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HIGH-ENERGY APPROXIMATION
FOR NUCLEUS-NUCLEUS SCATTERING

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Высокоэнергетическое приближение для ядро-ядерного рассеяния

Метод высокоэнергетического приближения адаптируется к ядро-ядерным столкновениям при энергиях в десятки МэВ на нуклон и выше. Внимание направлено на получение эйкональных фаз в аналитическом виде, чтобы сделать возможным качественный анализ физики процессов, а также для выполнения численных расчетов. Показано, что явный вид эйкональной фазы, предложенной для реалистической формы потенциала Вудса–Саксона, удобен для последующих приложений. Анализируется применимость подхода Глаубера–Ситенко для малых углов рассеяния, а также исследуется роль отклонения траектории движения от прямой линии за счет действия кулоновских сил. Приведены методические расчеты и сравнения с экспериментальными данными.

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High-Energy Approximation for Nucleus-Nucleus Scattering

The high-energy method for potential scattering is adapted for the nucleus-nucleus collisions at energies of several dozen MeV/nucleon and higher. Attention is paid to analytic forms of the eikonal phases to make possible a qualitative consideration of physics of processes and perform fast numerical calculations. It is shown that the closed form of the eikonal phase suggested for the realistic Woods–Saxon type potential is a hopeful one for further applications. Applicability of the Glauber–Sitenko approach for scattering at small angles is analyzed, and a role of the Coulomb deviation of the straight-ahead trajectory of motion is investigated. The methodical calculations and comparison with experimental data are made.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

1 Introduction

Nuclear collisions at comparatively high energies $E \gg U$ and small wavelengths $kR \gg 1$ are highly sensitive to parameters of the interaction potential and nuclear structure. In this regard, elastic scattering makes the basis for understanding more complicated processes, including reactions of nucleon transfer. Within the framework of initial conditions, one can construct appropriate models by using the so-called eikonal wave functions

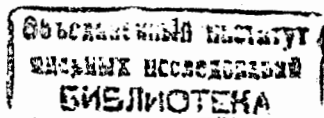
$$\Psi^{(\pm)} = \exp \left\{ i\vec{k}\vec{r} - \frac{i}{\hbar v} \int_{\pm\infty}^z [V(r) \mp iW(r)] d\lambda \right\}. \quad (1.1)$$

where $U(r) = V(r) + iW(r)$ is the potential; $r = \sqrt{b^2 + \lambda^2}$, the distance between centers of colliding nuclei; and b , the impact parameter. This expression can be derived (see, for instance, [1]) if in the quasiclassical function $\exp(iS(\vec{r})/\hbar)$, one expands the action $S(\vec{r}) = \int [E - U(\vec{r} - \vec{k}_c \lambda)]^{1/2} d\lambda$ in small parameter $U/E \ll 1$, leaving the terms of zeroth and first order. Function $\vec{k}_c(\vec{r})$ defines the trajectory of motion which in the case under consideration is regarded as a straight line along momentum \vec{k} at asymptotics. Minor deviations of a genuine trajectory can be taken into account if in (1.1), one replaces the vector \vec{k} by $\vec{k} = \vec{k} \mp \vec{q}_c/2$, where $\vec{q}_c = 2k \sin(\theta_c/2)$ and $\theta_c \simeq U(R_c)/E$ is the classical angle of deviation corresponding to the radius of closest approach of nuclei [1].

In this paper, we study applicability of the high-energy approximation (HEA) developed in [2], [3] for small angles of scattering (the Glauber-Sitenko approach) to elastic nucleus-nucleus collisions. This approach is applied successfully in problems of proton-nucleus scattering when, as a rule, the nuclear potential in the Gaussian form is used and the Coulomb interaction is of minor importance. However, in the case of nucleus-nucleus scattering, the role of Coulomb forces is rather significant, and the small-angle approximation requires a special investigation. Moreover, in scattering of nuclei, one should deal with extended nuclear potentials, mainly with a typical Woods-Saxon potential, but for the latter, one no suitable analytic form of the eikonal phase has been found. Recently, in [4], we have obtained a simple approximate expression for the phase but for the symmetrized Woods-Saxon potential (the form of the symmetrized Fermi function) written as follows

$$U(r) = U_0 u_{SF}(r), \quad u_{SF}(r) = \frac{\sinh(R/a)}{\cosh(R/a) + \cosh(r/a)}. \quad (1.2)$$

Below we will use it to study the range of its applicability for heavy-ion scattering as well as a possibility to use there the small-angle approximation. Effects of Coulomb distortion of the trajectory of motion will be analyzed, and the respective methodical calculations and comparisons with experimental data will be discussed, as well.



