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REVIEW OF INELASTIC LOSSES OF UCN AND QUANTUM MECHANICS OF THE DE BROGLIE WAVE PACKET

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1 Introduction

The anomalous loss coefficient $\eta = 3 \cdot 10^{-5}$ of UCN in the Be bottle [1], which is 2 orders of magnitude higher than the theoretical one: $\approx 3 \cdot 10^{-7}$, requires an explanation. We tried to explain this phenomenon by properties of the neutron itself, i.e., by properties of its wave-function structure. Thus our goal is to present here the hypothesis and to show experimental results aimed at its verification. However, before doing that, it is useful to briefly overview all the possible inelastic scattering processes leading to UCN losses for to see, whether it is necessary indeed to devise something extraordinary to explain the anomaly.

2 Review of all the inelastic loss processes

First of all we should remind the definition of the loss coefficient. Reflection of UCN of energy k^2, where k is a wave-number, from a wall with the potential $u = 4\pi N_0 b$, where b is the coherent scattering amplitude of the wall nuclei, and N_0 is the atomic density, is described by a reflection amplitude R. For total reflections ($k^2 < u$) in absence of losses $|R| = 1$. Because of losses the $|R| < 1$, and a loss coefficient is defined as $\mu = 1 - |R|^2$. The coefficient μ is proportional to the reduced loss coefficient $\eta = \mathrm{Im}u/\mathrm{Re}u$, which is equal to the ratio of the imaginary and real parts of the coherent scattering amplitude b_c: $\eta = \mathrm{Im}b_c/\mathrm{Re}b_c$, where $\mathrm{Im}b_c$ according to the optical theorem is: $\mathrm{Im}b_c = k\sigma_l/4\pi$, and σ_l is the total loss cross section, which includes absorption and many inelastic scattering cross sections. The particular inelastic process i gives its partial contribution to the loss coefficient $\eta_i = k\sigma_i/4\pi\mathrm{Re}b_c$. In the following we shall write in the denominator b_c instead of $\mathrm{Re}b_c$, because $\mathrm{Im}b_c \ll \mathrm{Re}b_c$.

In general, the cross section of inelastic scattering, and the related loss coefficient are representable in the form:

$$\sigma = 4\pi|b'|^2 k_{eff}/k_0 \to \eta = |b'|^2 k_{eff}/b_c \equiv |b'/b_c|^2 b_c k_{eff}, \tag{1}$$

where k_{eff} is an effective wave-vector of the neutron heated via considered inelastic process, and b' denotes a coherent, incoherent or magnetic scattering amplitude.

In consideration of all the inelastic processes it is necessary to remember two experimental facts:

1. the recent experiments in ILL [2] have shown that with probability of the order 10^{-6} there is some process of UCN stepwise small heating up to the energy which is approximately twice of the primary one,

2. the experiments [3] had shown that there were no continuous broadening of the spectrum, i.e. there were no heating by $2 \cdot 10^{-12}$ eV at a single collision with the walls.

Both facts have nothing to do with the UCN anomaly. Especially the first one because it has been observed in bottles with large losses, and because its magnitude is smaller than the anomaly. However it is useful to take both facts into account in estimation of probabilities for different inelastic processes.

With the relations (1) we can easily find the probability of stepwise small heating. Indeed, if $b' \approx b_c$ and $k_{eff} \approx k_{lim} \equiv \sqrt{u}$, the partial loss coefficient becomes $\eta = b_c k_{lim}$, which, in particular for Cu walls, is approximately $7 \cdot 10^{-7}$ in pretty good agreement with the observations.

2.1 Phonons

2.1.1 The coherent phonon cross section

$$\sigma_{c,inel}=|b_c|^2\sum_\tau\int\left[\frac{m}{M}\frac{\kappa^2}{2\omega_q}\right]d^3q\delta(q-\kappa-\tau)n(\omega_q/T)\delta(\omega-\omega_q)\frac{d^3k}{k_0},$$

where $\kappa=k_0-k$ is a momentum transferred, τ is a vector of the reciprocal lattice, $\omega=(k_0^2-k^2)/2=-\kappa^2/2+\kappa k_0$ is energy transferred, ω_q is the phonon energy, which for small q can be represented as cq with c being sound velocity, $n(x)=1/(\exp x-1)$ is the Bose-Einstein factor, T is temperature, m is the neutron mass, and M is the mass of the wall nuclei. Here and in the following we use the units $\hbar=m=1$.

