_\gamma^z(P_x, \mathbf{p}, \omega, x, \phi) = \\ & \left(\frac{eG_{if}}{2\sqrt{2}}\right)^2 \cdot \frac{8}{(2\pi)^7} \cdot \frac{1}{4\varepsilon^2} \cdot \frac{\varepsilon_\nu}{[1-xv]^2} \cdot \frac{1}{\omega} \cdot dx d\omega dP_x d\mathbf{p} \times \\ & \times \{ (1-x^2)\varepsilon v [v(\varepsilon+\omega)(g_V^2+3g_A^2) + y(\omega+v^2\varepsilon)(g_V^2-g_A^2)] + \\ & + \omega^2 [(g_V^2+3g_A^2) + yx(g_V^2-g_A^2)](1-vx) + \\ & + 2zg_A[(1-x^2)\varepsilon v[(g_V-g_A)(v^2\varepsilon+\omega) + (g_V+g_A)v y(\varepsilon+\omega)] + \\ & + \omega^2(1-vx)[(g_V-g_A)x + (g_V+g_A)y] \}. \end{aligned} \quad (29)$$

However, this expression itself is not applicable immediately, as it stands here, to calculate (23)-(28) because of the evidential unintegrable singularity at  $\omega \rightarrow 0$  which would entail the logarithmic divergency in integrating (24) over  $d\omega$ . As the quantities to be averaged,

$f(\omega), f(\omega)y(\omega, x)$ , in (23)-(28) show up to be independent on  $x$  when  $\omega \rightarrow 0$ , we are, as a matter of fact, to take care only of the true behaviour of the distribution  $W_\gamma^z(\mathbf{p}, P_x, \omega, x, \phi)$  at  $\omega \rightarrow 0$  upon integrating over  $dx$ . To describe consistently the  $\gamma$ -radiation at small  $\omega$ , when  $\alpha \ln(\Delta/\omega) \geq 1$ , i.e., infrared radiation, the processes incorporating arbitrary number of such "soft" photons are known to be allowed for [15]. Pursuing the method set forth in Refs. [9], [15], the true behaviour at  $\omega \rightarrow 0$  of the  $\gamma$ -radiation distribution, upon integrating over  $dx$ , is to be obtained accordingly [8] by replacing in (29) at  $\omega \rightarrow 0$

$$\frac{1}{\omega} \rightarrow \frac{1}{m} \left(\frac{m}{\omega}\right)^{(1-o)}, \quad o = \frac{2\alpha}{\pi} \left[ \frac{1}{v} \ln\left(\frac{\varepsilon + |\mathbf{p}|}{m}\right) - 1 \right]. \quad (30)$$

If anything, it might be instructive to recall that at such small  $\omega \rightarrow 0$ , when  $\alpha \ln(\Delta/\omega) \geq 1$ , we deal with the radiation of the unfixed, arbitrary large "soft" photons number, the directions of their emission being not fixed too [9], [15], that is, the classical  $\gamma$ -radiation sets on under such conditions. The consistent allowance for the infrared ("soft-photon")  $\gamma$ -radiation results, accordingly to [8], in the factor  $e^{B(\varepsilon, k_m)}$  in the formulae (6), (14), the Eqs. (29), (30) being deduced also rather straightforward by differentiating of (14) with respect to  $k_m$  and then setting  $k_m = \Delta - \varepsilon$  [8].

With accounting for (29), (30), calculating all the integrals (24) do not encounter any divergencies, and all the quantities  $\langle B \rangle^z(P_x, \mathbf{p}), \Delta B^z(P_x, \mathbf{p})$  are evaluated directly. It is expedient to point out that if fine-structure constant  $\alpha \rightarrow 0$ , i.e., when electromagnetic interactions disappear, all the mean values (24) calculated with the function  $W_\gamma^z(P_x, \omega, \mathbf{p}, x, \phi)$  (29), (30) will prove to be equal to the values  $F(0)$  of the averaged quantities  $F(\omega)$  at  $\omega = 0$ . Indeed, at  $\alpha = 0$  the quantity  $o$  in (30) drops to zero too, and the normalizing integral over  $d\omega$  in the denominator in Eq. (24) diverges logarithmically on the lower limit  $\omega \rightarrow 0$ . Consequently, if the nominator in (24) had got finite value, the whole expression (24) would have vanished. On presenting the function to be averaged as a power series in  $\omega$ ,  $F(\omega) \approx F(0) + \omega F'(0) + \dots$ , we become conscious that all the terms including  $\omega$  provide the finite contributions to the integral in the nominator in (24), whereas the term with  $F(0)$  proves just to be multiple of the normalizing integral residing in the denominator. With this infinite normalizing integral canceled, each expectation value (24) would reduce merely to the value  $F(0)$ . Thus, at  $\alpha \rightarrow 0$  we would leave with reduction  $\langle f(\omega) \rangle^z \rightarrow f(0) \equiv f_0$ ,  $\langle y(\omega) \rangle^z \rightarrow y(0) \equiv y_0$ ,  $\langle f(\omega)y(\omega) \rangle^z \rightarrow f(0)y(0) \equiv f_0 y_0$ ,  $\langle B \rangle^z = B_0$ , and, consequently, all the uncertainties would disappear,