2 Scattering amplitude

The high-energy approximation for *small scattering angles* was developed by Glauber and Sitenko [2], [3]. The corresponding amplitude can be derived from the standard expression for the scattering amplitude if for the exact function of relative motion, one substitutes its eikonal ansatz (1.1):

$$f(q) = -\frac{m}{2\pi\hbar^2} \int d\vec{r} e^{-i\vec{k}_f \vec{r}} U(r) \Psi_{\vec{k}_i}^{(+)} =$$

$$= -\frac{m}{2\pi\hbar^2} \int d\vec{r} U(r) \exp \left\{ i\vec{q} \vec{r} - \frac{i}{\hbar v} \int_{-\infty}^z U(\sqrt{b^2 + \lambda^2}) d\lambda \right\}, \quad (2.1)$$

where m is the reduced mass; $\vec{q} = \vec{k}_i - \vec{k}_f$, the vector of momentum transfer $q = 2k \sin(\theta/2)$; and θ , the scattering angle. This formula can be simplified if one takes the cylindrical system of coordinates, where the axis $oz \parallel \vec{k}_i$, the axis ox lies in the plane of vectors \vec{k}_i , \vec{k}_f , and $\vec{b} \perp \vec{k}_i$. Now the volume element is $d^3r = b db d\varphi dz$, and the scalar product is defined as follows:

$$\vec{q} \vec{r} = q_1 b \cos \varphi + q_2 z. \quad (2.2)$$

Here $q_1 = q \cos(\theta/2)$ is a transverse component; and $q_2 = q \sin(\theta/2)$, a longitudinal component of the momentum transfer. In the small-angle approximation, the second term $q_2 z$ in (2.2) is neglected, and one can set $q_1 = q$. If we consider that a major contribution to the amplitude (2.1) comes from the region $r \simeq z \simeq R$, then this approximation holds valid under the condition

$$\theta < \bar{\theta} = \sqrt{2/kR}. \quad (2.3)$$

In this case, the integral over φ is reduced to the Bessel function of zero order, and integration over z is made by parts with the use of the equality $U(r) = (d/dz) \int_{-\infty}^z U(r) d\lambda$; in this way, we arrive at the known result

$$f(q) = ik \int_0^\infty db b J_0(qb) \left(1 - e^{i\Phi_N + i\Phi_C} \right). \quad (2.4)$$

Here each of the nuclear and Coulomb eikonal phases has the form

$$\Phi(b) = -\frac{i}{\hbar v} \int_{-\infty}^\infty U(\sqrt{b^2 + \lambda^2}) d\lambda. \quad (2.5)$$

In [2] it is shown that at finite scattering angles when $q \neq 0$, the first integral in (2.4) with 1 inside brackets equals zero. Then it is interesting to note that the remaining part of integrand diverges at asymptotics as $b^{1/2}$, because at large b , the eikonals $\Phi(b)$, generally speaking, tend to zero. This problem is related with the small-angle approximation when one puts $\exp(iq_2 z) = 1$ and thus eliminates the factor of convergence in the integrand. However, in the optical model, the convergence is provided by the imaginary part of the potential $iW = -i|W|$.

An independent estimate of the range of applicability of this approach can be made proceeding from the HEA amplitude for *large scattering angles* $\theta \gg 1/kR$ (see [5])

$$f(\theta) = -\frac{m}{2\pi\hbar^2} \int d^3r \Psi_{\vec{k}_f}^{(-)*} U(r) \Psi_{\vec{k}_i}^{(+)}. \quad (2.6)$$

Note that the eikonal functions (1.1) do not contain outgoing (incoming) spherical waves in asymptotics. The use, in (2.6), of two rather than one distorted waves like in (2.2) somewhat compensates this drawback, which allows one to move into the region of large scattering angles.

