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PROPERTIES OF ISOVECTOR 1^+ STATES IN $A = 28$ NUCLEI AND NUCLEAR MUON CAPTURE

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Свойства изовекторных 1^+-состояний в ядрах с $A = 28$ и ядерный захват мюонов

Предложен метод учета в расчетах парциальных скоростей захвата мюонов ядрами существующей экспериментальной информации о силовых функциях ГТ- и изовекторных М1-переходов. Метод сводится к подбору ортогонального преобразования, действующего в подпространстве волновых функций возбужденных состояний, и не требует введения никаких модификаций операторов перехода. Матрица преобразования строится как произведение матриц отражения в плоскости. Все расчеты проводятся на основе многочастичной модели оболочек. Численные результаты получены для изовекторных состояний в ядрах с $A = 28$. При этом рассмотрены силовые функции ГТ- и изовекторных М1-переходов в ^{28}Si, вычислены времена жизни и отношения каналов γ-распада состояний 1^+ в ^{28}Al. Показано, что учет экспериментальной информации о свойствах изовекторных состояний впервые позволил правильно описать отношение каналов γ-распада состояния 1^+ с энергией 2,201 МэВ в ^{28}Al. Показано, что использование преобразованных волновых функций сильно меняет распределение парциальных скоростей разрешенного захвата мюонов ядром ^{28}Si по конечным 1^+-состояниям ядра ^{28}Al по сравнению с результатами расчетов, использующих собственные функции гамильтонианов многочастичной модели оболочек.

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Properties of Isovector 1^+ States in $A = 28$ Nuclei and Nuclear Muon Capture

The method how one can utilize the existing experimental information concerning GT and M1 strength functions into the calculations of the rates of nuclear muon capture is proposed. The method is reduced to adjusting of the orthogonal transformation in the subspace of wave functions of excited states and does not require any modifications of transition operators. The transformation matrix is created as a product of matrices of reflection in the plane. All calculations are made on the base of many-particle shell model. The numerical calculations are carried out for isovector transitions in the nuclei with $A = 28$. The GT and M1 strength functions in ^{28}Si are considered; for 1^+ states in ^{28}Al the life times and γ-decay branching ratios are calculated. This method allows one to describe correctly the branching ratio of 1^+ state with energy 2.201 MeV in ^{28}Al for the first time. It is shown that introduced transformation of wave functions changes considerably the distribution of partial allowed muon capture rates in ^{28}Si over 1^+ states of product nucleus ^{28}Al compared to results obtained with eigenfunctions of Hamiltonian of many-particle shell model.

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1 Introduction

In recent years, results have been published for two independent measurements of the coefficients of gamma-neutrino correlation between the momentum of a neutrino produced in the capture of negative polarized muons by the nucleus ^{28}Si and the momentum of a photon emitted in the γ-decay of an excited state of a product nucleus [1, 2]. The authors of both the papers considered the same allowed partial transition

$$\mu^- + {}^{28}\mathrm{Si}(0^+_{\mathrm{g.s.}}) \to \nu_\mu + {}^{28}\mathrm{Al}(1^+_3, 2201).$$

Comparison of the values of correlation coefficients obtained in these works with different theoretical calculations [3, 4, 5] led to the conclusion that a induced pseudoscalar weak interaction of a muon with nucleons of the nucleus is much suppressed as compared with that estimated from the hypothesis of partial conservation of the axial-vector current (PCAC). In Table 1 we present the values of g_P/g_A obtained from the comparison of the measured coefficients of angular correlation [1, 2] with different theoretical values. In another experiment the ratio of rates of muon capture from the states of hyperfine splitting of the mesoatom ^{23}Na was measured and comparison with calculations gave $g_P/g_A = 7.6 \pm 2.1$ near to the PCAC estimate [6]. Such a large difference in values of g_P/g_A is quite unexpected since the nuclei ^{28}Si and ^{23}Na are rather close to each other in mass number and thus one would expect that the constants of induced pseudoscalar interaction would be almost equal in both the cases. We note that in refs. [5] and [6] the nuclear matrix elements of the effective Hamiltonian of ordinary muon capture were computed within the multiparticle shell model [7] based on the full sd shell space and parametrization of the shell model Hamiltonian [10]. However, the partial transitions under consideration possess an important difference. In experiment [6], the allowed transition $3/2^+, 1/2 \to 1/2^+, 3/2$ has been considered. (The states of nuclei are classified by the total spin, parity, and the total isospin J^π, T.) According to calculations, this partial transition occurs mainly due to the one-body transition $d_{5/2} \to d_{5/2}$. As a result, the matrix element of the operator $j_0(\nu r)\,\sigma\, t^-$ ([101] matrix element in the notation of [8, 9]) turns out to be dominating and it determines the partial rates of muon capture by the nucleus, therefore the results of calculations are quite reliable. For the transition $0^+_{\mathrm{g.s.}} \to 1^+_3$, in the μ-capture on ^{28}Si there is no such a dominating matrix element since the one-body tran-

sition $d_{5/2} \to d_{3/2}$ having the largest amplitude in the one-body transition density is suppressed by the contrary transition $d_{3/2} \to d_{5/2}$. When the leading matrix element is absent, the amplitudes of nuclear muon capture are determined by interference of several small matrix elements including velocity-dependent ones. Therefore, the reliability of theoretical description decreases, and further studies the excited states of a product nucleus become necessarily.

Isovector states with spin and parity $J^\pi = 1^+$ in nuclei with $A = 28$ were theoretically studied in ref. [11] on the basis of the multiparticle shell model with the use of the full sd shell space. Energies of single-particle states and two-body matrix elements of the interaction between valence nucleons were taken from ref. [10]. The results obtained in ref. [11] can be summarized as follows. The calculated excitation energies and life times of the first 1^+ states are in agreement with the experimental data. However, the γ-decay branching ratio for the state 1^+ with energy 2.201 MeV in ^{28}Al is described worst.