We can neglect small vectors k_0 and q in the first δ-function. Then, after integration over k, we obtain

$$\sigma_{c,inel}=|b_c|^2\sum_\tau\int\frac{m}{M}n(\omega_q/T)\delta(\tau^2/2-\omega_q)\frac{d^3q}{k_0}=4\pi|b_c|^2\sum_\tau\frac{m}{M}n(\tau^2/2T)\frac{\tau^4}{4c^3k_0}.$$

To estimate the magnitude we can replace the sum over all τ by the integral:

$$\sigma_{c,inel}=4\pi|b_c|^2\int^\infty\frac{d^3\tau}{(2\pi)^3N_0}\frac{m}{M}n(\tau^2/k_T^2)\frac{\tau^4}{4c^3k_0}=4\pi|b_c|^2\int_0^\infty\frac{2\pi k_T^7\sqrt{x}dx}{(2\pi)^3N_0}\frac{m}{M}n(x)\frac{x^2}{4c^3k_0},\qquad(2)$$

where we integrated over angles, used the relation $T=k_T^2/2$, and changed variables $(\tau/k_T)^2=x$. Integration over x gives

$$\sigma_{c,inel}=4\pi|b_c|^2\frac{k_T}{k_0}F,\quad F=\frac{\pi}{2}\Gamma(7/2)\zeta(7/2)\frac{m}{M}\left(\frac{v_T}{c}\right)^3\left(\frac{a}{\lambda_T}\right)^3,\qquad(3)$$

where $\Gamma(n)$ and $\zeta(n)$ are Euler and Riemann functions: $\Gamma(7/2)=15\sqrt{\pi}/8=3.32$, $\zeta(7/2)=1.127$, $\Gamma(7/2)\zeta(7/2)\approx3.75$, and we used the relations, $k_T=2\pi/\lambda_T$, $k_T/c=v_T/c$, where v_T is the speed of thermal neutrons. All the magnitudes in (3) can be now used with their natural dimensionalities.

From (3) it follows that the coherent 1-phonon heating contributes to loss coefficient $\eta_{c,inel}=b_ck_TF$. For Be at $T=300$ K we have $v_T=1200$ m/s, $\lambda_T=1.8$ Å, $b_{Be}k_T=2.7\cdot10^{-4}$, and $F\approx1.1\cdot10^{-3}$, where for c we used the sound velocity of transverse vibrations: $c=8.8$ km/s. Thus $\eta_{c,inel}=3\cdot10^{-7}$, and it quickly decreases with temperature as $T^{7/2}$.

The smallest wavelength of the neutron after inelastic coherent scattering is $\lambda\approx a$ — interatomic distance. Thus the coherent phonon process does not give small energy heating.

2.2 Incoherent phonon cross section

$$\sigma_{inc,inel}=|b_{inc}|^2\int\frac{m}{M}\frac{\kappa^2}{2\omega_q}n(\omega_q/T)\frac{3\omega_q^2d\omega_q}{\omega_D^3}\delta(\omega_q-\omega)\frac{d^3k}{k_0}.$$

We can approximate $\kappa\approx k$, and after integration over k we obtain

$$\sigma_{inc,inel}=4\pi|b_{inc}|^2\int_0^\infty\frac{m}{M}n(\omega_q/T)\frac{3\omega_q^2d\omega_q}{\omega_D^3}\frac{\sqrt{2\omega_q}}{k_0},$$

and after integration over ω_q we obtain

$$=4\pi|b_{inc}|^2\frac{k_T}{k_0}F,\quad F=3\Gamma(7/2)\zeta(7/2)(m/M)(T/T_D)^3,$$

where T_D is the Debye temperature.

For Be the incoherent amplitude b_{inc} is very small, however for estimation we can treat all the inelastic scattering in the incoherent approximation and suppose that $b_{inc}=b_c$, then at room temperature, if we take for Be $T_D=1200$ K, we get $F\approx0.02$, thus $\eta_{inc,inel}\approx5.4\cdot10^{-6}$.

Heating to small energies $\omega_1\approx k_0^2/2$ is less by the factor $(k_0/k_T)^5\approx10^{-12}$, i.e., it is negligible.