As the integration contour, we again take a straight line along \vec{k}_i , and the cylindrical system

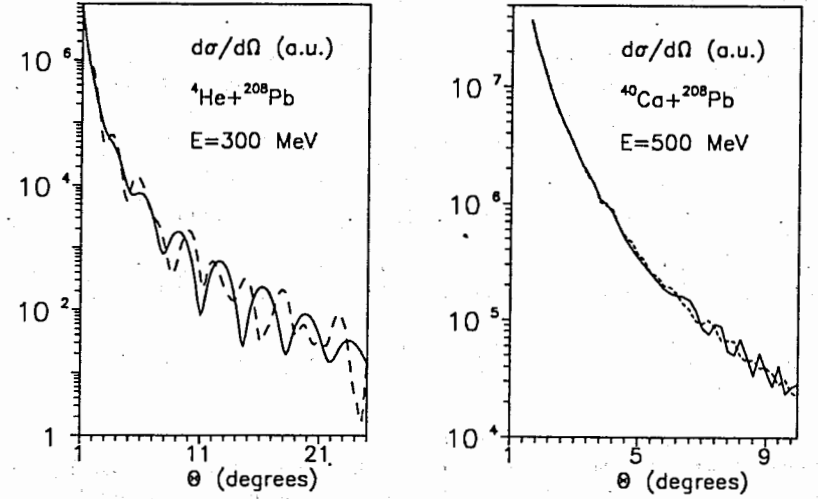


Figure 1: Scattering cross sections in the field of nuclear potential (1.2) with $V_0 = -50$ MeV, $r_0 = 1.04$ fm, $a = 0.75$ fm at $W_0 = -42$ MeV (on the left) and $W_0 = -13$ MeV (on the right). Solid lines - with the factor of convergence $F(q_2, b)$ in (2.7). Dotted lines - without that factor.

of coordinates. Then in distorted waves (1.1), the limits z_i and z_f coincide with each other, and using both terms in (2.2), we obtain

$$f(\theta) = -\frac{mU_0}{\hbar^2} \int_0^\infty db b J_0(q_1 b) e^{i\Phi} F(q_2, b), \quad (2.7)$$

where

$$F(q_2, b) = \int_{-\infty}^\infty dz e^{iq_2 z} u(\sqrt{b^2 + z^2}) \quad (2.8)$$

has a meaning of the form factor of the distribution function $u(r)$ of potential $U(r) = U_0 u(r)$ at a fixed impact parameter b . It is just the factor $F(q_2, b)$ that ensures the convergence of (2.7), and if it is set to equal 1, we arrive at the mentioned integral in the Glauber-Sitenko approach. Figure 1 shows how this factor influences the angular distribution of scattering in the field of the Saxon-Woods potential at different energies. It can be established that the difference between calculations with $F(q_2, b)$ and without (dotted lines) occurs at angles somewhat smaller than the estimate (2.3) for applicability of the small-angle approximation, $\theta({}^4\text{He}) \simeq 10^\circ$ and $\theta({}^{40}\text{Ca}) \simeq 4.5^\circ$.

3 Eikonal Phases of Coulomb and nuclear potentials

Representing a potential as $U(r) = U_0 u(r)$, where U_0 characterises its value; and $u(r)$ is a dimensionless function of its distribution in space, we can introduce the notation

$$i\Phi = \gamma I(b), \quad \gamma = -i \frac{U_0}{\hbar v}, \quad I(b) = 2 \int_0^\infty u(\sqrt{b^2 + \lambda^2}) d\lambda. \quad (3.1)$$

where the phase, or profile, integral $I(b)$ carries information only on geometrical parameters of the potential.