Experimental studies of the properties of isovector 1^+ states in nuclei having $A = 28$ were made by means reactions (e, e') in [12], (p, n) in [13], and $(^3\mathrm{He}, t)$ in [14]. The authors of ref. [12] compared the obtained experimental distributions of the strength of magnetic dipole transitions over the excitation energy (strength function of M1 transitions) with the distributions calculated within the shell model with the Hamiltonian of Wildenthal [10] and showed that the theoretical distribution of $B(\mathrm{M1})$ does not coincide with the experimental distribution. For the majority of states the calculated $B(\mathrm{M1})$ exceeds appreciably the experimental value but for the state with excitation energy 11.445 MeV, which corresponds to the third eigenstate of the shell-model Hamiltonian, the theoretical value is smaller than the experimental one. The theoretical value of the total strength of transitions is appreciably larger than its experimental value. The experimental strength function of Gamow-Teller (GT) σt^+ transitions was measured in ref. [13]. Like for M1 transitions, the theoretical total strength of GT transitions exceeds the experimental value, but the theoretical strength of the strongest transition is smaller than its experimental value. This is transition onto the state with energy 2.10 MeV in ^{28}P, which is described by the third eigenstate of the Hamiltonian.

The hypothesis of isospin conservation in nuclei allows us to combine the isovector 1^+ states in ^{28}Al, ^{28}Si and ^{28}P into isotopic triplets. The table

of correspondence can be found in ref. [11]. From this table it is seen that the level 1^+ with energy 2.201 MeV in ^{28}Al populated in the $\gamma\nu$-correlation experiment on ^{28}Si enters into one triplet with levels 11.445 MeV in ^{28}Si and 2.10 MeV in ^{28}P for which the theoretical transition strength turns out to be smaller than the experimental value. Transitions from the ground state of the nucleus ^{28}Si, whose isospin is zero, onto states of the isotopic triplets can be described by isotopic tensor operators of rank 1. Then it is possible to single out the dependence on isospin projections from matrix elements of these operators and thus to pass to isospin-reduced (triple-bar) matrix elements. If the electron scattering, (p, n) reactions and μ-capture were described by the same isospin-reduced matrix elements, they could be used, being determined from one process, for describing other processes. However, these operators differ from each other. In (p, n) reaction the square matrix element $\langle 1^+, 1 \| \sigma t \| 0^+, 0 \rangle$ can be measured; in M1 transition — the square of $\langle 1^+, 1 \| (g_s^{IV} \sigma + g_l^{IV} l)\, t \| 0^+, 0 \rangle$; and the amplitude of partial μ-capture contains the matrix elements $\langle 1^+, 1 \| j_0(\nu r)\, \sigma\, t \| 0^+, 0 \rangle$, $\langle 1^+, 1 \| j_2(\nu r)\, [Y_2 \sigma]_1\, t \| 0^+, 0 \rangle$, $\langle 1^+, 1 \| j_1(\nu r)\, [Y_1 \nabla]_1\, t \| 0^+, 0 \rangle$, and $\langle 1^+, 1 \| j_1(\nu r)\, Y_1\, (\sigma \nabla)\, t \| 0^+, 0 \rangle$. Therefore, the quantities $B(\mathrm{M1})$ and $B(\mathrm{GT})$ can be used for the description of μ-capture indirectly only, as a tool of checking the quality of reproduction of characteristics of the given partial transitions. As a result, the fact that the calculated strength of M1 and GT transitions onto the third eigenstate of the Hamiltonian is considerably smaller than the experimental value, though the theoretical total strength of transitions significantly exceeds its experimental value is a clear evidence for the theoretical model being a failure for the description of that level. One should take into account that the wave functions of states were calculated by diagonalization of the shell-model Hamiltonian in the full sd-shell space; and at present it is not clear how this description can be improved in the framework of consistent theory.

In this situation it will be interesting to use the existing experimental information concerning GT and M1 strength functions in the calculations of muon capture. For that purpose we propose to introduce the phenomenological corrections into the results of calculations of muon capture by means of an orthogonal transformation of the wave functions of excited states with $J^\pi, T = 1^+, 1$. A key point is that the transformation parameters should be chosen so that the strength functions of GT and M1 transitions calculated with the transformed wave functions coincide in shape, in other

words up to a constant factor, with the experimental GT and M1 strength functions. Since the transformation is orthogonal, the transformed wave functions will be orthogonal to each other and normalized, like the initial functions. The space of excited states will neither narrow nor broaden and the total strength of GT and M1 transitions will be conserved. As result of transformation of wave functions transition strength will be redistributed between excited states only. Therefore the problems concerning theoretical values of the total transition strength being higher than the experimental ones are out of scope of present paper. Since the new wave functions are constructed as linear combinations of the functions with the same values of the total moment, parity, and isospin ($J^\pi, T = 1^+, 1$), then for them $J^\pi, T = 1^+, 1$, too. This method allows us to include the bulk of the available experimental information on the strength functions of GT and M1 transitions in the calculations of characteristics of muon capture.

2 Mathematical details

The orthogonal transformation of the wave functions of the excited states is searched in order to reproduce the shape of the experimental strength functions of GT and M1 transitions from the 0^+ ground state to the 1^+ excited states.