TABLE 1. The values of  $(\langle B \rangle^+ - B_0)/B_0$  in %.

	-0.80	-0.40	-0.20	-0.10	-0.05	0.05	0.10	0.20	0.40	0.80
0.55	0.00	-0.04	-0.12	-0.28	-0.59	0.66	0.34	0.19	0.10	0.03
0.70	-0.01	-0.09	-0.25	-0.57	-1.20	1.34	0.70	0.38	0.21	0.08
0.80	-0.02	-0.12	-0.33	-0.74	-1.57	1.74	0.91	0.49	0.27	0.11
0.90	-0.02	-0.15	-0.40	-0.90	-1.90	2.11	1.10	0.59	0.33	0.14
1.00	-0.03	-0.17	-0.47	-1.05	-2.20	2.45	1.28	0.69	0.39	0.17
1.25	-0.04	-0.23	-0.62	-1.37	-2.86	3.20	1.66	0.89	0.50	0.24

$\Delta f = \Delta(fy) = \Delta B = 0$ . So, accordingly to the physical meaning, the formula (25) would reduce to (19) when  $\alpha$  was equal to zero, and we should arrive at the result dealing with the idealized case where  $\gamma$ -radiation was dropped-out, as was presumed in Ref. [3].

One can see the results of our calculations of the quantities  $\langle B \rangle^\pm, \Delta B^\pm$  presented in Tables 1-4. Just in case, Table 5 shows what  $P_x$  value corresponds to given  $y_0, \epsilon$  values. The first row in each Table contains the values of the quantity  $y_0$ , (12), and the  $e^-$  energies (in MeV) are given in the first column. So, every magnitude in Tables corresponds to the certain pair of the  $y_0, \epsilon$  values. In our presented results, the  $y_0$  value varies from  $-0.8$  up to  $+0.8$ , alike in Ref. [3]. In Tables 1,2 the quantities

$$\frac{B^\pm - B_0}{B_0} \cdot 100 \quad (31)$$

are displayed, i.e., the deviations of the quantities  $\langle B \rangle^\pm$  (25) from  $B_0$  (19) in percent.

For instance, from Tables 1 and 2, one can find out that at  $y_0 = 0.2, \epsilon = 1\text{MeV}$  the  $B^+$  value exceeds  $B_0$  by 0.69%, whereas the  $B^-$  value at the same  $\epsilon, P_x$  is less than  $B_0$  by 0.44%. The differences displayed in Tables 1,2 substantially grow when the value  $|y_0|$  decreases. It comes to light from these Tables that the differences  $\langle B \rangle^\pm - B_0, \langle B \rangle^+ - \langle B \rangle^-$  prove to be not negligible in so far as the accuracy 1% or better in  $B$  determination goes. So, the results presented in these Tables 1,2 make us realize that there is no reason to take for granted that the accuracy  $\sim 0.4\%$  in  $B$  obtaining is attainable in experimental data processing [3], with the  $\gamma$ -radiation left out. Next, the quantities

$$\frac{\Delta B^\pm}{\langle B \rangle^\pm} \cdot 100, \quad (32)$$

TABLE 2. The values of  $(\langle B \rangle^- - B_0)/B_0$  in %.

	-0.80	-0.40	-0.20	-0.10	-0.05	0.05	0.10	0.20	0.40	0.80
0.55	0.02	0.08	0.17	0.34	0.67	-0.65	-0.32	-0.15	-0.06	-0.01
0.70	0.03	0.13	0.29	0.58	1.17	-1.18	-0.58	-0.28	-0.12	-0.02
0.80	0.03	0.15	0.34	0.70	1.40	-1.43	-0.71	-0.34	-0.15	-0.03
0.90	0.04	0.17	0.38	0.79	1.59	-1.64	-0.81	-0.40	-0.18	-0.04
1.00	0.04	0.18	0.42	0.86	1.74	-1.81	-0.90	-0.44	-0.20	-0.05
1.25	0.05	0.21	0.49	1.01	2.03	-2.14	-1.07	-0.52	-0.24	-0.07

TABLE 3. The values of  $\Delta B^\pm / \langle B \rangle^\pm$  in %.