At $r \gg R_c = r_c(A_1^{1/3} + A_2^{1/3})$ nuclei interact like point charges $Z_1 e$ and $Z_2 e$. In this case $U_0 \equiv U_B = Z_1 Z_2 e^2 / R_c$ and $u_{pc} = R_c / r$, whereas the profile integral and eikonal phase are equal to

$$I_{pc}(b) = 2R_c \int_0^{L-\infty} \frac{d\lambda}{\sqrt{b^2 + \lambda^2}} = -2R_c \ln \frac{b}{2L} = -2R_c \ln(kb) + 2R_c \ln(2kL), \quad (3.2)$$

$$\Phi_{pc}(b) = 2\eta \ln \frac{b}{2L} = 2\eta \ln(kb) - \Phi_a, \quad \Phi_a = 2\eta \ln(2kL). \quad (3.3)$$

Here Φ_a appears because of the charge screening at large distances L , and $\eta = Z_1 Z_2 e^2 / \hbar v$ is the Sommerfeld parameter. The respective amplitude of Coulomb scattering [2] is as follows:

$$f_{pc}(q) = -\frac{ik}{(2L)^{2i\eta}} \int_0^\infty db b^{1+2i\eta} J_0(qb) = -\frac{2k\eta}{q^2} e^{-2i\eta \ln(q/2k) + 2i\sigma_0 - i\Phi_a}, \quad (3.4)$$

where $\sigma_0 = \arg \Gamma(1 + i\eta)$ is the Coulomb phase.

For heavy-ion scattering the usually applied Coulomb potential is that for the field of the uniformly charged sphere having radius R_u and density $\rho(r) = \rho_0 \Theta(R_u - r)$. It has the form

$$U_{uc}(r) = U_B \frac{1}{2} (3 - r^2/R_u^2) \Theta(R_u - r) + U_B \frac{R_u}{r} \Theta(r - R_u), \quad (3.5)$$

where $U_B = Z_1 Z_2 e^2 / R_u$. The corresponding eikonal phase is equal to

$$\Phi_{uc}(b) = \begin{cases} 2\eta \left[\ln(kR_u) + \ln(1 + \sqrt{1 - b^2/R_u^2}) - \frac{1}{3} \sqrt{1 - b^2/R_u^2} (4 - b^2/R_u^2) \right], & b \leq R_u \\ 2\eta \ln(kb) - \Phi_a, & b > R_u. \end{cases} \quad (3.6)$$

The radius of a sphere R_u can be connected with the root-mean-square radius of the realistic charge-density distribution by the relation $R_u = \sqrt{\frac{5}{3}} R_{rms}$. For instance, for the Fermi-like density distribution (2.6) with radius R_c and diffuseness a_c we have $R_u = R_c \sqrt{1 + (7/3)(\pi a_c / R_c)^2}$.

In calculating the amplitudes of nucleus-nucleus collision (2.4), there arise the problem with integration owing to divergent terms $\ln(kb)$ in the Coulomb eikonal phase. This difficulty can be overcome if one adds and subtracts the eikonal function $\exp(i\Phi_{pc})$ in brackets in the integrand (2.4). Then we obtain, respectively, the sum of the amplitudes of Coulomb scattering on a pointlike charge and a corrected nuclear scattering

$$f(q) = f_{pc}(q) + ik \int_0^\infty db b J_0(qb) e^{i\Phi_{pc}} \left(1 - e^{i\Phi_N + i\delta\Phi_{uc}} \right), \quad (3.7)$$

where the addition $\delta\Phi_{uc} = \Phi_{uc} - \Phi_{pc}$ to the nuclear eikonal phase no longer contains the logarithmic term $\ln(kb)$ at large b . Besides, here, the amplitude $f_{pc}(q)$ gets separated, which is known explicitly (3.4). In the integral, the growth of the function $\exp(i\Phi_{pc})$ with b is compensated by the decrease of the expression in brackets. From (3.7) it also follows that the ratio of differential cross section to the Rutherford one $d\sigma/d\sigma_R$ will always be equal to 1 at small scattering angles.