Two phonon process has an additional factor F_2, which for emission and absorption is equal to $F_2=(m/M)(E_0/\omega_D)^3<10^{-16}$, and for 2 absorptions is equal to $F_2=(m/M)(T/T_D)^3\approx0.002$ (for Be).

2.3 Surface waves

The surface waves are important for solid and liquid, like fombline oil, walls. To find the heating by surface waves we solve the Schrödinger equation

$$[i\partial_t+\Delta-u(\boldsymbol{r},t)]\psi(\boldsymbol{r},t)=0,\qquad(4)$$

with the potential $u=u_0\vartheta(z>\zeta_0\cos(\boldsymbol{q}\boldsymbol{r}_\parallel-\Omega t))$, where we introduced the Heviside function $\vartheta(x)$ which is equal to 1 or 0 when inequality in its argument is satisfied or not. For small amplitude ζ_0 of vibrations we can use the linear expansion: $u(\boldsymbol{r},t)\approx u_0\theta(z>0)+u_0\zeta_0\cos(\boldsymbol{q}\boldsymbol{r}_\parallel-\Omega t)\delta(z)$, and treat the second term as a perturbation for unperturbed potential $u_0\vartheta(z>0)$.

The scheme for solving the equation is the following: we go to the reference frame moving along the surface wave with the speed $c_R=\Omega/q$, which is the Raleigh speed. Then we find the diffraction from a frozen wavy relief. After that we transform the result back to laboratory frame and average it over the spectrum.

After transmission to the moving reference frame we solve the stationary equation

$$[E+\Delta-u_0\vartheta(z>0)-u_0\zeta_0\cos(\boldsymbol{q}\boldsymbol{r}_\parallel)\delta(z)]\psi(\boldsymbol{r},E)=0,\qquad(5)$$

where $E=(\boldsymbol{k}_0+\boldsymbol{c}_R)^2$. The solution by perturbation theory is

$$\psi=\psi_0+\int G(\boldsymbol{r},\boldsymbol{r}',E)\delta u(\boldsymbol{r}')\psi_0(\boldsymbol{r}',E)d^3r',\qquad(6)$$

where G is the Green function of the unperturbed equation

$$[E+\Delta-u_0\vartheta(z>0)]G(\boldsymbol{r},\boldsymbol{r}',E)=\delta(\boldsymbol{r}-\boldsymbol{r}'),\qquad(7)$$

which is represented in the form

$$G(\boldsymbol{r},\boldsymbol{r}',E)=\frac{1}{(2\pi)^2}\int d^2p_\parallel\exp(i[\boldsymbol{r}-\boldsymbol{r}']\boldsymbol{p}_\parallel)G(p_\perp,z,z').\qquad(8)$$

Here $p_\perp=\sqrt{E-p_\parallel^2}$ and $G(p_\perp,z,z')$ under the integral is the one dimensional Green function, which is the solution of the Schrödinger equation

$$[p_\perp^2+d^2/dz^2-u_0\vartheta(z>0)]G(p_\perp,z,z')=\delta(z-z'),\qquad(9)$$

and is equal to

$$G(p_\perp, z, z') = \frac{1}{2ip_\perp}[\vartheta(z > z')\psi_1(p_\perp, z)\psi_2(p_\perp, z') + \vartheta(z' > z)\psi_1(p_\perp, z')\psi_2(p_\perp, z)],$$

where $\psi_{1,2}(p_\perp, z)$ are the two linearly independent solutions

$$\psi_1(p, z) = \vartheta(z < 0)[\exp(ipz) + \rho(p)\exp(-ipz)] + \vartheta(z > 0)\tau(p)\exp(ip'z) \quad (10)$$

$$\psi_2(p, z) = \vartheta(z < 0)\exp(-ipz) + \vartheta(z > 0)[\exp(-ip'z) - \rho(p)\exp(ipz)]/\tau(p') \quad (11)$$

of the one-dimensional Schrödinger equation

$$[p^2 + d^2/dz^2 - u_0\vartheta(z > 0)]\psi_{1,2}(p, z) = 0. \quad (12)$$

In the expressions (10,11) we used the notations $p' = \sqrt{p^2 - u_0}$, $\rho(p) = (p - p')/(p + p')$, and $\tau(p) = 2p/(p + p')$.