	-0.80	-0.40	-0.20	-0.10	-0.05	0.05	0.10	0.20	0.40	0.80
0.55	0.39	1.53	4.48	10.34	22.01	24.53	12.85	6.96	3.86	1.68
0.70	0.37	2.14	5.92	13.45	28.44	31.82	16.62	9.00	5.05	2.28
0.80	0.41	2.45	6.67	15.05	31.70	35.58	18.54	10.02	5.64	2.64
0.90	0.45	2.70	7.29	16.37	34.39	38.76	20.15	10.87	6.14	2.96
1.00	0.48	2.92	7.82	17.50	36.67	41.51	21.54	11.60	6.56	3.24
1.25	0.57	3.37	8.97	19.74	41.18	47.09	24.34	12.98	7.38	3.82

rendering the width  $\Delta B^\pm$  of the  $B$  genuine values distribution (dispersion) around the expected (mean) value  $\langle B \rangle^\pm$ , are displayed in Tables 3,4. Certainly, being in possession of the observed  $w_{exp}^\pm(p, P_x)$  (13) [3] only, we are not in position to judge about the genuine quantity  $B$  with the accuracy better than the correspondent  $\Delta B^\pm(y_0, \epsilon)$  magnitude, as was set forth before. Yet, as seen in Tables 3,4, even the smallest uncertainties  $\Delta B$  at  $|y_0| = 0.8$  amount to  $\sim 1\%$ , and the  $\Delta B$  values increase very fast when  $|y_0|$  decreases, likewise the quantities in Tables 1,2 behave. The data in Tables make us aware that the ambiguities in  $B$  estimation because of the large dispersion  $\Delta B^\pm$  come out to be far more significant than ones on account of the distinction between  $\langle B \rangle^+, \langle B \rangle^-$ . So, it is not sensible to ascertain the quantity  $B$  in (6) from experimental data processing [3] at the  $p, P_x$  values corresponding to the small values  $|y_0|$ .

Of course, it is no wonder that the values in Tables 1,2 and especially in 3,4 augment

TABLE 4. The values of  $\Delta B^- / \langle B \rangle^-$  in %.

	-0.80	-0.40	-0.20	-0.10	-0.05	0.05	0.10	0.20	0.40	0.80
0.55	1.19	3.29	6.47	12.47	24.31	22.99	11.05	5.04	1.96	0.56
0.70	1.01	3.37	7.07	14.04	27.75	27.43	13.38	6.32	2.66	0.65
0.80	0.96	3.40	7.27	14.56	28.85	29.02	14.20	6.77	2.92	0.71
0.90	0.94	3.43	7.40	14.89	29.55	30.13	14.77	7.08	3.10	0.77
1.00	0.93	3.45	7.50	15.13	30.02	30.96	15.18	7.31	3.24	0.83
1.25	0.94	3.51	7.72	15.48	30.68	32.38	15.87	7.61	3.46	0.95

sharply when  $y_0$  tends to zero,  $y_0 \rightarrow 0$ , the physical reason of such  $\langle B \rangle, \Delta B$  behaviour being quite visible. Indeed, when  $y_0 \approx 0$ , that is  $|p| \approx P_x$ , the imposition of the term  $x\omega$  in  $y(\omega)$  (21) originates the significant value of the ratios  $(y - y_0)/y_0, \Delta y / \langle y \rangle$  at any  $\omega$ , even tiny small one. In this case, any  $\gamma$ -radiation destroys absolutely the antineutrino kinematics which would be valid without electromagnetic interactions. In turn, the  $(\langle B \rangle^{\pm} - B_0)/B_0$  values increase significantly, and the enhancement of the  $\Delta B^{\pm} / \langle B \rangle^{\pm}$  values can be even arbitrary large at  $|y_0| \rightarrow 0$ . Of course, under such circumstances, there is nothing to say about expectation (mean) values themselves.