As for the nuclear eikonal phase Φ_N , one should take into consideration that the nucleus-nucleus potentials are characterized by large radii $R = R_1 + R_2$. Therefore the typical extended potential used in calculations is a Woods-Saxon potential with the Fermi distribution $u_F(r) = [1 + \exp((r - R)/a)]^{-1}$. It is, however, known that there is a number of physical and

mathematical arguments (see, e.g., [6]) for using, instead of $u_F(r)$, its symmetrized form (2.6), i.e. $u_{SF}(r) = \sinh(R/a) / [\cosh(R/a) + \cosh(r/a)]$. Indeed, the distribution $u_{SF}(r)$ at large R actually coincides with $u_F(r)$, but unlike the latter, it has no nonphysical "cusp" at $r = 0$ that produces mathematical difficulties in a number of problems. In [7], a single attempt was perhaps made to derive an analytic expression for the eikonal phase with u_F -distribution. The result was an infinite sum of residues at poles of the Fermi function. However, at $b = 0$ it gave $I_F(0) = 0$, though it should be $I_F(b = 0) \approx 2R$. The authors of [7] added this term to their sum, but the obtained ansatz is still not valid for nucleus-nucleus scattering, where powerful Coulomb tails force one to integrate over b up to distances larger than R ; here the ansatz does not work, and $I(b)$ becomes growing rather than decreasing.

Thus, because there is no an appropriate recipe, many authors fit the Fermi function by a sum of Gaussians [8] with varying coefficients. At large R , this is a difficult problem, but since the main contribution to nucleus-nucleus scattering comes from a small part of the potential at its periphery, the fit in this region can be made with one-two terms of the series of Gaussians. The problem is that one must be sure in selection of the precise distance in a surface region where the fit should be made. Besides, analytic properties of the eikonal integrals with the Fermi and Gaussian functions on a complex plane b drastically differ from each other.

Our subsequent calculations are based on an approximate formula for $I(b)$ derived in [4] for the symmetrized Fermi function with the use of approximate separation of variables b and λ in the integrand u_{SF} of the respective eikonal integral (3.8). The obtained result can be written in a certain form if one introduce variables $\zeta = \lambda/a$ and $\beta = b/R$. Then, the integral $I(b)$ is expressed in the form

$$I(b) \equiv I(\beta R) = 2RI(\beta), \quad (3.8)$$

where $I(\beta)$ depends only on the ratio $C = R/a$ of two input parameters, the radius R and diffuseness a , as follows:

$$I(\beta) = \frac{1}{C} \int_0^\infty \frac{\sinh C d\zeta}{\cosh C + \cosh \sqrt{(\beta C)^2 + \zeta^2}} \approx \frac{\sinh C}{\cosh C + \cosh \beta C} P(\beta, C), \quad (3.9)$$

$$P(\beta, C) = \frac{1}{C} \frac{1}{\sqrt{1-x}} \ln \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}}, \quad x = \frac{2}{\kappa} \frac{1}{1 + \frac{\cosh C}{\cosh \beta C}} \left\{ 1 + \frac{\kappa - 1}{\cosh \beta C} \right\}. \quad (3.10)$$

Here κ is a parameter determined from the χ^2 -fit of the approximate expression in the r.h.s. of (3.9) to the numerically calculated integral $I(\beta)$ in the region $0 < \beta < 2$ and $5 < C < 20$, which gives

$$\log \kappa = 0.47909 + 0.15025 C - 0.001938 C^2. \quad (3.11)$$

Since $\cosh C \gg \kappa \gg 1$, and thus $x \ll 1$, then expanding P in x and considering that the main contribution to nucleus-nucleus scattering comes from the region $b \approx R$ (or $C \approx 1$), we obtain

$$P_a(1, C) \approx \frac{1}{C} \ln 4\kappa = \frac{1}{C} [2.48945 + 0.34597 C - 0.0046 C^2]. \quad (3.12)$$

From Fig.2 it is seen that in the presence of the Coulomb field, the region of angles $\theta < \theta_c \approx U_B/E$ (4° for 4He and 10° for ${}^{40}Ca$) is that of the Rutherford scattering. Further, with increasing angles up to $\theta < \theta_c + \bar{\theta} \approx 14^\circ$ (for 4He) and $\approx 15^\circ$ (for ${}^{40}Ca$) agreement of cross sections is observed when the nuclear eikonal phase is calculated with the help of numerical integration of $I(\beta)$ and by the analytic formula (3.9) with different "gathering functions" $P(\beta, C)$ (3.10) and $P_a(1, C)$ (3.12). Thus, we can conclude that the explicit form of the nuclear profile integral (3.9) proves to be valid in problems of nucleus-nucleus scattering.