2.1 Transformation of wave functions and transition amplitudes

Let ϕ_k $(k = 1, \ldots, N)$ be the set of the excited state wave functions. They are supposed to be normalized and mutually orthogonal as usual. The new set of functions is created by a unitary transformation U

$$\psi_k = U_{k,k'}\phi_{k'} \qquad (k = 1, 2, \ldots, N). \tag{1}$$

Owing to the transformation U being unitary, the functions ψ_k are normalized and orthogonal. The matrix element of transition operator $\mathcal{O}$ between the ground state wave function Φ and the excited state with wave function ψ_k equals

$$\langle\psi_k|\mathcal{O}|\Phi\rangle = U^*_{k,k'}\langle\phi_{k'}|\mathcal{O}|\Phi\rangle = \langle\phi_{k'}|\mathcal{O}|\Phi\rangle U^\dagger_{k',k}. \tag{2}$$

Thus, the vector of transition amplitudes from the state Φ into transformed states ψ_k is obtained by the unitary transformation $U^\dagger$ from the vector of

transition amplitudes into the initial states ϕ_k; the transformation $U^\dagger$ is Hermitian conjugate transformation U. Equation (2) simplifies the problem of finding transformation (1) since it reduces the search of a transformation of the vector consisting of multiparticle wave functions to the search of a transformation of a much simpler vector composed of the transition amplitudes, i.e. ordinary numbers. In the majority of cases, the transition amplitudes are real numbers, and therefore, the elements of matrix U can be demanded to be real numbers whereas the matrix itself will orthogonal.

2.2 The structure of transformation matrix

An orthogonal matrix of dimension N could be fixed by $N(N-1)/2$ free parameters. Therefore, if we consider 9 or 10 excited states, we should fix 36 or 45 parameters. As a result, there are 9 or 10 linear equations of type (2) for determining 36 or 45 unknown variables and the problem becomes undetermined. Therefore one would like to use matrices of less general structure.

The simplest orthogonal transformation of a vector is the reflection in the plane [15, 16]. Any vector v can be decomposed in a sum of two vectors $v = u + w$, u belonging to a certain plane and w being perpendicular to this plane. The reflection in the plane alters the direction of the vector w and converts the vector v into the vector $v' = u - w$. The length of the vector v whose square is $(|v|^2 = |u|^2 + |w|^2 = v_1^2 + v_2^2 + \ldots + v_N^2 = v_k v_k)$, is under this transformation conserved, $|v| = |v'|$. Therefore, any two given vectors u and v of the same length $(|u| = |v|)$ can be transformed into each other by the reflection in the plane that crosses the origin of coordinates perpendicular to the vector $u - v$. The equation of the plane is

$$(u_i - v_i)x_i = 0. \tag{3}$$

In this case the transformation matrix is

$$R_{i,j} = \delta_{i,j} - 2\frac{(u_i - v_i)(u_j - v_j)}{|u - v|^2}. \tag{4}$$

Matrix R is symmetric, $R_{i,j} = R_{j,i}$, and orthogonal, $\tilde{R}R = RR = I$; I is the unit matrix. For any pair of equal length vectors u and v the reflection transformation is unique because the parameters of transformation

are coordinates of vector $u - v$. The number of parameters equals to the dimension of the vector space where the transformation is carried out.

To construct the matrix of transformation of wave functions of excited states, it suffices to know two vectors consisting of transition amplitudes. One vector should contain amplitudes calculated within the multiparticle shell model; the other should consist of transition amplitudes extracted from the experimental data.

3 Strength functions of GT and M1 transitions

We compute transformation matrices using two vectors consisting of amplitudes of transition onto the chosen excited states. The square of length of each vector equals the transition strength (either experimental or theoretical) summed up over all the selected excited states.

We start with the vector composed of theoretical amplitudes. Since we are looking for the transformation that reproduces only the shape of the experimental strength function, we may normalize this vector to unity. Let $(t_1, \ldots, t_N)$ be a vector composed of calculated amplitudes of GT transitions normalized by the condition $t_k t_k = 1$. Relative signs of coordinates of the vector t are controlled by relative phases of wave functions of excited states.

3.1 On experimental amplitudes. Selection of best transformation

The situation with experimental amplitudes is much complicated. Experiment produces only the transition strength, the squared absolute value of a transition amplitude. If we denote the strength of the GT transition onto a k-th 1^+ excited state by e_k and compose the vector $\tilde{e} = (e_1, \ldots, e_N)$, of them, the total strength of transitions onto chosen states is a sum of coordinates, $s^2(e) = e_1 + \ldots + e_N$. Knowing the strength of transition e_k, we compute the absolute value of its amplitude $|f_k| = \sqrt{e_k}$, the sign of f_k being indefinite. Therefore we should consider all possible distributions of signs of amplitudes inside the vector f. For every distribution of signs in a vector composed of experimental amplitudes of GT transitions the reflection

matrix (4) exists

$$R(f,t)_{i,j} = \delta_{i,j} - 2\,\frac{(f_i - s(e)t_i)(f_j - s(e)t_j)}{|f - s(e)t|^2}. \qquad (5)$$

The vector of theoretical amplitudes transformed by this matrix gives the distribution of the strength of GT transitions coincident in shape with the experimental GT strength function. Therefore, using only one strength function, one cannot choose the optimal transformation.

The criterion arises naturally if we consider the GT strength function together with the strength function of M1 transitions. The transformation of wave functions (1) gives rise to the transformation of transition amplitudes (2), therefore the amplitudes of M1 transitions must be transformed by the matrix $R(f,t)$. Difference in the structure of GT and M1 transition operators results in a linear independence of vectors composed of the amplitudes of GT and M1 transitions. This indepedence keeps both for experimental and theoretical vectors. A new M1 strength function obtained with $R(f,t)$ will not, in general, coincide with the experimental M1 strength function because any orthogonal transformation preserves the scalar product of two vectors. Since the scalar product of vectors composed of theoretical GT and M1 amplitudes is not likely to equal that of experimental GT and M1 vectors, at the complete reproduction of the shape of the GT strength function, the shape of the M1 function will be reproduced with an error. The transformation giving the smallest root-mean-square error will be considered the best one.

To diminish the difference in shape of the obtained theoretical and experimental M1 strength functions, a further step is one more transformation of type (4) acting in a subspace orthogonal to the plane composed of the theoretical and experimental vectors of GT amplitudes. Such transformation will conserve the obtained distribution of the strength of GT transitions over excitation energies.