Substitution of the primary wave function

$$\psi_0(\boldsymbol{r}, t) = \psi_1(k_{0\perp}, z)\exp(i[\boldsymbol{k}_0 + \boldsymbol{c}_R]\boldsymbol{r}_\parallel)$$

into (6), gives the perturbed part of the wave function for diffraction

$$\delta\psi(\boldsymbol{r}, t) = \exp(i\boldsymbol{k}\boldsymbol{r}_\parallel)f(\boldsymbol{k}_0 \to \boldsymbol{k})[\vartheta(z < 0)\exp(-ik_\perp z) + \vartheta(z > 0)\exp(ik'_\perp z)],$$

where $\boldsymbol{k}_\parallel = -\boldsymbol{q} + \boldsymbol{k}_{0\parallel} + \boldsymbol{c}_R$, $k_\perp = \sqrt{(k_0 + c_R)^2 - (-\boldsymbol{q} + \boldsymbol{k}_{0\parallel} + \boldsymbol{c}_R)^2} \approx \sqrt{2c_R q}$,

$$f(\boldsymbol{k}_0 \to \boldsymbol{k}) = u_0\xi_0[1 + \rho(k_\perp)][1 + \rho(k_{0\perp})]/2k_\perp = -2ik_{0\perp}\xi_0\frac{k_{0\perp} - k'_{0\perp}}{k_\perp + k'_\perp}.$$

The probability of diffraction is

$$W(\boldsymbol{k}_0 \to \boldsymbol{k}) = \frac{k_\perp}{k_{0\perp}}|f(\boldsymbol{k}_0 \to \boldsymbol{k})|^2 = 4k_\perp k_{0\perp}|\xi\frac{k_{0\perp} - k'_{0\perp}}{k_\perp + k'_\perp}|^2 \approx u_0|\xi|^2\frac{k_{0\perp}}{k_\perp}, \quad (13)$$

where $k_{0\perp}$, $k_\perp$ are the normal to the reflecting surface components of primary and scattered waves, and $k_\perp \approx \sqrt{2\Omega}$. Now we suppose, as in the phonon case, that $|\xi|^2 = 1/2M\Omega$, and the spectrum of the surface waves is the 2-dimensional Debye spectrum. Then after averaging over the spectrum we get

$$W = \eta_{sw} \approx \frac{m}{M}\int\limits_{u_0/2}^{\infty}\frac{2\Omega d\Omega}{\Omega_{sD}^2}\frac{u_0^{3/2}}{2k_\perp\Omega}n(\Omega/T) \approx \frac{m}{M}\frac{Tu_0}{T_{sD}^2} < 10^{-6}. \quad (14)$$

The main contribution to the integral comes from low Ω, where $n(\Omega/T) \approx T/\Omega$ and integral diverges. To avoid the divergence the integration was performed down to $2\Omega = u_0$. For lower Ω expression (13) should be approximated by $4k_{0\perp}k_\perp\xi_0^2$ which provides the convergence for the integral.

The last number in (14) is correct for Be at room temperature, if we take surface Debye temperature $T_{sD} = 0.8T_{D,Be} \approx 900$ K.

2.4 Liquids

In liquids there are no pure elastic scattering. If we don't take into account the optical potential of the liquid and treat the quasi-elastic scattering in the same way as for thermal neutrons, then the cross section is

$$\sigma_{qe} = \frac{2}{k_0}|b_c|^2\int d^3k\frac{\kappa^2 D}{(k^2 - k_0^2)^2 + (\kappa^2 D)^2}.$$

The integration over the quasi-elastic peak gives the magnitude of the order $4\pi|b_c|^2$. We can suppose that all this cross section means losses. Then its contribution to loss coefficient is $\eta_{qe} \approx k_0 b_c < 10^{-6}$.

The formula is correct not only for liquids, but also for surfaces contaminated with hydrogen [7], if the hydrogen atoms are freely diffusing along the surface. In that case the magnitude of the effect is $\eta_{H,dif} = C(\sigma_H/\sigma_c)b_c k_0$, where C and σ_H are the hydrogen concentration and cross section. Thus there can be an enhancement factor $F = C(\sigma_H/\sigma_c)$, which is important for sufficiently high C. However diffusion processes should broaden UCN spectrum in the bottles, and in experiment the heating was seen to be a step wise.