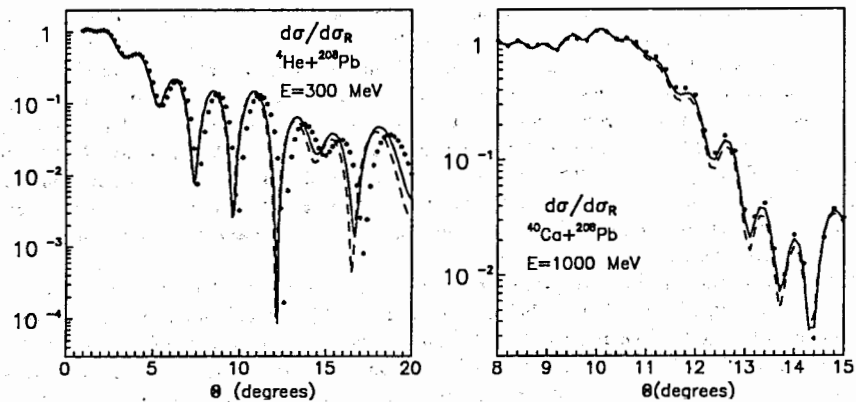


Figure 2: Scattering cross sections in the field of the Coulomb potential (3.5) and symmetrized Woods-Saxon potential (1.2) at $V_0 = -50$ MeV, $W_0 = -42$ MeV, $r_0 = r_c = 1.04$ fm, $a = 0.75$ fm. The eikonal integral in (3.9) $\mathcal{I}(\beta)$ was computed both numerically (dots) and with the analytic formula in the r.h.s. of (3.9) with $P(\beta, C)$ (3.10) (solid lines) and with $P_a(1, C)$ (3.12) (dashed lines).

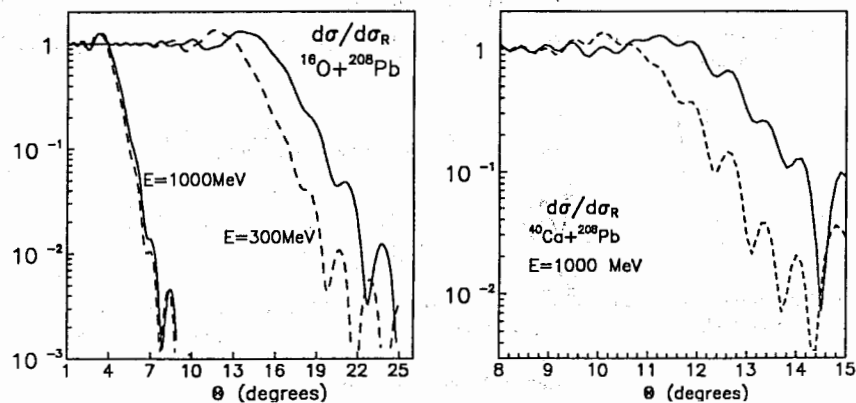


Figure 3: The influence of the Coulomb shift of the trajectory on the cross section of nuclear elastic scattering at different energies. Parameters of potentials are the same as in Fig.2; the nuclear eikonal was computed with the help of analytic formula (3.9) with $P_a(1, C)$ (3.12). Dashed curves — without the trajectory shift; solid curves — with that shift.

4 Results of calculations and conclusions

When the scattering of heavy ions by nuclei is considered in the framework of HEA, an important problem is to take into account the deviation of their trajectory from a straight line owing to the long-range Coulomb potential. The point is that the distance from the scattering center to