3.2 Calculations of strength functions of GT and M1 transitions

Theoretical and experimental strength functions of GT and M1 transitions to be used in what follows are reported in Table 2. Calculations are based on the Hamiltonian of ref. [10]. Multiparticle wave functions, energies of

states, and transition amplitudes have been calculated by programs of ref. [7].

In the first part of Table 2, we present the calculated characteristics of the first 10 excited states with $J^\pi, T = 1^+, 1$: the excitation energies, amplitudes and reduced probabilities of GT and M1 transitions from the lowest state with $J^\pi, T = 0^+, 0$ (the ground state of ^{28}Si nucleus) onto them. The excitation energies are reckoned from the ^{28}Si ground state.

The wave functions with $J^\pi, T = 1^+, 1$ and isospin projection 1 describe 1^+ states in the nucleus ^{28}P that are observed as resonances in a charge-exchange reaction $^{28}\mathrm{Si}(p,n)^{28}\mathrm{P}$ at intermediate energies [13]. The cross section of that reaction is proportional to $B(\mathrm{GT})$. The energies of resonance states and probabilities $B(\mathrm{GT})$ obtained from the experimental study of reaction $^{28}\mathrm{Si}(p,n)^{28}\mathrm{P}$ [13] are listed in the second part of Table 2. There are shown all the states with excitation energies lower than 6 MeV detected experimentally. The energies are reckoned from the ground state of ^{28}P. States with energies higher than 6 MeV take a small part of observed GT transition strength, and besides, their exact values of spins are undetermined. The total strength of GT transitions discovered experimentally in the interval of excitation energies up to 12.6 MeV amounts to 2.595, whereas that for states listed in Table 2 amounts to 2.301. The calculated value of the total strength of GT transitions for the first 10 states equals 3.492.

The wave functions with $J^\pi, T = 1^+, 1$ and isospin projection 0 describe isovector 1^+ states in ^{28}Si. These states are excited in electron inelastic scattering, from experimental cross sections of which the reduced probabilities $B(\mathrm{M1})$ are extracted. In the last part of Table 2, we report experimental data on the strength of isovector M1 transitions measured in [12]. We indicate all the isovector states discovered in the interval of excitation energies from 10.5 to 15.5 MeV. Here the following remark is to the point concerning the state with energy 10.64 MeV presented in the first column of Table 2: As a matter of fact, instead of that state, experimentally observed are two states with energies 10.597 and 10.725 MeV. They are usually considered as superpositions of one isovector and one isoscalar 1^+ states [17]. It can be shown that the total strength of M1 transitions onto these two states is a sum of strengths of M1 transitions onto pure isovector and isoscalar 1^+ states. The calculations within the multiparticle shell model with the Hamiltonian [10] show that the strength of the isoscalar

M1 transition is few ten times as small as that of the isovector transition. As a result, we can neglect the strength of the isoscalar M1 transition and determine the energy of the pure isovector state as the weighted average of energies of those two states, $(E_1 \cdot B_1(\mathrm{M1}) + E_2 \cdot B_2(\mathrm{M1}))/(B_1(\mathrm{M1}) + B_2(\mathrm{M1}))$. The obtained energy of the isovector state and the total strength of transitions onto it are shown in Table 2. The sum of experimental values of $B(M1)$ listed in Table 2 equals 7.360 nuclear magnetons; whereas the theoretical value is 8.623. Calculations were carried out with the use of free gyromagnetic ratios, without effective magnetic charges.

The experimentally observed states are described by eigenfunctions of the shell-model Hamiltonian. The correspondence is established by the comparison the energies of observed states to the eigenvalues of the Hamiltonian. The comparison gets simplified if we consider the excitation energies measured from that of the first excited state. Values $E_k - E_1$ are given in Table 2 too. Then, correspondence between the eigenfunctions of the multiparticle shell model Hamiltonian and states observed experimentally is practically unique, which evidences in favor of a high accuracy of the description of energies of excited states within the nuclear model [10].

As is seen from Table 2, not all the Hamiltonian eigenfunctions are associated with the states observed experimentally. In particular, the 5-th function corresponds to neither experimental state. The model explains this fact by very small theoretical values of $B(\mathrm{GT})$ and $B(\mathrm{M1})$. In what follows we will not consider this state since it is not observed experimentally at all. The state corresponding to the 6-th function was found in the reaction (p,n), but its analog with energy 13.31 MeV in ^{28}Si is not observed in the reaction (e,e') though is observed in the reaction (p,p') [18]. This fact is explained in [12] as follows: the contributions of spin and orbital magnetic currents to the M1 transition amplitudes compensate each other. The shell model calculations with Hamiltonian [10] carried out in ref. [12] reproduces this fact well.

Thus, we can conclude that the shell model describes qualitatively the main features of the strength distribution of GT and M1 transitions over excitation energies in the sense that small theoretical values of $B(\mathrm{GT})$ and $B(\mathrm{M1})$ correspond to small experimental cross sections. However, the theoretical distributions of the strength of transitions over the states which absorb a considerable part of the total strength of transitions are highly different from the experimental distributions. In particular, the calculated

B(GT) and B(M1) for transitions into the third isovector 1^+ states are noticeably smaller than the experimental values, but the theoretical values of the total strength of GT and M1 transitions are much larger than experimental ones. In refs. [13, 12] figures are presented with the theoretical and experimental strength functions where one can see that the theoretical strength functions are fragmented to a greater extent than the experimental ones. It is just a further configuration mixing we have introduced to eliminate that discrepancy.

The obtained matrix of the whole transformation, the product of two reflection matrices, is given in Table 3 with the numbers of eigenfunctions of the Hamiltonian between which the mixing occurs. The matrix is symmetric, because the reflections are made in two perpendicular planes, and thus the corresponding reflection matrices commute with each other. Basically, the main diagonal of the matrix contains the numbers near 1.0 in absolute value. The states with numbers 6 and 8 whose experimental partners have been observed only in (p,n) reactions change places. In other cases the principal component is conserved, and other states are admixed to it with small amplitudes.