2.5 Spin waves

Spin waves can be important for ferromagnetic walls. The spin wave cross section in the Heisenberg model is

$$\sigma_{spin} = \frac{2}{k_0}r_0^2\sum_\tau\int d^3k d^3q|F(\kappa)|^2 Sn(\omega_q/k_T^2)\delta(\boldsymbol{k} - \boldsymbol{q} - \boldsymbol{\tau} - \boldsymbol{k}_0)\delta(k^2 - k_0^2 - \omega_\kappa),$$

where $r_0 = \gamma e^2/m_e c^2$, $\boldsymbol{\tau}$ is the vector of the reciprocal lattice, S is the spin of the atom, and $F(\kappa)$ is form factor, which in our case can be approximated by 1. Spin waves in zero external magnetic field are characterized by energy $\omega_\kappa = D\kappa^2$ with the constant D, which can be represented as m/m_{eff}, with $m_{eff} \approx 0.01m$ of the neutron mass.

For $\tau = 0$ the integral is zero. For $\tau \neq 0$ we can neglect q and k_0 in the first δ-function. Thus the integration over k, q gives

$$\sigma_{spin} = \frac{2S}{k_0}r_0^2\sum_\tau\int d^3q n(\omega_q/k_T^2)\delta(\tau^2 - Dq^2) = 4\pi r_0^2 S\left(\frac{m_{eff}}{m}\right)^{3/2}\sum_\tau n(\tau^2/k_T^2)\frac{\tau}{k_0}.$$

Approximation of the sum over τ by integral gives

$$\sigma_{spin} = 4\pi r_0^2 S\left(\frac{m_{eff}}{m}\right)^{3/2}\int\frac{d^3\tau}{(2\pi)^3 N_0}n(\tau^2/k_T^2)\frac{\tau}{k_0} =$$

$$4\pi r_0^2 S\left(\frac{m_{eff}}{m}\right)^{3/2}\frac{2\pi k_T^4}{(2\pi)^3 k_0 N_0}\zeta(2)\Gamma(2) = 4\pi r_0^2\frac{k_t}{k_0}F, \quad F = 2\pi\frac{\pi^2}{6}\left(\frac{m_{eff}}{m}\right)^{3/2}\left(\frac{a}{\lambda_t}\right)^3,$$

where λ_T is the wave length of thermal neutrons, a is the interatomic distance. The magnitude of F is near 10^{-2} for room temperatures, therefore η_{sw} is nearly the same as for phonons.

2.6 Gas scattering

There are some losses of UCN due to scattering on residual gas. The loss coefficient due to these losses can be estimated as $\eta_{gas} = n_0\sigma_g L$, where n_0 is the gas density, and

$$\sigma_{gas} = 4\pi\frac{|b|^2}{(1 + m/M)^2}\sqrt{\frac{m}{M}}\sqrt{\frac{4}{\pi}\frac{k_T}{k_0}}. \quad (15)$$

We suppose that $b_{gas} = b_c$ of the walls. Thus

$$\eta_{gas} = b_c k_T F, \quad F \approx 2b_c n_0 L\lambda_0\sqrt{\frac{m}{M}}.$$

For $F = 10^{-2}$, $b_c = 10^{-12}$ cm, $M = 25m$, and $L = 10$ cm gas density should be

$$n_0 = F\sqrt{\frac{M}{m}}\frac{1}{2b_c L\lambda_0} = 5\cdot 10^{14}\,\mathrm{cm}^{-3},$$

which is equivalent to the gas pressure at ambient temperature $\approx 10^{-2}$ Torr. Usually residual gas pressure in UCN traps is considerably lower.

2.6.1 Gas model of the walls

We can imagine the wall to be composed of gas molecules with high density. Then from (15) it follows $\eta_{gas,w} \approx b_c k_T\sqrt{m/M}$. Since $b_c k_T \approx 3.5\cdot 10^{-4}$, then scattering by gas with mass $M = 25m$ is near $7\cdot 10^{-5}$.

2.7 Cluster model

2.7.1 The wall of clusters

If we consider the substance to be a collection of gas clusters, every one containing n atoms, then the elastic scattering cross section of a cluster is proportional to $(b_c n)^2$, their density is N_0/n, and inelastic scattering for a gas cluster contains the factor $\sqrt{m/nM}$ as it follows from (15). In that model the effective η_{cl} is $\eta_n = \sqrt{n}\eta_{gas}$. The clusters have thermal velocity $v_{T,n} = v_{T,1}/\sqrt{n}$. Thus, for neutrons to acquire the velocity 5 m/s it is necessary to have clusters of $n \geq 10^4$ nucleons. They have dimension less than the neutron wavelength, so the scattering on them can be considered in s-wave approximation. The value of η_n can be sufficiently large to explain any magnitude of observed η. However this model seems not to be plausible.