This outcome makes us realize quite clear that there is none single shred of physical reason to make use of the experimental data of neutron  $\beta$ -decay at small values  $|y_0| \sim 0.1$  in tenable studying the asymmetry factor  $B$  of antineutrino angular distribution. Having at our disposal only the probability of polarized neutron  $\beta$ -decay with given  $p, P_x$ ,  $\gamma$ -radiation being left out, we are in position to reconstruct the antineutrino angular distribution only approximately, on the average and, subsequently, to estimate the quantity  $B$  on the average as well, with the accuracy restricted accordingly to the results offered in Tables, never better. The events related to the values  $|y_0| \leq 0.1$  being incorporated in the analysis simply just, without prescribing any relevant weights, obtaining the antineutrino asymmetry coefficient  $B$  is thought to be untenable and to loose any faith, as a matter of fact. Processing all the experimental data beyond these small  $|y_0|$  values, we can pretend to acquire the semiquantitative  $B$  estimation with an accuracy about a few per cent. At best, with allowance for the events with  $|y_0| \approx 0.8-1.0$  only, the accuracy better than 1%

TABLE 5. The values of  $P_x$  in MeV.

	-0.80	-0.40	-0.20	-0.10	-0.05	0.05	0.10	0.20	0.40	0.80
0.55	-0.39	-0.09	0.05	0.13	0.17	0.24	0.28	0.35	0.50	0.80
0.70	0.00	0.24	0.36	0.42	0.45	0.51	0.54	0.60	0.72	0.95
0.80	0.22	0.42	0.52	0.57	0.59	0.64	0.66	0.71	0.81	1.01
0.90	0.43	0.58	0.66	0.70	0.72	0.76	0.78	0.82	0.90	1.06
1.00	0.62	0.74	0.80	0.83	0.84	0.87	0.89	0.92	0.98	1.10
1.25	1.11	1.12	1.13	1.14	1.14	1.14	1.15	1.15	1.16	1.18

is thought to be attainable in restoring the antineutrino asymmetry coefficient  $B$  from the  $e^-$  and proton momentum distribution [3].

## 6. Concluding remarks.

Thus, there is no reason to gloss over the  $\gamma$ -radiation effect on  $B$  attainment and bolster up the achievement of the very high,  $\approx 0.4\%$ , accuracy of the  $B$  measurement proclaimed in [3], alas.

It is to realize and to score under that the ambiguities entrained by the unregistered  $\gamma$ -radiation in acquiring the  $B$  value, i.e., the differences  $|B^+ - B^-|, |B^{\pm} - B_0|$  and the dispersion  $\Delta B^2$ , can be neither removed nor even lessened by improving statistics of the experiment [3], or by ingenious ameliorating any devices within the actual experimental set up [3]. The quantities in Tables have emerged in course of our treatment because the antineutrino kinematics can be reconstructed from the measurements of  $p, P_x$  values [3] on the average only, so far as  $\gamma$ -radiation being not allowed for. In every decay event, the antineutrino kinematics could be reconstructed in unique manner, if the momentum of  $\gamma$ -radiation was registered beside the  $p, P_x$  values. Consequently, the experiment should be arranged to register the triple coincidences in polarized neutron decay, that is the events where the quantities  $\omega, x$  (21) of the accompanying  $\gamma$ -radiation would be registered simultaneously with certain  $p, P_x$  values. Then, there would be exist the one-to-one correspondence between such observed triple distribution and the antineutrino angular distribution where the desideratum quantity  $B$  resides. Hard as is such experiment to carry out, there is no reason stickle that it is feasible, if necessary indeed.

What can be inferred from our calculations (see Tables) is that the ambiguities caused by  $\gamma$ -radiating effect in obtaining  $B$  could be reduced to  $\leq 1\%$ , if the  $B_0$  value had been extracted in [3] from processing experimental data at  $|y_0| \geq 0.8$  only. For that matter, it might be sensible to re-process in this line the data in [3].

Let us recall also that, as mentioned above in Sec. 3, Eq. (14)-(16) the experimental data obtained in [3] could be employed to acquire immediately the  $g_A$  value residing in (2). Of course, the explicit expression of the quantities  $\tilde{C}(P_x, p, g_V, g_A), C(P_x, p, g_V, g_A)$  in (14) are to be pulled out at length in much the same way as it was done previously in the work [8] for the coefficients in Eq. (6). And also, vice versa, having at our disposal the  $g_A$  value obtained in [2], we can insert it in (14) asking if the experimental data [3] would be reproduced thereby, such recapitulation of the  $g_A$  value acquired from various experiments argued in Ref. [16]. In the light of all aforesaid about how important is to ascertain, with the desirable high accuracy, the quantity  $B$  itself and the  $g_A$  value as well, both the subjects of great conceptual interest and significance, the investigations carried out in Refs. [3, 4] are realized, beyond questions, to be developed and set forward as fast as possible.

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