Once the transformation matrix is known, the amplitudes of GT and M1 transitions and then B(GT) and B(M1) can be calculated by eq. (2) (see Table 4). The calculated B(GT) differ from the experimental ones given in Table 2 by constant factor 0.66, i.e. the shape of experimental strength function of GT transitions is reproduced exactly. The ratios of experimental B(M1) to the ones from Table 4 are in the interval from 0.67 to 1.5. Consequently, as it was expected, the shape of strength functions of M1 transitions is reproduced approximately.

4 γ-decay of 1^+ states in ^{28}Al

As additional test of obtained wave functions of excited states we compute the life time and branching ratios of electromagnetic decay of 1^+ states in ^{28}Al.

The transformation of wave functions (1) changes both the matrix elements of γ-transition between 1^+ states and states with other spins and parities given by eq. (2) and the matrix elements of γ-transitions between

the $1^+, 1$ states

$$\langle\phi_k|\mathcal{O}|\phi_l\rangle \to \langle\psi_k|\mathcal{O}|\psi_l\rangle = U_{l,l'}\langle\phi_{k'}|\mathcal{O}|\phi_{l'}\rangle U^{\dagger}_{k',k}. \qquad (6)$$

As a result, all theoretical values of the life time and branching ratios for γ-decay of isovector 1^+ states become altered.

Table 5 shows the calculated life times of isovector 1^+ states together with known experimental data [17, 11]. The life times are calculated with wave functions of Hamiltonian [10] and with wave functions (or γ-transition amplitudes) transformed by matrix from Table 3. The nuclear final states with $J^\pi = 0^+$, 1^+, 2^+ and 3^+ are taken into account. We have used the known experimental energies of excited states because the rates of electromagnetic transitions depend strongly from the transition energy [19] and therefore the nuclear model accuracy equals to 0.1 – 0.2 MeV is unsufficient for 1 – 2 MeV transition energies. The influence of this error decreases with increasing of γ-transition energy. Probably, the experimental excitation energies have been used in calculations of ref. [11] too. The life-times calculated only with theoretical excitation energies are shown in Table 5 for completeness. The comparison of two versions of calculations reveals that even small errors in the energies of the excited states lead to considerable discrepancies in calculated values of life times.

The branching ratios for γ-decay of 1^+ states in ^{28}Al are presented in Table 6. The comparison the calculation results to experimental data shows that additional configuration mixing in wave functions of the 1^+ states accelerates the transitions onto the 2^+_1 state with energy 0.031 MeV from the $1^+_{1,3}$ states and slows down the transition from the 1^+_2 state. As result the description of life time of $1_{1,2}$ levels becomes worse. The life time of 1^+_3 state obtained in the calculation with transformed wave functions equals to $44 \cdot 10^{-15}$ s^{-1}. This value is rather closed to $(38.3 \pm 2.8)10^{-15}$ s^{-1} obtained in ref. [1]. Simultaneously, the correct relation is reproduced between the intensities of transitions from the $1^+_{3,4}$ states onto the states 2^+_1 and 0^+_1. Calculations by the multiparticle shell model fail to describe this branching ratio [11, 20].

The study of γ-decay branching ratios of excited 1^+ states in ^{28}Al is important for analyzing experiments on the muon partial capture by the nucleus ^{28}Si, including correlation experiments [1, 2]. Of much importance can be γ-transitions to 2.201 MeV 1^+ state from high-lying 1^+ states that can be populated in the μ-capture.

In connection to analysis of life times and branching ratios next comment should be given. The procedure used here to take into account the experimental data on strength functions assumes silently that the ground states are well described by multiparticle shell model and the differences between the theoretical and experimental strength functions originate from the wave functions of excited states only. By this assumption the attempt to cover the shortcomings in the description of ground states by the transformation of excite state wave functions is made. The total strength of GT (M1) transitions can be calculated by the averaging of certain two-body operator over the ground state. Therefore the appreciable excess of calculated total transition strength over experimental one indicates the uncompleteness of the description of ground state.

5 Rates of the muon capture for partial allowed transitions

The rate of ordinary capture of muons (OMC) accompanied by an partial transition $J_i \to J_f$ is given [8, 9] by

$$\Lambda^{\rm OMC} = V \sum_{J=|J_f-J_i|}^{J_f+J_i} [M_J^2(-J) + M_J^2(J+1) + M_J^2(-J-1) + M_J^2(J)]. \quad (7)$$

Independent nuclear amplitudes $M_J(\kappa)$ describe the capture of a muon from state $s_{1/2}$ when a neutrino is created in state, characterized by spherical quantum number κ, and the nucleus acquires the total angular momentum J. General formulae for computing these amplitudes are given in [8, 9]. The equation for computing factor V is given in [9]. For the allowed transition $0^+ \to 1^+$, nonzero amplitudes are the following:

$$\begin{aligned} M_1(-1) &= \sqrt{\frac{2}{3}}\Big\{-(G_A - \frac{1}{3}G_P)\,[101] + \frac{\sqrt{2}}{3}\,G_P[121] \\ &\quad -\frac{g_A}{M}[011p] + \sqrt{\frac{2}{3}}\,\frac{g_V}{M}[111p]\Big\}, \\ M_1(2) &= \sqrt{\frac{2}{3}}\Big\{-\frac{\sqrt{2}}{3}G_P\,[101] + (G_A - \frac{2}{3}G_P)\,[121] \\ &\quad +\sqrt{2}\,\frac{g_A}{M}[011p] + \sqrt{\frac{1}{3}}\,\frac{g_V}{M}\,[111p]\Big\}, \end{aligned} \quad (8)$$