2.7.2 Gas of clusters

The more reasonable is a suggestion that the storage volume contains the dust consisting of such clusters. The losses of neutrons in such a gas of clusters can be represented $\eta_{cl} = C\sigma_{cl}L$, where L is the bottle dimension. If clusters are made of n atoms with the same amplitude b as the walls, then $\eta_{cl} = 4\pi b^2 C n^{3/2} L(k_T/k_0)\sqrt{m/M} = Fbk_T$, where $F = 2Cn^{3/2}Lb\lambda_0\sqrt{m/M}$. Thus, for a given F we should have the concentration of clusters in the volume:

$$C = Fk_0\sqrt{M/m}/4\pi bLn^{3/2}.$$

For $F = 10^{-2}$, $M/m \approx 25$, $L = 10$ cm, $b = 10^{-12}$ cm, and $n = 10^4$ we obtain $C \approx 10^9$. The pressure of such a gas is nearly 10^{-7} Torr.

2.7.3 Gas of large clusters

In the case of large clusters with dimensions $d = 100$ Å UCN are totally reflected from them and in average increase their energy by w^2 after every collision, where w is the thermal cluster velocity. We can again use the relation $\eta_{cl} = C\sigma_{cl}L$ with $\sigma_{cl} = \pi d^2$. Thus the density of such a gas should be $C = \eta_{cl}/\sigma_{cl}L$. For $d = 100$ Å, $\eta_{cl} = 10^{-5}$, $L = 10$ cm, and we have $C = 10^6/\mathrm{cm}^3$. Such clusters contain 10^5 atoms and their velocity is near 1 m/s.

2.7.4 Large clusters on wall's surface

It is also possible to imagine the dust with dimensions $d \approx 100$ Å on the wall surfaces. In that case neutron can be totally reflected from a dust particle, and probability of inelastic scattering

at every collision with the wall can be estimated as a probability of interaction with the dust particle. Probability of inelastic scattering is $\eta_{dust} = C_s d^2$, where C_s is two dimensional density of the dust particles. If $\eta_{dust} = 10^{-5}$, then $C = 10^7$ particles per cm^2. The thermal velocity of such particles is near 1 m/s, thus almost every collision with a dust particle heats the neutron to limiting energy.

2.8 Acoustics

Acoustical vibrations have classical effect starting from frequencies 10^9 Hz, because at such frequencies the most important is the relative motion of the wall's surface with respect to the neutron.

2.8.1 Ultrasound

Let us suppose that the wall is trembling with the amplitude A and frequency ω. If $T = 2\pi/\omega > 1/\sqrt{u}v \approx 2\cdot 10^{-9}$ s, where v is neutron velocity, then interaction with the wall can be treated as classical, and the neutron energy after collision is in average equal to $E =< (v + 2w\cos(\omega t))^2 > \approx E_0 + 2w^2$, where $w = A\omega$. If $w = 1$ m/s, and $\omega \equiv \omega_0 \approx 10^9$ rad/s, then A should be ≈ 10 Å.

Let us suppose, that $A(\omega) = A_0\omega_0/\omega$, and that neutron is heated at every collision with probability 10^{-5}. Suppose the spectrum to be $g(\omega) = 3\omega^2/\omega_1^3$. Then we should have

$$(\omega_0/\omega_1)^3 = 10^{-5} \to \omega_1 \approx 20\omega_0.$$

The pressure of the sound is

$$p_{ac} = \rho\int_0^{\omega_1} A^2(\omega)\omega^2 g(\omega)d\omega = \rho A_0^2\omega_0^2 = 1\mathrm{N/cm}^2,$$

where ρ — the density of substance was taken 10 g/cm^3. This energy is 10^{-4} of the density of the thermal energy.