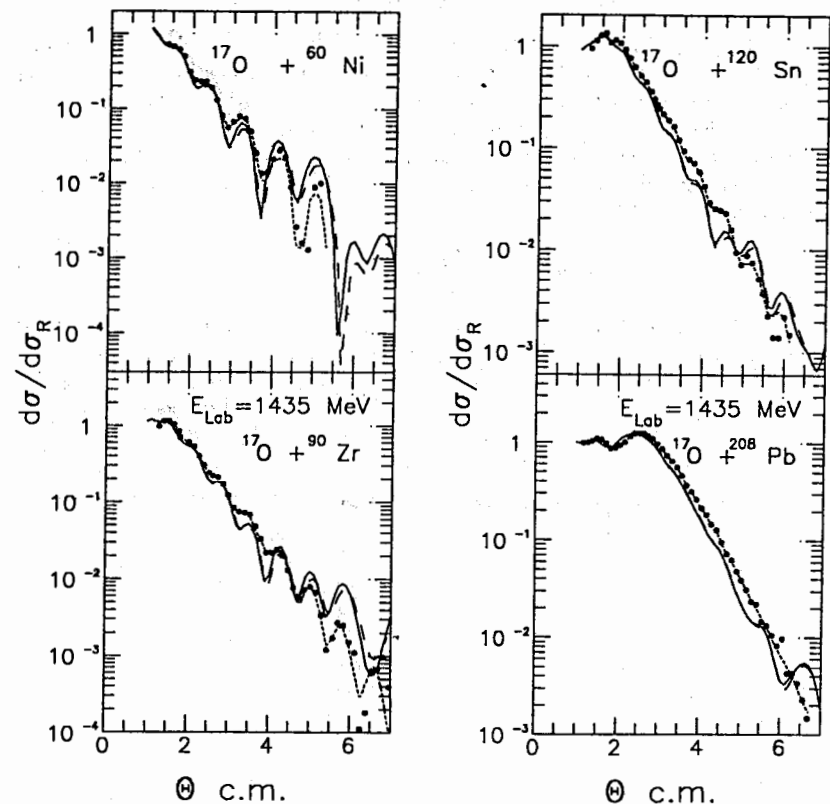


Figure 4: Comparison of the differential cross sections of elastic scattering calculated in the framework of HEA for small angles (solid lines — at the nuclear eikonal with $P(\beta, C)$, dashed lines — numerical calculation of the nuclear eikonal integral) with the cross sections obtained by numerical solution of the wave equation (dotted lines) and experimental data (solid dots) taken from [10].

the point of closest approach b_c of an incident nucleus with a target-nucleus differs, generally speaking, from the respective impact parameter b at the asymptotics $z = -\infty$. This is manifested in the nuclear eikonal since its profile integral (see (3.8), (3.9)) sharply changes in the periphery at $b \sim R$. This effect can be taken into account (see, for instance, [9]) by replacing the impact parameter b in the nuclear eikonal by b_c and the probability flux v by $(b/b_c)v$. In the Coulomb field, we have

$$b_c = k^{-1} \left\{ \eta + \sqrt{\eta^2 + (kb)^2} \right\}. \quad (4.1)$$

The results of calculations are presented for scattering of ^{16}O and ^{40}Ca by ^{208}Pb at different energies (Fig.3). It is seen that for $E \sim 60$ MeV/nucleon and at higher energies the effect of distortion of the trajectory is weak but gets significant at lower energies. The shift of the trajectory increases the angle by an order of magnitude $\theta_c \sim U_B/E$, and thus, the range of applicability of small-angle approximation is expanded. If one joins the estimate (2.3) for θ , the Rutherford scattering angle θ_c , and the trajectory deflection angle (in the case of the Coulomb field θ_c), it can be defined as follows:

$$\theta < 2\theta_c + \bar{\theta} = 2 \frac{U_B}{E} + \sqrt{2/kR}. \quad (4.2)$$

Figure 4 demonstrates the HEA cross sections of the heavy ions ^{17}O scattering as compared with the numerical solution of the Schroedinger equation (dotted lines) and experimental data (dots). Parameters of the potential and experimental data are taken from [10]. It can be ascertained that they agree qualitatively, and at small angles, also quantitatively (solid lines for the eikonal with $P(\beta, C)$, dashed lines are in the case of numerical calculations of the nuclear eikonal). Discrepancies appear for $\theta > 1/kR + \theta_c$ at large scattering angles, where computations should be performed on the basis of definition of the amplitude (2.6), by developing appropriate methods (see, for instance, [11], [12] and references therein).

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