which consist of products of effective weak form factors

$$\begin{aligned} G_A &= g_A(q^2) - [g_V(q^2) + g_M(q^2)]\,\frac{E_\nu}{2M}, \\ G_P &= [g_P(q^2) - g_A(q^2) - g_V(q^2) - g_M(q^2)]\,\frac{E_\nu}{2M} \end{aligned} \quad (9)$$

with multiparticle nuclear matrix elements

$$\begin{aligned} [101] &= \sqrt{\frac{3}{4\pi\,(2J_f+1)}}\,\langle J_f \| \sum_{k=1}^{A} \varphi_\mu(r_k)\, j_0(\nu r_k)\, \sigma_k\, t_k^- \| J_i\rangle, \\ [121] &= \sqrt{\frac{3}{4\pi\,(2J_f+1)}} \\ &\quad \cdot\langle J_f \| \sum_{k=1}^{A} \varphi_\mu(r_k)\, j_2(\nu r_k)\, [Y_2(\hat{r}_k)\,\sigma_k]_1\, t_k^- \| J_i\rangle, \\ [111p] &= \sqrt{\frac{3}{4\pi\,(2J_f+1)}} \\ &\quad \cdot\langle J_f \| \sum_{k=1}^{A} \varphi_\mu(r_k)\, j_1(\nu r_k)\, [Y_1(\hat{r}_k)\,\nabla_k]_1\, t_k^- \| J_i\rangle, \\ [011p] &= \sqrt{\frac{1}{4\pi\,(2J_f+1)}} \\ &\quad \cdot\langle J_f \| \sum_{k=1}^{A} \varphi_\mu(r_k)\, j_1(\nu r_k)\, Y_1(\hat{r}_k)\,(\nabla_k, \sigma_k) t_k^- \| J_i\rangle. \end{aligned} \quad (10)$$

Here $\varphi_\mu(r)$ is the wave function of bound muon. The main difficulty in analyzing the partial OMC rates comes from uncertainties in values of these nuclear matrix elements. The most important is, as a rule, the matrix element [101] that in the limit of vanishing neutrino energy is proportional to the GT matrix element $\langle J_f \| \sigma t^- \| J_i\rangle$ known from the theory of β^+-decay. Nuclear matrix elements (10) depend on the neutrino energy and thus depend on the energy of a final state of product nucleus. Therefore, during the calculation of the rates we will transform not the nuclear matrix elements (10) and not the amplitudes (8) but the one-body transition densities (OBTD). The latest are the matrix elements of tensor product of creation and destruction operators between multiparticle wave functions [7, 21]

$$D(\Delta J, \Delta T, \alpha\alpha', f, i) = \frac{\langle J_f, T_f \| [a_\alpha^\dagger \otimes \tilde{a}_{\alpha'}]^{(\Delta J, \Delta T)} \| J_i, T_i\rangle}{\sqrt{(2\Delta J+1)(2\Delta T+1)}}. \quad (11)$$

Matrix elements are reduced with respect to the spin and isospin. According to definition (11), the OBTD are transformed by rule (2) under the action of transformation (1). The calculations of the OMC rates with initial and transformed wave functions of excited states are given in Table 7 together with the values of matrix elements (10). Nuclear matrix elements (10) are calculated using the constant approximation for φ_μ [9]. The amplitudes (8) are obtained with $g_A = -1.263$ and $g_P/g_A = 7.0$. Of the greatest interest are the cases of μ-capture when the 1_3^+ state with energy 2.201 MeV studied in experiments [1, 2] is excited. The calculation with transformed functions shows that this state is most populated in the μ-capture and the capture rate is larger than the total rate of capture on all the other 1^+ states. The matrix elements [101] and [011p] are the largest ones; other matrix elements are considerably smaller. The matrix element [101] is related with the GT matrix element which can be tested in (p,n) reaction. There are no methods of direct verification of the matrix element [011p]. It is shown in recent paper [22] that the g_P/g_A value extracted from the data of correlation experiments [1, 2] is extremely sensitive to theoretical value of calculated ratio $\dfrac{[011p]}{[101]}$.

The experimental values of $B(\mathrm{GT})$ and $B(\mathrm{M1})$ have been used as the parameters of the orthogonal transformation of kind (5). Therefore, we should consider a influence of errors in the values of $B(\mathrm{GT})$ and $B(\mathrm{M1})$ on the obtained results. We estimate the root-mean-square deviation in the computed rates of μ-capture using Eq. (7). The error in $B_k(\mathrm{GT})$ denoted by $\delta B_k(\mathrm{GT})$ influences only the quantity $S = M_1^2(-1) + M_1^2(2)$; thus for every partial transition

$$\delta S \approx \sqrt{\sum_{k=1}^{N}\left[\left(\frac{\partial S}{\partial B_k(\mathrm{GT})}\,\delta B_k(\mathrm{GT})\right)^2 + \left(\frac{\partial S}{\partial B_k(\mathrm{M1})}\,\delta B_k(\mathrm{M1})\right)^2\right]}, \qquad (12)$$

and the error in the capture rate equals, respectively,

$$\delta\Lambda = V\delta S.$$

Partial derivatives in (12) can be calculated either analytically or by the finite-difference approximation

$$\frac{df}{dx} \approx \frac{f(x+h) - f(x-h)}{2h}.$$

We have used this method for computing the errors displayed in Table 7.

6 Conclusions

The method how one can utilize existing experimental information concerning GT and M1 strength functions in the calculations of the rates of nuclear muon capture is proposed in this paper. The method is reduced to the orthogonal transformation in the subspace of wave functions of excited states. The transformation parameters should be chosen so that the strength functions of GT and M1 transitions calculated with transformed wave functions coincide in shape with experimental GT and M1 strength function. It is important that this method does not require any modifications of transition operators because GT and M1 transition operators differ from the ones of effective Hamiltonian of nuclear muon capture. The transformation is created as a product of the matrices of the reflection in the plane. All calculations are made within the many-particle shell model. The numerical calculations are carried out for isovector transitions in the nuclei with $A = 28$. The GT and M1 strength functions in ^{28}Si are considered; for 1^+ states in ^{28}Al the life times and γ-decay branching ratios are calculated. It is shown that method proposed in this paper allows to describe correctly the branching ratio of 1^+ state with energy 2.201 MeV in ^{28}Al for the first time.