The intensity of the sound is measured in deciBells: $I = 10\log(P/P_0)$, where P_0 is the reference pressure: $p_0 = 10^{-16}\mathrm{N/cm}^2$. Thus the intensity of our vibrations is 160 dB. However the frequencies 10^9 are too high to be considered as acoustical. So to estimate how much of acoustical energy is contained in pure acoustical vibrations we should limit ourselves to frequencies $\omega_2 \approx 10^5$. This range contains only $(\omega_2/\omega_1) \approx 10^{-16}$ of the full energy, thus the fraction of total supersound energy in acoustical range is of the order 0 dB.

2.8.2 Acoustical sound

The real acoustical sound is limited in the range up to 10^4 Hz, and the motion of the interface with such frequencies should be considered classically. If amplitude of the motion is A, the velocity of the interface is $w = A\omega$, and neutron acquires an energy $\propto w^2$ at every collision with the walls. If neutron should survive 10^{-5} collisions, it should be $w^2 = 10^{-5}u \approx 10^{-12}$ eV. In that case the UCN spectrum in the trap should continuously broaden, which was not observed in experiment [3].

3 Quantum mechanics of the de Broglie wave packet

In previous section we considered all possible channels for UCN heating. The presence of ultrasound seems unreasonable. The hypothesis of fine dust or cluster gas is not yet checked, however it may be rejected too, so we should be ready to seek for another explanation of the UCN anomaly.

One hypothesis was formulated in [4]. We suggested there that the explanation is related to structure of the wave function of the free neutron, which is described by the de Broglie wave-packet [5, 4]:

$$\psi(\boldsymbol{r},t) = \sqrt{\frac{s}{2\pi}}\frac{\exp(-s|\boldsymbol{r}-\boldsymbol{v}t|)}{|\boldsymbol{r}-\boldsymbol{v}t|}e^{i\boldsymbol{v}\boldsymbol{r}-i\omega t}, \qquad (16)$$

where s is the parameter, determining the width of the packet in the momentum space and the inverse width in the coordinate space, $\boldsymbol{v}$ is the central wave-vector, and $\omega = (v^2 - s^2)/2$. The wave-packet (16) is fundamentally different from the superposition of plane waves used in conventional descriptions. The wave-packet (16) is normalizable, non-spreading, and satisfies the Schrödinger equation everywhere except one point:

$$(i\partial/\partial t + \Delta/2)\psi(\mathbf{r},t) = -\sqrt{2\pi s}e^{i(v^2+s^2)t/2}\delta(\mathbf{r}-\mathbf{r}(t)) \qquad (17)$$

which can be considered as a source of the wave function, or neutron itself.

The wave-number spectrum of the packet (16)

$$\psi(\boldsymbol{r},t) = \sqrt{\frac{s}{2\pi}}\int\frac{d^3p/2\pi^2}{(\boldsymbol{p}-\boldsymbol{v})^2+s^2}e^{i\boldsymbol{p}\boldsymbol{r}-i\Omega(\boldsymbol{p})t}, \qquad (18)$$

where $\Omega(\boldsymbol{p}) = [p^2 - (\boldsymbol{p}-\boldsymbol{v})^2 - s^2]/2$, has a long tail extending far away from the central wave-vector $\boldsymbol{v}$, and the anomalous losses of UCN can be attributed to nontunneling transmission of neutrons over the potential barrier. This transmission always takes place even if the height u of the barrier is considerably higher than the kinetic energy $v^2/2$ of the neutron.

The losses described by the over barrier penetration are given by

$$W = \frac{(4\pi c)^2}{(2\pi)^3}\int_{-\infty}^{\infty}\theta(|p_\perp| > v_{\rm lim})\frac{p_\perp}{|p_\perp|}\frac{[1-|R(p_\perp)|^2]|\sqrt{p_\perp^2-u}|}{[(\mathbf{p}-\mathbf{v})^2+s^2]^2}d^3p, \qquad (19)$$

where θ is the function equal to unity when the inequality in its argument is satisfied, and to zero in the opposite case, and

$$R(p_\perp) = \frac{|p_\perp| - |\sqrt{p_\perp^2-u}|}{|p_\perp| + |\sqrt{p_\perp^2-u}|}$$

is the reflection amplitude from potential step u of the plane wave with the normal component of the wave vector equal to $p_\perp$. The calculation gives

$$W \approx \frac{s}{\sqrt{u}}. \qquad (20)$$

If we compare this value with the averaged loss coefficient

$$\overline{\mu}(v) = \frac{2\eta}{z^2}(\arcsin z - z\sqrt{1-z^2}) \approx \frac{4}{3}\eta\frac{v}{\sqrt{u}} \quad \text{for small } z, \qquad (21)$$

where $z = v/\sqrt{u}$, and $\eta \approx 3\cdot 10^{-5}$ [1]. Comparing (20) and (21) we get the information about s:

$$s = 4\eta v/3 \approx 4v\cdot 10^{-5}. \qquad (22)$$

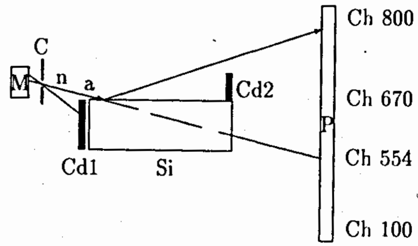

FIG. 1. The experimental layout: M - monochromator, C - collimator, n - neutron beam, Cd1 - cadmium shield at the entrance surface of the sample to prevent the part of the direct beam splitted off at the edge a, Cd2 - second cadmium shield to eliminate the part of the direct beam, which propagates without interaction with the sample surface, Si - silicon mirror sample, P - position sensitive detector (PSD), the numbers on the right side are related to channels on PSD at the glancing angle of 0.400 degree

The wave packet description leads to consequences that are important not only for UCN. For instance, the total reflection of thermal neutrons from plane mirrors should always be accompanied with small fraction of incident neutrons being refracted. This prediction can be and was verified experimentally.

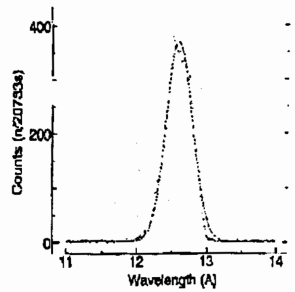

FIG. 2. Spectrum of the incident neutrons

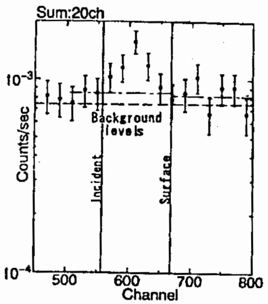

FIG. 3. Neutron counts by PSD when the silicon mirror is at a subcritical angle of 0.381 degree

We performed the experiment [6], scheme of which is shown in fig. 1. The well monochromatized and collimated neutrons of wave length $\lambda = 12$ Å and $\Delta\lambda/\lambda \approx 0.01$ (see spectrum in fig. 2) were reflected from a thick Si mirror under glancing angles θ below the limiting one $\theta_l = 0.577$ degree, and the intensity, transmitted through side surface (fig. 3) and registered in channels 550-670 of position sensitive detector P (fig. 1) was measured. The spectrum of these neutrons was analyzed by transmission through In foils.

Our results at present can be formulated as follows: We see the transmitted neutrons that have the same energy as in the incident beam. The fraction of them is near 10^{-4}, which is in good agreement with the predicted magnitude if parameter s in (16) does not depend on velocity v of the neutron, as shown in (22), but is supposed to be a constant of the order $\cdot 10^{-4}v_c$ with some wave number $v_c = \sqrt{u}$ and u close to the Be potential u_{Be}. The results are checked again and again with higher statistics and improved collimation and monochromatization. Some new experiments are planned at ILL reactor in Grenoble with different glancing angles, and with the incident neutron beam less contaminated by higher energy neutrons. The limits for the possible false effects in them will be even more narrow. If the observed effect with improved experimental conditions decreases below 10^{-5}, we shall decide that the de Broglie wave-packet cannot explain the UCN anomaly.

4 Conclusion

The most important feature of the recent experiments [2] in Grenoble is a discovery of a stepwise heating of stored UCN slightly above the limiting energy. However this result was obtained for vessels with low storage time. The observed effect can have no relations to the anomaly observed in clean and cold Be bottle. It can be explained, in particular, by the dust of little clusters with dimensions up to several hundreds of angstroms. However, the small heating can also be explained by properties of the de Broglie wave-packet. Indeed, our theory permits us to calculate the nontunneling overbarrier transmission, but it does not tell us how do neutron behaves inside medium. Because its kinetic energy is lower than potential, the neutron may be at rest at every point in the media, and it can be kicked out by the motion of surrounding matter, which leads to the stepwise heating.

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