It is shown that used transformation of wave functions changes considerably the distribution of partial allowed muon capture rates in ^{28}Si over 1^+ states of product nucleus ^{28}Al compared to results obtained with eigenfunctions of Hamiltonian of many-particle shell model.

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Table 1: Values of g_P/g_A, obtained by comparison the measured angular correlation coefficients to the theoretical ones.

Calculations (References)	Experiment	
	[1]	[2]
[3]	3.4 ± 1.0	5.3 ± 2.0
[4]	2.0 ± 1.6	4.2 ± 2.5
[5]	-2.8 ± 1.6	0.0 ± 3.2

Table 2: Properties of 1^+ isovector states in nuclei with $A = 28$. GT and M1 strength functions, calculated within sd-shell model; the information from the reactions $^{28}\mathrm{Si}(p,n)^{28}\mathrm{P}$ and $^{28}\mathrm{Si}(e,e')^{28}\mathrm{Si}$. $B(\mathrm{GT})$ and $B(\mathrm{M1})$ – reduced probabilities of the GT and M1 transitions. $b(\mathrm{GT})$ and $b(\mathrm{M1})$ – transition amplitudes, normalized by conditions $B(\mathrm{GT}) = b^2(\mathrm{GT})$ and $B(\mathrm{M1}) = b^2(\mathrm{M1})$.

Calculation results										
k	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$E_k - E_1$	0.0	0.38	0.71	1.83	2.16	2.56	3.56	3.80	4.21	4.70
E_k	10.81	11.19	11.52	12.64	12.97	13.37	14.37	14.61	15.02	15.51
$b(\mathrm{GT})$	0.822	0.262	0.862	−0.783	0.014	−0.355	0.798	−0.574	−0.426	0.342
$b(\mathrm{M1})$	1.232	0.733	1.750	−1.178	0.162	−0.091	0.957	−0.712	0.629	0.538
$B(\mathrm{GT})$	0.676	0.069	0.744	0.613	0.000	0.126	0.637	0.330	0.182	0.117
$B(\mathrm{M1})$	1.518	0.538	3.064	1.387	0.026	0.008	0.917	0.507	0.395	0.290
The experimental distribution of GT transition strength obtained in $^{28}\mathrm{Si}(p,n)^{28}\mathrm{P}$ [13]										
$E_k - E_1$	0.0	0.34	0.85	1.69		2.62	3.34	3.77	4.30	4.64
E_k	1.25	1.59	2.10	2.94		3.87	4.59	5.02	5.55	5.91
$B(\mathrm{GT})$	0.198	0.109	0.956	0.146		0.163	0.410	0.137	0.092	0.090
err.	0.002	0.002	0.005	0.003		0.002	0.004	0.041	0.004	0.003
The distribution of M1 transition strength obtained in $^{28}\mathrm{Si}(e,e')^{28}\mathrm{Si}$ [12]										
$E_k - E_1$	0.0	0.26	0.80	1.69			3.39		4.50	4.86
E_k	10.64	10.90	11.45	12.33			14.03		15.15	15.50
$B(\mathrm{M1})$	0.30	0.90	4.42	0.87			0.37		0.23	0.26
err.	0.04	0.02	0.20	0.06			0.02		0.02	0.03

Table 3: Transformation matrix.

	(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)	(10)
(1)	0.952	0.024	-0.293	-0.013	-0.080	-0.032	-0.003	0.006	0.001
(2)	0.023	0.988	0.146	0.004	0.030	0.014	-0.005	-0.003	-0.001
(3)	-0.293	0.146	-0.885	0.021	-0.153	-0.149	0.248	0.042	0.011
(4)	-0.013	0.004	0.021	0.903	-0.340	-0.050	-0.258	-0.002	-0.005
(6)	-0.080	0.030	-0.153	-0.340	-0.216	-0.194	-0.877	-0.004	-0.018
(7)	-0.032	0.014	-0.150	-0.050	-0.194	0.961	-0.116	0.002	-0.002
(8)	-0.003	-0.005	0.248	-0.258	-0.877	-0.116	0.299	-0.011	-0.015
(9)	0.006	-0.003	0.043	-0.002	-0.004	0.002	-0.011	0.999	-0.000
(10)	0.001	-0.001	0.011	-0.005	-0.018	-0.002	-0.015	-0.000	1.000

Table 4: GT and M1 transition strength distributions in $^{28}\mathrm{Si}$ calculated with transformed wave functions of excited states.

k :	(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)	(10)
$B(\mathrm{GT})$	0.300	0.165	1.451	0.222	0.247	0.622	0.208	0.140	0.137
$B(\mathrm{M1})$	0.445	1.044	4.641	0.764	0.258	0.620	0.234	0.286	0.330

Table 5: Life-times of 1^+ states in ^{28}Al (10^{-15} s).

k	E_k	(a)	(b)	E'_k	(a′)	(b′)	(c)	expt.
1	1.373	239	152	10.810	184	117	235	320 ± 50
2	1.620	465	531	11.192	285	279	590	120 ± 60
3	2.201	66	44	11.519	70	48	65	65 ± 35
4	3.105	21	12	12.643	17	10	22	
5	3.542	9.7	9.8	12.970	9.1	9.2	7.9	
6	4.115	0.94	7.0	13.771	1.1	8.6	0.9	
7	4.846	0.69	0.88	14.374	0.64	0.80	0.7	
8	5.017	1.2	0.48	14.605	1.0	0.41		
9	5.435	0.94	0.93	15.024	0.77	0.76		
10	5.919	1.8	1.8	15.507	1.5	1.5		

(a) – Calculated with Wildenthal Hamiltonian [10];
(b) – calculated with transformed wave functions of 1^+ states;
(c) – results of calculations of ref. [11], utilizing Hamiltonian of ref. [10];
expt. – experimental data. Cited from ref. [11]
Known experimental values of excitation energy of 1^+ states have been used in (a,b) calculations. The excitation energies calculated with sd-shell model Hamiltonian of ref. [10] have been used in the calculations (a′, b′). In that case all excitation energies are measured from the ^{28}Si ground state.

Table 6: Branching rations for γ-decay of 1^+ states in ^{28}Al.

J^π_f:		$3^+_{\text{g.s.}}$	2^+_1	0^+_1	3^+_1	1^+_1	1^+_2	2^+_2	2^+_3	1^+_3	2^+_4	0^+_2	1^+_4
$E_i \setminus E_f$:		0.000	0.031	0.972	1.014	1.372	1.620	1.623	2.139	2.201	2.486	3.012	3.105
1.372	(a)	4.4	73.2	22.4									
	(b)	2.9	75.8	21.3									
	(c)	4.4	74	22									
	expt.	4.7 *3*	55 *1*	40 *1*									
1.620	(a)	3.8	77.6	18.0		0.5							
	(b)	4.2	58.7	36.7		0.3							
	(c)	4.7	93	2.1									
	expt.	6	92	< 2									
2.201	(a)	0.2	2.1	80.4		14.5		2.6					
	(b)	0.0	63.3	35.2		1.4							
	(c)	0.2	2.2	80.1		14		2.7					
	expt.	< 6	79 *3*	16 *2*		< 3		< 5					
3.105	(a)	0.5	41.4	30.5	0.1	0.4	3.7	17.7	1.3	0.3	4.1		
	(b)	0.2	76.1	2.6	0.0	1.2	4.9	7.4	3.7	0.1	3.7		
	(c)	0.5	43	32	0.2	0.4	3.8	18	1.4	0.3			
	expt.		75 *3*							25 *3*			
3.542	(a)	1.6	30.2	25.7		11.9	1.5	22.2	2.6	0.0	4.1		
	(b)	1.6	30.4	25.8		11.1	1.6	22.3	2.7	0.1	4.2		
4.115	(a)	0.0	52.3	36.0		3.2	2.7	0.1	4.4	0.1	0.6		
	(b)	0.1	9.9	2.1		30.0	0.1	18.5	26.1	5.0	6.3		

Table 6: continued

J_f^π:		$3^+_{g.s.}$	2^+_1	0^+_1	3^+_1	1^+_1	1^+_2	2^+_2	2^+_3	1^+_3	2^+_4	0^+_2	1^+_4
$E_i \setminus E_f$:		0.000	0.031	0.972	1.014	1.372	1.620	1.623	2.139	2.201	2.486	3.012	3.105
4.846	(a)	0.1	77.0	1.2		4.9	4.5	2.1	1.2	5.1	0.2	2.0	1.1
	(b)	0.2	71.8	0.3		4.6	5.3	4.3	0.3	8.2	0.8	3.1	0.4
5.017	(a)	0.3	36.5	29.0		7.9	0.8	4.9	3.4	0.0	12.1	1.3	0.3
	(b)	0.1	56.4	36.1		0.4	0.9	0.7	0.3	1.4	0.2	0.0	2.3
5.435	(a)	1.1	1.1	14.0	0.3	11.0	0.0	61.5	0.2	0.3	4.0	0.4	0.6
	(b)	1.1	1.0	12.0	0.3	8.2	0.3	61.7	0.6	4.0	4.0	0.5	0.7
5.919	(a)	13.2	15.7			34.2	6.8	3.6	10.6	0.2	0.2	3.3	0.4
	(b)	12.0	15.1			36.1	6.3	3.7	10.5	0.5	0.3	3.5	0.2

(a) – Calculated with wave functions of Hamiltonian [10];
(b) – calculated with transformed wave functions;
(c) – results of calculations of ref. [11], with Hamiltonian [10];
expt. – experimental data, cited according to ref. [11].
Known experimental values of energy of excited states are used in calculations.

Table 7: ^{28}Si. The partial OMC rates (in $10^3\,s^{-1}$) and nuclear matrix elements, calculated with initial (a) and transformed (b) one-body transition densities.

		k - number of excited 1^+ state									
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Λ_k^{OMC}	(a)	29.87	3.13	34.06	26.09	0.02	3.05	20.59	11.49	8.42	3.54
	(b)	12.81	7.58	63.57	11.17		8.45	18.69	7.27	6.56	4.17
err.		0.18	0.16	2.36	0.46		0.37	0.23	2.13	0.22	0.14
[101]	(a)	0.039	0.012	0.041	-0.037	0.001	-0.017	0.039	-0.028	-0.021	0.017
	(b)	0.027	0.019	-0.057	-0.022		0.024	0.039	0.023	-0.018	0.018
[121]	(a)	-0.006	0.000	-0.005	0.006	0.001	-0.002	0.001	0.001	0.006	0.004
	(b)	-0.004	-0.001	0.007	0.005		-0.002	0.002	-0.001	0.005	0.004
[111p]	(a)	-0.004	0.011	0.012	0.003	0.005	0.017	-0.012	0.008	0.002	-0.001
	(b)	-0.008	0.012	-0.007	-0.004		-0.010	-0.017	-0.009	0.002	0.001
[011p]	(a)	-0.016	-0.007	-0.017	0.017	0.001	0.007	-0.019	0.015	0.012	-0.005
	(b)	-0.011	-0.010	0.025	0.010		-0.013	-0.020	-0.008	0.011	-0.005

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